Anglo - Chinese School
(Independent)

FINAL EXAMINATION 2022

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS

PAPER 2

Thursday **6th October 2022** **1 hour 30 minutes**

ADDITIONAL MATERIALS:

Answer Paper (6 sheets)
Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS

The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1. [Maximum mark: 6]

(a) Simplify $\dfrac{a^2-2ab+b^2+b-a}{a^2-b^2}$. [3]

$$=\frac{a^2-2ab+b^2+b-a}{a^2-b^2}$$
$$=\frac{(a-b)^2+b-a}{a^2-b^2}$$
$$=\frac{(a-b)^2-(a-b)}{a^2-b^2}$$
$$=\frac{(a-b)[(a-b)-1]}{(a-b)(a+b)}$$
$$=\frac{(a-b)-1}{(a+b)}$$

Students are very careless and many are weak in algebraic manipulation.
Many students made the following mistake:

$$\frac{(a-b)^2+b-a}{(a-b)(a+b)}$$
$$=\frac{a-b+b-a}{a+b}$$
$$=\frac{0}{a+b}$$
$$=0$$

(b) Subtract $\dfrac{4}{x^2-4}$ from $\dfrac{1}{2-x}-\dfrac{1}{x+2}$, expressing your answer as a single fraction in its simplest form. [3]

$$\frac{1}{2-x}-\frac{1}{x+2}-\frac{4}{x^2-4}$$
$$=-\frac{1}{x-2}-\frac{1}{x+2}-\frac{4}{x^2-4}$$
$$=-\left[\frac{1}{x-2}+\frac{1}{x+2}\right]-\frac{4}{x^2-4}$$
$$=-\left[\frac{x+2+x-2}{x^2-4}\right]-\frac{4}{x^2-4}$$
$$=-\frac{2x}{x^2-4}-\frac{4}{x^2-4}$$
$$=\frac{-2x-4}{x^2-4}=\frac{-2(x+2)}{(x-2)(x+2)}=\frac{-2}{x-2}$$

Students were careless with the negative signs.

Some students have problem understanding which term to subtract.

2. [Maximum mark: 8]

Joe bought x number of books, each at the same price, for a total cost of $336.

(a) Write down an expression for the cost of each book in terms of x. [1]

$$\frac{336}{x}$$

Joe sold 20 of them for $480, and the rest at a loss of $4 per book.

(b) Write down an expression for the total amount, in dollars, he received for all the books. [2]

Selling price of the remaining books = $\left(\frac{336}{x}-4\right)$

Number of books left = $x-20$

Total mount collected = $480+(x-20)\left(\frac{336}{x}-4\right)$

Students don't understand the term "sold 20 of them for $480" They think each book is being sold for $480.

(c) Given that Joe made a profit of $184 altogether, form an equation in x and show that it reduces to $x^2-94x+1680=0$. [2]

$$480+(x-20)\left(\frac{336}{x}-4\right)-336=184$$

$$(x-20)\left(\frac{336}{x}-4\right)=40$$

$$336-4x-\frac{6720}{x}+80=40$$

$$-4x-\frac{6720}{x}+376=0$$

$$-4x^2+376x-6720=0$$

$$x^2-94x+1680=0$$

Very poorly done.
Many students associated the answer in (b) to be $184 which is the profit only. Hence they have difficult showing the expression.

(d) Hence, solve the equation $x^2 - 94x + 1680 = 0$ and state the cost price of each book. [3]

$x^2 - 94x + 1680 = 0$

$x = \dfrac{-(-94) \pm \sqrt{(-94)^2 - 4(1)(1680)}}{2}$

$x = 24$

$x = 70$

Price of each book = $\dfrac{\$336}{70}$ or $\dfrac{\$336}{24}$

Price of each book = \$4.80 or \$14

There is no reason why students should reject either of the answer. Many students rejected the answer \$4.80 because it is not a whole number.

3. [Maximum mark: 12]

(a) Evaluate $\log_3\left(\dfrac{2.15 + e^2}{0.25^{-2}}\right)$, leaving your answer correct to 2 significant figures. [3]

$\log_3\left(\dfrac{2.15 + e^2}{0.25^{-2}}\right)$

$= \log_3(0.596191)$

$= \dfrac{\lg 0.596191}{\lg 3}$

$= -0.47$

There is no need to apply log rules to this question, except change of base.

The whole expression in the bracket should be keyed into the calculator to obtain 0.596191. But students manipulated and simplified because they are stuck with e, which is a constant

(b) Given that $\left(144p^4\right)^{\frac{3}{2}} \div \left(216p^{-3}\right)^{-\frac{1}{3}} = 2^x 3^y p^z$, evaluate x, y and z. [4]

$\left(144p^4\right)^{\frac{3}{2}} \div \left(216p^{-3}\right)^{-\frac{1}{3}}$

$= (2^4 3^2 p^4)^{\frac{3}{2}} \div (2^3 3^3 p^{-3})^{-\frac{1}{3}}$

$= (2^6 3^3 p^6) \div (2^{-1} 3^{-1} p^1)$

$= 2^{6-(-1)} 3^{3-(-1)} p^{6-1}$

$= 2^7 3^4 p^5$

Hence,

$x = 7$

$y = 4$

$z = 5$

Very poorly attempted.
Students very confused with laws of indices and they did not change the numbers to base 2 and base 3 even though the right hand side of the equation has already given a clue that the numbers must be in base 2 and 3 so that they can compare the powers.

(c) Simplify $\dfrac{6^{2w}+2(3^{2w})}{4^{w+1}+8}$. [3]

$$\frac{6^{2w}+2(3^{2w})}{4^{w+1}+8}$$
$$=\frac{2^{2w}3^{2w}+2(3^{2w})}{2^{2w+2}+8}$$
$$=\frac{3^{2w}(2^{2w}+2)}{2^{2w}(4)+8}$$
$$=\frac{3^{2w}(2^{2w}+2)}{4(2^{2w}+2)}$$
$$=\frac{3^{2w}}{4}$$

Many misconceptions with the law of indices. For example:

$6^{2w}=2\times 3^{2w}$

$6^{2w}=6^{2}\times 6^{w}$

These are very basic and fundamental concepts of indices, students should not make these mistakes.

(d) Find the range of values of x if $7^{2x^2-5x-2}<7$. [3]

$7^{2x^2-5x-2}<7$

$2x^2-5x-2<1$

$2x^2-5x-3<0$

$(x-3)(2x+1)<0$

$-\dfrac{1}{2}<x<3$

It is recommended that students draw the curve and shade the necessary region to visualise the values needed.

Some students give the wrong inequalities because they did not draw the graph above.

4. [Maximum mark: 8]

(a) Solve the equation $3^{y+1} = 4^y$. [4]

$3^y(3) = 4^y$

$\frac{3^y}{4^y} = \frac{1}{3}$

$\left(\frac{3}{4}\right)^y = \frac{1}{3}$

$y\lg\left(\frac{3}{4}\right) = \lg\left(\frac{1}{3}\right)$

$y = \frac{\lg\left(\frac{1}{3}\right)}{\lg\left(\frac{3}{4}\right)}$

$y = 3.82$

$3^{y+1} = 4^y$

$\lg 3^{y+1} = \lg 4^y$

$(y+1)\lg 3 = y\lg 4$

$y\lg 3 + \lg 3 = y\lg 4$

$y\lg 4 - y\lg 3 = \lg 3$

$y(\lg 4 - \lg 3) = \lg 3$

$y = \frac{\lg 3}{\lg 4 - \lg 3}$

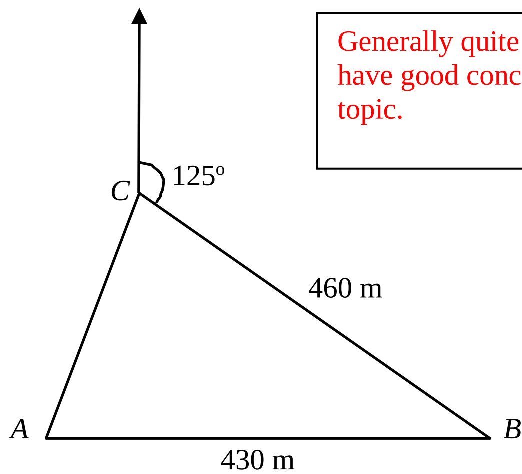
$y = 3.82$

(b) Solve the equation $\log_3(x) + \log_3(x-3) = \log_9(9x^2)$. [4]

$\log_3(x) + \log_3(x-3) = \log_9(9x^2)$
$\log_3(x) + \log_3(x-3) = \log_3(3x)$
$\log_3[(x)(x-3)] = \log_3(3x)$
$x^2 - 3x = 3x$
$x^2 - 6x = 0$
$x(x-6) = 0$
$x = 6$
$x = 0$

5. [Maximum mark: 12]

In the figure, A, B and C are three points on a horizontal field. A is due west of B, the bearing of B from C is 125°, AB = 430 m and BC = 460 m.



Generally quite well done. Most students have good conceptual understanding of this topic.

(a) Find

(i) the distance between A and C, [3]

Angle $\measuredangle CBA = 90^o - (180^o - 125^o)$

Angle $\measuredangle CBA = 35^o$

$AC^2 = 430^2 + 460^2 - 2(430)(460)\cos 35^o$

$AC^2 = 72443.45$

$AC = 269$

(ii) $\measuredangle ACB$, [2]

$$\frac{\sin 35}{269.15} = \frac{\sin ACB}{430}$$

$\sin ACB = 0.91636$

$ACB = 66.4^o$

(iii) the bearing of C from A, [2]

Angle CAB $= 180^o - 66.4^o - 35^o$ Angle CAB $= 78.6^o$ Bearing of C from A $= 90^o - 78.6^o$ Bearing of C from A $= 011.4^o$

(iv) the area of ΔABC. [2]

Area $= \frac{1}{2}(430)(460)\sin 35^o$ Area $= 56727$

At a certain instant, a hot air balloon is at a point which is directly above C.

(b) Given that the angle of elevation of the hot air balloon from *B* is 5.2º, find the angle of elevation of the hot air balloon from A. [3]

$\tan 5.2^o = \frac{height}{460}$ $height = 41.8632$ $\tan\theta = \frac{41.8632}{269.15}$ $\theta = 8.84^o$

6. [Maximum mark: 9]

(a) Find the smallest positive integer value of m given that the straight line $y = mx - 5$ intersects the curve $y = x^2 - 2m$ at two distinct points. [5]

$mx - 5 = x^2 - 2m$

$x^2 - mx + 5 - 2m = 0$

$(-m)^2 - 4(1)(5 - 2m) > 0$

$m^2 + 8m - 20 > 0$

$(m + 10)(m - 2) > 0$

$m < -10$ or $m > 2$

Smallest positive integer is 3

Must show the final inequality ($m < -10$ or $m > 2$) first before stating the smallest positive integer of m.

Many students forgot to state the smallest value.

(b) Prove that $3x^5 - 25x^3 + 60x = 0$, has only one real root. [4]

$3x^5 - 25x^3 + 60x = 0$

$x(3x^4 - 25x^2 + 60) = 0$

$x(3y^2 - 25y + 60) = 0$

$D = (-25)^2 - 4(3)(60)$

$D = -95$

Since D < 0, there is only 1 real root, which is $x = 0$

There should not be any x in the calculation of the discriminant.

It is recommended that students explain clearly the result obtained. Just stating that D = −95 is not sufficient. There should be a statement explaining what this means.

7. [Maximum mark: 14]

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 3x + \frac{60}{x} - 35$. Some corresponding values of x and y are given in the following table:

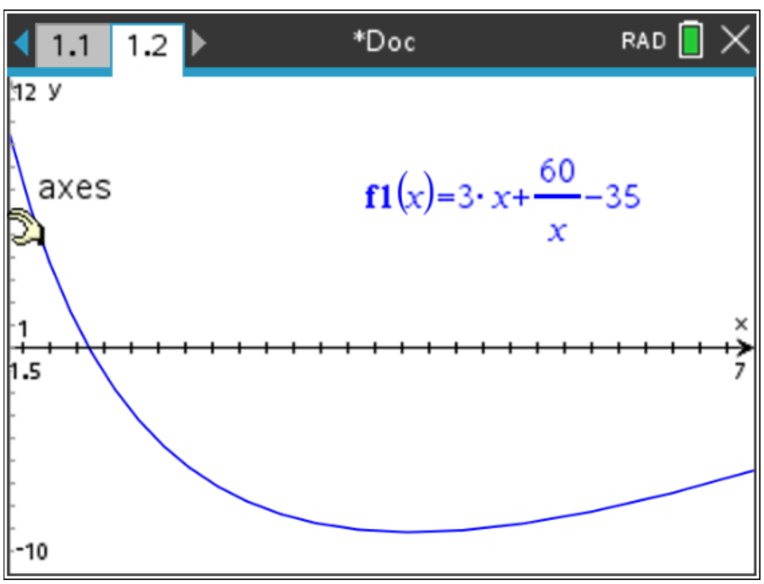
x	1.5	2	2.5	3	4	5	6	7
y	9.5	a	–3.5	–6	b	–8	–7	–5.4

(a) Find the value of a and of b. [2]

$a = 1$

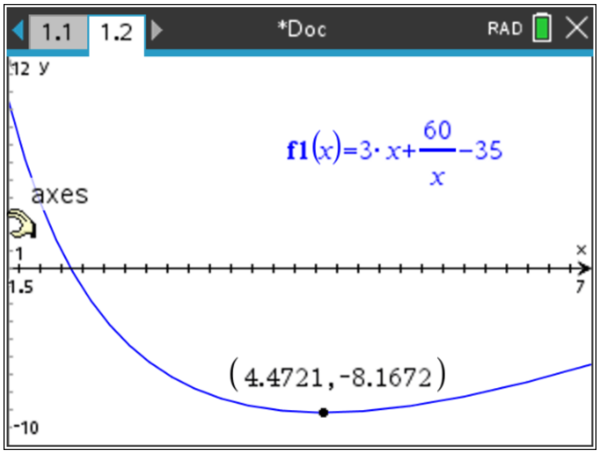
$a = -8$

(b) Taking 2 cm to represent 1 unit on the x-axis and 1 cm to represent 1 unit on the y-axis, draw the graph of $y = 3x + \frac{60}{x} - 35$ for $1.5 \le x \le 7$. [4]



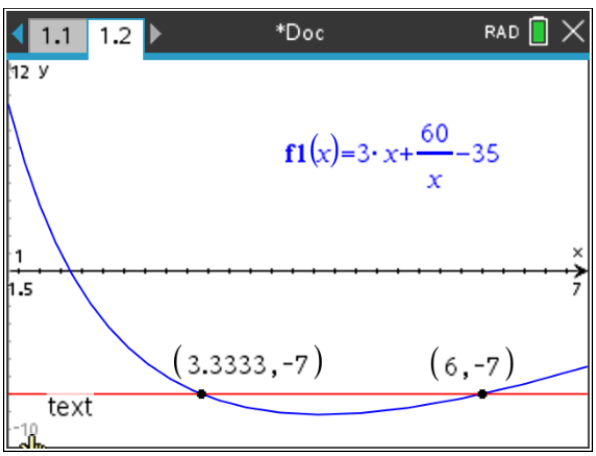
Use your graph to find

(c) the least value of y, [1]



Answer: -8.17

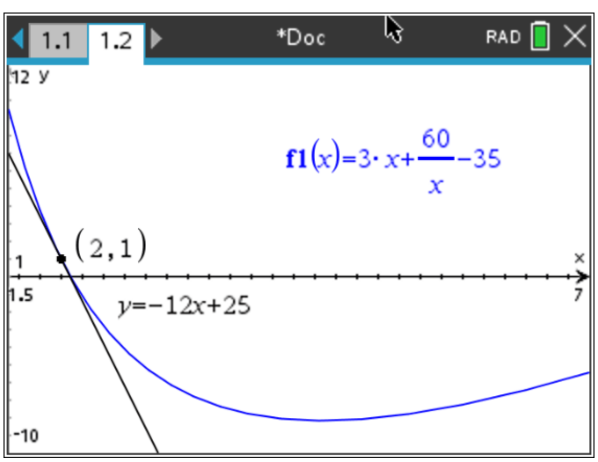
(d) the range of values of x for which $y \le -7$, [2]



Answer: $3.33 \le x \le 6$

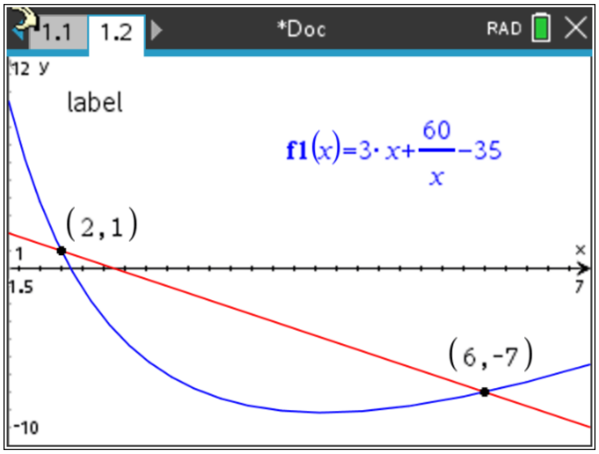
Proper inequality must be shown. Since the question says $y \le -7$, the final answer's inequality must come with the = sign.

(e) the gradient of the curve at the point $x = 2$ by drawing a suitable straight line, [2]



Answer: Gradient = -12

(f) the solution of the equation $5x+\frac{60}{x}-40=0$ by drawing a suitable straight line. [3]



Answer: 2 or 6

Results must be obtained from the graph.
No marks awarded if obtained by calculation.

8. [Maximum mark: 10]

(a) It is given that

$$\log_a x = 12$$
$$\log_b x = 60$$

(i) Express a in terms of b. [3]

$x = a^{12}$

$x = b^{60}$

$a^{12} = b^{60}$

$a = b^5$

Answer must be in simplest form. Students should not leave answers as: $a = b^{\frac{60}{12}}$

(ii) If $\log_{abc} x = 6$, prove that $\log_c x = 15$. [4]

$\log_{abc} x = 6$

Students are to prove that $\log_c x = 15$
They should not assume this result in the beginning and use it in their working to prove it is true.

$\log_x abc = \frac{1}{6}$

$\log_x a + \log_x b + \log_x c = \frac{1}{6}$

$\frac{1}{12} + \frac{1}{60} + \log_x c = \frac{1}{6}$

$\log_x c = \frac{1}{15}$

$\log_c x = 15$

(b) Using the two single digits a and b, explain clearly whether there exists a number, ab, such that the sum of ab and its reverse ba is a prime number. [3]

Number is = $10a + b$

The a and b here has no relation to the a and b in Part (a)!

Reverse number is = $10b + a$

Sum of the two numbers = $11b + 11a$

Sum of the two numbers = $11(b + a)$

Since the sum of the two numbers is always a multiple of 11, the sum will **NEVER** be a prime

-- END OF PAPER 2 --