RAFFLES INSTITUTION
H2 Mathematics (9758)
2024 Year 6

<u>2024 Year 6 Timed Practice Revision Practice Paper 3</u>
<u>(Source: 2021 Year 6 Term 3 Common Test)</u>

The solution will be released in Ivy on 10 June (Monday)

Total Marks: 100 **Duration: 3h**

**** Note that Qn 4 (Complex Numbers) will NOT be examined in the coming Timed Practice. So, you should complete this paper within 2 hrs 47 mins.**

1 An arithmetic series has first term a and common difference d, where a and d are non-zero. If the first, fifth and eleventh terms of the arithmetic series are equal to the first, second and third terms of a geometric series respectively, find the exact sum of the first ten terms of the geometric series, in the form ka, where k is a simplified rational number. [4]

2 **(i)** By writing $\dfrac{2}{(2r+1)(2r+3)}$ in partial fractions, find an expression for $\displaystyle\sum_{r=2}^{n}\frac{2}{(2r+1)(2r+3)}$ in terms of n. [3]

(ii) Hence find the exact value of $\dfrac{2}{19}\times\dfrac{1}{21}+\dfrac{2}{21}\times\dfrac{1}{23}+\dfrac{2}{23}\times\dfrac{1}{25}+\cdots$. [2]

3 For a curve with equation $x^2-3xy+y^3=8$, find

(i) the x-coordinate of the point at which the tangent is parallel to the x-axis, [4]

(ii) the equation(s) of the normal(s) at the point(s) where $y=2$. [3]

4 The polar form of a complex number z is given by $z=r\mathrm{e}^{\mathrm{i}\theta}$, where $r>0$ and $0<\theta\leq\pi$, and the complex number $w=\left(\dfrac{1}{3}-\dfrac{\sqrt{3}}{3}\mathrm{i}\right)z$.

(i) Find w in exact polar form in terms of r and θ. [3]

(ii) Given that $\dfrac{z^5}{w^*}$ is real and positive, find the possible value(s) of θ in exact form, leaving your answer(s) in terms of π. [4]

5 The function f is defined by

$$\mathrm{f} : x \mapsto 2x^2 + 5x - 3, \quad x \in \mathbb{R}, \quad x \le a.$$

(i) State the greatest value of a such that the function f^{-1} exists. [1]

For the rest of the question, use the value of a found in part **(i)**.

(ii) Find f^{-1} in a similar form. [3]

(iii) Find the exact solution of $\mathrm{f}^{-1}(x) = x$. [3]

(iv) The function g is defined for specific integer values of x as follows.

x	-5	-4	-1	0	2
$\mathrm{g}(x)$	6	4	$2\sqrt{3}$	0	-3

Find the value of b, where $\mathrm{f}^{-1}\mathrm{g}(b) = -3$. [1]

6 The curve C has equation $y = \dfrac{2x^2 - 10x + 17}{x - 3}$, $x \in \mathbb{R}$, $x \ne 3$.

(i) Without using a calculator, find the set of values of y that C can take. [4]

(ii) Sketch C, labelling the relevant features in exact form. [5]

7 To build a Virtual Reality (VR) experience, a virtual world is created to contain geometric models. With reference to the origin *O*, every point in the virtual world is represented as coordinates (*x*, *y*, *z*). Perspective projection is a technique used to project points in the virtual world onto a virtual screen while respecting the scaling properties of objects at various distances. Getting this right in VR helps in the perception of depth and scale. In addition, only points that lie within a zone in front of the eye positioned at *O*, known as the viewing frustum (see **Fig. 1**), will be projected onto the virtual screen.

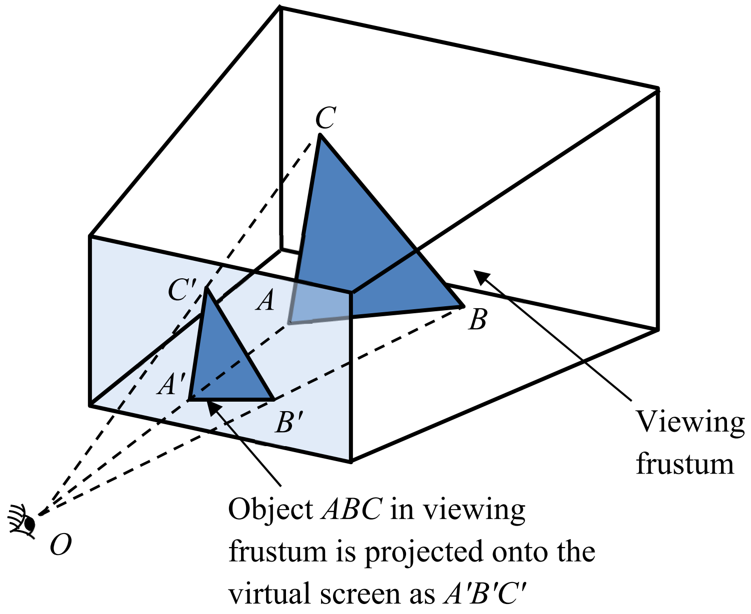

Fig. 1

In **Fig. 1**, a triangular object ABC in the viewing frustum with $A(36,36,12)$ and $B(24,48,72)$ is projected onto the virtual screen to give an image of $A'B'C'$. It is given that the coordinates of A' are $(3,3,1)$ and the line that passes through A' and B' is parallel to the vector $2\mathbf{i}+\mathbf{j}-2\mathbf{k}$.

(i) Show that the coordinates of B' are $(1,2,3)$. [4]

(ii) Given that the coordinates of C' are $(1,4,2)$, find the equation of the virtual screen in scalar product form. [2]

(iii) Find the exact area of triangle $A'B'C'$. [2]

(iv) If the length of projection of $\overrightarrow{BC}$ onto the normal of the virtual screen is $\dfrac{57}{\sqrt{29}}$ units, find the possible position vector(s) of C, leaving your answer(s) in exact form. [4]

Section B: Statistics [48 marks]

8 The continuous random variable Y has the distribution $\mathrm{N}\left(\mu, \sigma^2\right)$. It is known that $\mathrm{P}\left(Y < -a\right) = \mathrm{P}\left(Y > 5a\right) = 0.1$. Express μ and σ in the forms ka and ma respectively, where k and m are constants to be determined. [4]

9 A card game called "Happy Family" is played with 28 cards, consisting of 7 sets of 4 cards. Each set consists of a father, a mother, a son and a daughter from the same family. The family names are Painter, Postman, Plumber, Butcher, Carpenter, Singer and Teacher. So for example, the complete set of Teacher family cards consists of father Teacher, mother Teacher, son Teacher and daughter Teacher.

The objective of the game is to collect as many complete sets of family cards as possible. At the end of the game, all the cards are collected. Each player will either be left with no cards or complete sets of family cards. The winner is the one with the most number of complete sets of family cards.

A and B play a game of "Happy Family".

(a) At the end of the game, A has exactly 3 complete sets of family cards and one of the sets is the Teacher family cards. How many possible combinations of complete sets of family cards can B have? [1]

(b) If A is the winner at the end of the game, how many possible combinations of complete sets of family cards can A have? [2]

(c) Given that A has exactly 3 particular complete sets of family cards at the end of the game, and he arranges these 12 cards in a circle. How many different ways can these cards be arranged so that no two mothers are next to each other? [3]

10 For events A and B, it is given that $\mathrm{P}(A) = 0.38$, $\mathrm{P}(A \mid B) = 0.5$ and $\mathrm{P}(A \cup B) = 0.52$.

(i) Find $\mathrm{P}(B)$. [3]

For a third event C, it is given that $\mathrm{P}(C) = 0.6$ and $\mathrm{P}\left(\left(A' \cup B'\right) \mid C\right) = 0.85$.

(ii) Find $\mathrm{P}(A \cap B \cap C)$. [3]

(iii) State, with a reason, whether the events A and C are mutually exclusive. [2]

11 In a game, a player tosses a fair die, whose faces are numbered from 1 to 6. If the player obtains a 6, he tosses the die a second time, and in this case, his score is the absolute difference of 6 and the second number. Otherwise, his score is the number obtained in the first toss. Let the player's score be denoted by X.

(i) Show that $\mathrm{P}(X=1)=\dfrac{7}{36}$ and tabulate the probability distribution of X. [3]

(ii) Find the exact value of $\mathrm{E}(X)$. [1]

Mr Lim plays this game 4 times.

(iii) Find the probability that he obtained a score of 5 no more than 2 times. [2]

(iv) Find the probability that his total score is less than 2 in the 4 games that he played.[2]

12 A chocolate manufacturing company produces chocolate bars. The packaging of the chocolate bar states that the mass of each bar is 200 g.
The production manager has been receiving complaints that the mass is overstated and he wants to carry out a hypothesis test.

(i) Explain whether the manager should carry out a 1-tail test or a 2-tail test. State appropriate hypotheses for the test, defining any symbols you use. [2]

The masses, x grams, of a random sample of 30 chocolate bars are summarised as follows.

$$n=30 \qquad \sum(x-200)=-54 \qquad \sum(x-200)^2=550$$

(ii) Calculate unbiased estimates of the population mean and variance of the mass of chocolate bars. [2]

(iii) Carry out this test, at the 1% level of significance. [2]

The company replaces all the machines producing the chocolate bars. The supplier of the machines claims that the mass of each chocolate bar will have a normal distribution with mean 200 grams and standard deviation 2 grams.

(iv) The manager takes a random sample of n chocolate bars, and the sample mean mass is found to be 199.2 grams. A test is carried out, at the 5% level of significance, to determine whether the mean mass of the chocolate bars is indeed 200 grams. Given that the null hypothesis is not rejected, find the maximum value of n. [4]

13 **In this question, you should state clearly all the distributions that you use, together with the values of the appropriate parameters.**

A cup of cheese tea is prepared with two ingredients, cream cheese foam and tea. The barista preparing the cheese tea is not allowed to remove cream cheese foam nor tea once they have been poured into the cup. The amounts, in ml, of cream cheese foam and tea in one cup of cheese tea made by an experienced barista have independent normal distributions with means and standard deviations as shown in the table.

	Mean amount (ml)	Standard deviation (ml)
Cream cheese foam	72	5
Tea	421	4

(i) Given that the amount of cheese tea in each cup is expected to be at least 490 ml, find the probability that a randomly chosen cup of cheese tea made by the experienced barista meets this expectation. [2]

A cup of cheese tea is considered well-made if the amount of cream cheese foam is between 65 ml and 75 ml and the amount of tea is between 416 ml and 424 ml.

(ii) Find the probability that, for ten randomly chosen cups of cheese tea made by the experienced barista, more than five cups are well-made. [3]

The cost price of cream cheese foam and tea is \$20 per litre and \$1.20 per litre respectively. A cup of cheese tea is sold at \$4.90.

(iii) Without factoring in other costs incurred in producing the cheese tea, find the probability that the profit for 100 randomly chosen cups of cheese tea made by the experienced barista is at least \$295. State any assumption(s) needed for your calculation. [5]

A trainee barista was engaged at the shop. The amount of cheese tea in a cup she made was measured to have mean 498 ml and standard deviation 30 ml. The amount of cheese tea in any cup may be assumed to be independent of the amount of cheese tea in any other cup.

(iv) Find the probability that the mean amount of cheese tea of 40 randomly chosen cups of cheese tea made by the trainee barista does not exceed 500 ml. [2]