2020 Y3 Express AMath Final Exam P2 Worked Solutions

1 Length of sides $= \sqrt{\frac{3+2\sqrt{2}}{3-2\sqrt{2}}}$

$$= \sqrt{\frac{3+2\sqrt{2}}{3-2\sqrt{2}} \times \frac{3+2\sqrt{2}}{3+2\sqrt{2}}}$$

$$= \left(3+2\sqrt{2}\right)cm$$

Perimeter of square $= \left(12+\sqrt{128}\right)cm$

2 (i) at the start of the experiment, $x = 0$.

$$y = 40\left(1.4^0\right) = 40$$

(ii) $1000 = 40\left(1.4^x\right)$

$$25 = \left(1.4^x\right)$$

$$\lg 25 = x \lg 1.4$$

$$x = \frac{\lg 25}{\lg 1.4}$$

= 10 hrs (to the nearest hour)

3 When $x = 1$,

$$(1)^3 + 3(1)^2 - 2(1) + 16 - 0 = 0 + c(3)$$

$$c = 6$$

When $x = 2$,

$$x^3 + 3x^2 - 2x + 16 - b(x-2)^2(x-1) = ax^2(x-1) + c(x+2)$$

$$(2)^3 + 3(2)^2 - 2(2) + 16 - 0 = a(2)^2 + 6(4)$$

$$a = 2$$

When $x = 0$,

$$16 + 4b = 6(2)$$

$$b = -1$$

4 **(i)** LHS $= \dfrac{\sec^2\theta - 1}{\tan\theta + \tan^3\theta}$

$= \dfrac{\tan^2\theta}{\tan\theta\left(1+\tan^2\theta\right)}$

$= \dfrac{\tan\theta}{\sec^2\theta}$

$= \dfrac{\sin\theta}{\cos\theta}.\dfrac{\cos^2\theta}{1}$

$= \sin\theta\cos\theta$

(ii) $\dfrac{\sec^2\theta - 1}{\tan\theta + \tan^3\theta} = 2\cos^2\theta$

$\sin\theta\cos\theta = 2\cos^2\theta$

$\sin\theta\cos\theta - 2\cos^2\theta = 0$

$\cos\theta\left(\sin\theta - 2\cos\theta\right) = 0$

$\cos\theta = 0$ (N.A) $\qquad$ $\sin\theta - 2\cos\theta = \theta$

$\tan\theta = 2$

Ref $\angle$ of θ=63.4°

$\theta = 63.4^o \; or \; 243.4^o$

5 **(a)** $0.9 - \cos\left(\dfrac{x}{2} - 30^o\right) = 0$

$\cos\left(\dfrac{x}{2} - 30^o\right) = 0.9$, $\qquad$ $-30^o \le \dfrac{x}{2} - 30^o \le 60^o$

Ref $\angle$ of $\dfrac{x}{2} - 30^o = \cos^{-1}(0.9)$

= 25.842°

$\dfrac{x}{2} - 30^o = 25.842^o \;\; or \;\; -25.842^o$

$x = 8.3^o \;\; or \;\; 111.7^o$

(b) $2\cos^2 x + \dfrac{3}{co\sec x} = 0$

$$2\cos^2 x + 3\sin x = 0$$

$$2\left(1-\sin^2 x\right) + 3\sin x = 0$$

$$2 - 2\sin^2 x + 3\sin x = 0$$

$$2\sin^2 x - 3\sin x - 2 = 0$$

$$(2\sin x + 1)(\sin x - 2) = 0$$

$$\sin x = -\frac{1}{2} \;\; or \;\; \sin x = 2\,(N.A.)$$

Ref $\angle = \dfrac{\pi}{6} rad$

$$x = \frac{7\pi}{6} rad \;\; or \;\; \frac{11\pi}{6} rad$$

6a) For $(2-3p)x^2 + (4-p)x + 2 > 0$, $D < 0$ and $a > 0$

$$(4-p)^2 - 4(2-3p)(2) < 0$$

$$p^2 + 16p < 0$$

$$p(p+16) < 0$$

$$-16 < p < 0$$

Since $a > 0$, $2 - 3p > 0$

$$p < \frac{2}{3}$$

Answer: $-16 < p < 0$

(b) $4x^2 - 3x + 2 = -7 - kx$

$$4x^2 + (k-3)x + 9 = 0$$

For line not to intersect curve, $D < 0$.

$$(k-3)^2 - 4(4)(9) < 0$$
$$-9 < k < 15$$
$$k = 14$$

7 (a) $P(0,5) \;\; and \;\; R(10,0)$

(b) Since Q lies on the perpendicular bisector, it is equidistant from P and R.

$$\sqrt{(7-0)^2+(q-5)^2}=\sqrt{(7-10)^2+(q-0)^2}$$

$$49+q^2-10q+25=9+q^2$$

$$q=6.5$$

Alternatively,

Mid-point of *PR* $(5, 2.5)$

Gradient perpendicular to *PR* = $-\dfrac{1}{\left(\dfrac{5-0}{0-10}\right)}=2$

Equation of perpendicular bisector:

$$y-2.5=2(x-5)$$

$$y=2x-7.5$$

$$q=2(7)-7.5=6.5$$

(C) *PQ* =*QR* since perpendicular bisector passes through *Q***.**

Mid-point of *QS* = $\left(\dfrac{7+3}{2},\dfrac{6.5-\dfrac{3}{2}}{2}\right)=(5,2.5)$

Mid-point of *PR* **=** Mid-point of *QS* (opposite sides are parallel)

PQRS is a rhombus.

Alternatively,

$$PQ=\sqrt{(7-0)^2+(6.5-5)^2}=\frac{\sqrt{205}}{2}$$

$$QR=\sqrt{(7-10)^2+(6.5-0)^2}=\frac{\sqrt{205}}{2}$$

$$QS=\sqrt{(10-3)^2+\left(0+\frac{3}{2}\right)^2}=\frac{\sqrt{205}}{2}$$

$$PS=\sqrt{(3-0)^2+\left(-\frac{3}{2}-5\right)^2}=\frac{\sqrt{205}}{2}$$

Since the four sides are equal, it is a rhombus.

(d) Area of $PQRS = \frac{1}{2}\begin{vmatrix} 0 & 3 & 10 & 7 & 0 \\ 5 & -1.5 & 0 & 6.5 & 5 \end{vmatrix}$

$$= \frac{1}{2}\left|(0+0+65+35)-(15-15+0+0)\right|$$

$$= 50 units^2$$

8 **(ai)** $5^{2x} = 2^{x-1}$

$\lg 5^{2x} = \lg 2^{x-1}$

$2x \lg 5 = (x-1)\lg 2$

$x = \frac{-\lg 2}{2\lg 5 - \lg 2}$

$= -0.274$ $(to\ 3s.f.)$

(aii) $2(16^y)+2=5(4^y)$

$2(4^{2y})+2=5(4^y)$

Let $a = 4^y$,

$2a^2-5a+2=0$

$(2a-1)(a-2)=0$

$a=\frac{1}{2}\ or\ a=2$

$2^{2y}=\frac{1}{2}\ or\ 2^{2y}=2$

$y=-\frac{1}{2}\ or\ y=\frac{1}{2}$

(b) $7^x(7)-\frac{8}{7^x}+1=0$

Let y = 7^x

$7y-\frac{8}{y}+1=0$

$7y^2+y-8=0$

$(7y+8)(y-1)=0$

$$y = -\frac{8}{7}(N.A.)\ or\ y = 1$$

$$7^x = 1 \Rightarrow x = 0$$
