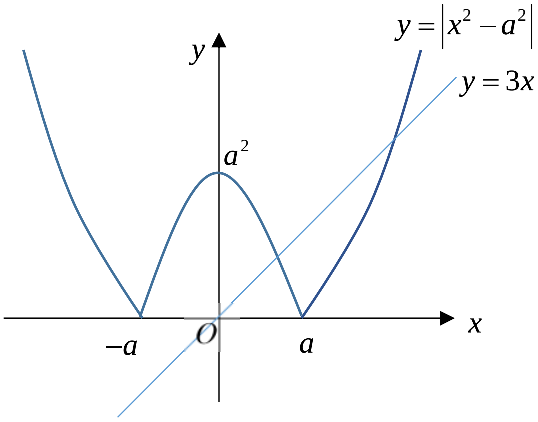
2024 JC2 H2 Maths Prelim Paper 2 Solutions

1((a)	
(b)	At point of intersection where 0 < x < a, $-(x^2-a^2)=3x$ $x^2-a^2=-3x$ $x^2+3x-a^2=0$ $x=\dfrac{-3\pm\sqrt{9+4a^2}}{2}$ $x=\dfrac{-3+\sqrt{9+4a^2}}{2}$ (since $x > 0$)

2(a)	$\int \sin px\cos qx\,\mathrm{d}x$ $=\frac{1}{2}\int 2\sin px\cos qx\,\mathrm{d}x$ $=\frac{1}{2}\int \sin(p+q)x+\sin(p-q)x\,\mathrm{d}x$ $=\frac{1}{2}\left[-\frac{1}{p+q}\cos(p+q)x-\frac{1}{p-q}\cos(p-q)x\right]+C$ <u>Alternatively,</u> $\int \sin px\cos qx\,\mathrm{d}x$ $=\frac{1}{2}\int 2\cos qx\sin px\,\mathrm{d}x$ $=\frac{1}{2}\int \sin(q+p)x-\sin(q-p)x\,\mathrm{d}x$ $=\frac{1}{2}\left[-\frac{1}{q+p}\cos(q+p)x+\frac{1}{q-p}\cos(q-p)x\right]+C$
(b)	$\int x\sin mx\,\mathrm{d}x=-\frac{1}{m}x\cos mx+\int\frac{1}{m}\cos mx\,\mathrm{d}x$ $\qquad u=x \quad \frac{\mathrm{d}v}{\mathrm{d}x}=\sin mx$ $=-\frac{1}{m}x\cos mx+\frac{1}{m^2}\sin mx+C$ $\qquad \frac{\mathrm{d}u}{\mathrm{d}x}=1 \quad v=-\frac{\cos mx}{m}$

(c)	$\int_0^{\pi} x\sin mx\,\mathrm{d}x$ $=\left[-\frac{1}{m}x\cos mx+\frac{1}{m^2}\sin mx\right]_0^{\pi}$ $=\left[-\frac{1}{m}\pi\cos m\pi+\frac{1}{m^2}\sin m\pi\right]-\left[-\frac{1}{m}(0)\cos 0+\frac{1}{m^2}\sin(0)\right]$ $=-\frac{1}{m}\pi\cos m\pi$ When m is odd, $\int_0^{\pi} x\sin mx\,\mathrm{d}x=-\frac{1}{m}\pi(-1)=\frac{1}{m}\pi$ When m is even, $\int_0^{\pi} x\sin mx\,\mathrm{d}x=-\frac{1}{m}\pi(1)=-\frac{1}{m}\pi$ $k = 1$ or – 1

3(i)	$\mathrm{f}'(x)=\mathrm{e}^{\tan^{-1}x}\left(\frac{1}{1+x^2}\right)$ $(1+x^2)\mathrm{f}'(x)=\mathrm{f}\,(x)$ Alternatively, $\mathrm{f}\,(x)=\mathrm{e}^{\tan^{-1}x}$ $\ln\mathrm{f}\,(x)=\tan^{-1}x$ Differentiate with respect to x $\frac{1}{\mathrm{f}\,(x)}\mathrm{f}'\left(x\right)=\frac{1}{1+x^2}$ $(1+x^2)\mathrm{f}'\left(x\right)=\mathrm{f}\,\left(x\right)$
(ii)	$(1+x^2)\mathrm{f}'(x)=\mathrm{f}\,(x)$ Differentiate with respect to x $(1+x^2)\mathrm{f}''(x)+2x\mathrm{f}'(x)=\mathrm{f}'(x)$ Differentiate with respect to x $(1+x^2)\mathrm{f}'''(x)+2x\mathrm{f}''(x)+2x\mathrm{f}''(x)+2\mathrm{f}'(x)=\mathrm{f}''(x)$ $(1+x^2)\mathrm{f}'''(x)+4x\mathrm{f}''(x)+2\mathrm{f}'(x)=\mathrm{f}''(x)$ When $x = 0$, $y=\mathrm{f}\,(0)=\mathrm{e}^{\tan^{-1}0}=1$ $(1+0)\mathrm{f}'(x)=1\Rightarrow\mathrm{f}'(x)=1$ $(1+0)\mathrm{f}''(x)+0=1\Rightarrow\mathrm{f}''(x)=1$ $(1+0)\mathrm{f}'''(x)+0+2\left(1\right)=1\Rightarrow\mathrm{f}'''(x)=-1$ $y=\mathrm{f}\,(x)=\mathrm{e}^{\tan^{-1}x}=1+x+\frac{1}{2!}x^2-\frac{1}{3!}x^3+\ldots\qquad\approx 1+x+\frac{1}{2}x^2-\frac{1}{6}x^3$
(iii)	$\int_0^{0.5}\mathrm{f}\,(x)\,\mathrm{d}x\approx\int_0^{0.5}1+x+\frac{1}{2}x^2-\frac{1}{6}x^3\,\mathrm{d}x\approx 0.6432$ (4s.f.)
(iv)	The series is a good approximation for f(x) if x is close to 0. Since x =1 is not close to 0, it is not suitable to be used to estimate $\mathrm{e}^{\frac{\pi}{4}}$.

4(i)	$\dfrac{7}{n-2}-\dfrac{5}{n-1}-\dfrac{2}{n}=\dfrac{7(n-1)n-5(n-2)n-2(n-2)(n-1)}{(n-2)(n-1)n}$ $=\dfrac{7n^2-7n-5n^2+10n-2n^2+6n-4}{(n-2)(n-1)n}$ Obtain $\dfrac{9n-4}{(n-2)(n-1)n}$ (Shown) <u>Alternative : By partial fractions</u> Let $\dfrac{9n-4}{(n-2)(n-1)n}=\dfrac{A}{n-2}+\dfrac{B}{n-1}+\dfrac{C}{n}$ $9n-4=A(n-1)n+B(n-2)n+C(n-2)(n-1)$ Subst $n=2$, obtain $A=7$ Subst $n=1$, obtain $B=-5$ Subst $n=0$, obtain $C=-2$ Obtain $\dfrac{9n-4}{(n-2)(n-1)n}$ (Shown)
(ii)	$\displaystyle\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}=\sum_{n=3}^{N}\left[\frac{7}{(n-2)}-\frac{5}{(n-1)}-\frac{2}{n}\right]$ $=\left[\begin{array}{l}\dfrac{7}{1}-\dfrac{5}{2}-\cancel{\dfrac{2}{3}}\\+\dfrac{7}{2}-\cancel{\dfrac{5}{3}}-\cancel{\dfrac{2}{4}}\\+\cancel{\dfrac{7}{3}}-\cancel{\dfrac{5}{4}}-\cancel{\dfrac{2}{5}}\\+\cancel{\dfrac{7}{4}}-\cancel{\dfrac{5}{5}}-\cancel{\dfrac{2}{6}}\\+\ldots\ldots\ldots\\+\dfrac{7}{N-4}-\cancel{\dfrac{5}{N-3}}-\cancel{\dfrac{2}{N-2}}\\+\cancel{\dfrac{7}{N-3}}-\cancel{\dfrac{5}{N-2}}-\dfrac{2}{N-1}\\+\cancel{\dfrac{7}{N-2}}-\dfrac{5}{N-1}-\dfrac{2}{N}\end{array}\right]$ $=8-\left[\dfrac{7}{N-1}+\dfrac{2}{N}\right]$
(iii)	$\displaystyle\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}=8-\left[\frac{7}{N-1}+\frac{2}{N}\right]$ As $N\to\infty$, $\dfrac{7}{N-1}\to 0$ and $\dfrac{2}{N}\to 0$, $\displaystyle\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}\to 8$ which is finite. Hence $\displaystyle\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n}$ is convergent and the sum to infinity is 8.

(iv)

$$\sum_{n=3}^{N}\frac{9n-4}{(n-2)(n-1)n} = \frac{23}{(1)(2)(3)}+\left(\frac{32}{(2)(3)(4)}+\frac{41}{(3)(4)(5)}+\dots\dots\dots\frac{9N-4}{(N-2)(N-1)N}\right)$$

$$\sum_{n=2}^{N}\frac{9n+14}{n(n+1)(n+2)}$$

$$=\left(\frac{32}{(2)(3)(4)}+\frac{41}{(3)(4)(5)}+\dots.+\frac{9N-4}{(N-2)(N-1)(N)}\right)+\frac{9N+5}{(N-1)(N)(N+1)}+\frac{9N+14}{N(N+1)(N+2)}$$

$$=\sum_{n=4}^{N+2}\frac{9n-4}{(n-2)(n-1)n} \quad \text{Note : replace } n \text{ by } n-2$$

$$=\sum_{n=3}^{N+2}\frac{9n-4}{(n-2)(n-1)n}-\frac{23}{(1)(2)(3)}$$

$$=8-\left[\frac{7}{N+2-1}+\frac{2}{N+2}\right]-\frac{23}{6}$$

$$=\frac{25}{6}-\left[\frac{7}{N+1}+\frac{2}{N+2}\right]$$

5(i)

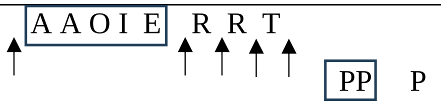
$$\overrightarrow{OA}=\begin{pmatrix}5\\0\\0\end{pmatrix},\ \overrightarrow{OB}=\begin{pmatrix}-4\\-3\\0\end{pmatrix},\ \overrightarrow{OC}=\begin{pmatrix}-5\\0\\6\end{pmatrix}$$

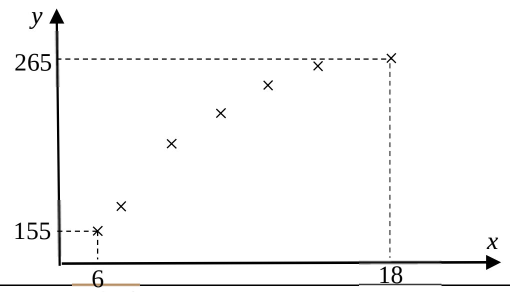
Since *ABCD* is a parallelogram,

$$\begin{aligned}\overrightarrow{AD} &= \overrightarrow{BC}\\ \overrightarrow{OD}-\overrightarrow{OA} &= \overrightarrow{OC}-\overrightarrow{OB}\\ &= \begin{pmatrix}-5\\0\\6\end{pmatrix}-\begin{pmatrix}-4\\-3\\0\end{pmatrix}=\begin{pmatrix}-1\\3\\6\end{pmatrix}\\ \overrightarrow{OD} &= \begin{pmatrix}-1\\3\\6\end{pmatrix}+\begin{pmatrix}5\\0\\0\end{pmatrix}\\ &= \begin{pmatrix}4\\3\\6\end{pmatrix}\end{aligned}$$

	$\overrightarrow{BA}=\begin{pmatrix}5\\0\\0\end{pmatrix}-\begin{pmatrix}-4\\-3\\0\end{pmatrix}=\begin{pmatrix}9\\3\\0\end{pmatrix}$ $\overrightarrow{BA}\bullet\overrightarrow{BC}=\begin{pmatrix}9\\3\\0\end{pmatrix}\bullet\begin{pmatrix}-1\\3\\6\end{pmatrix}=0$ Since *ABCD* is a parallelogram and $AB\perp BC$, *ABCD* is a rectangle.
(ii)	$\overrightarrow{BA}\times\overrightarrow{BC}=\begin{pmatrix}9\\3\\0\end{pmatrix}\times\begin{pmatrix}-1\\3\\6\end{pmatrix}=3\begin{pmatrix}3\\1\\0\end{pmatrix}\times\begin{pmatrix}-1\\3\\6\end{pmatrix}=6\begin{pmatrix}3\\-9\\5\end{pmatrix}$ Normal to the plane *ABC* $=\underset{\sim}{n}=\begin{pmatrix}3\\-9\\5\end{pmatrix}$. $\overrightarrow{OA}\bullet\begin{pmatrix}3\\-9\\5\end{pmatrix}=\begin{pmatrix}5\\0\\0\end{pmatrix}\bullet\begin{pmatrix}3\\-9\\5\end{pmatrix}=15$ Vector equation of plane *ABC* is $\underset{\sim}{r}\bullet\begin{pmatrix}3\\-9\\5\end{pmatrix}=15$ Cartesian equation of plane *ABC* is $\underline{\underline{3x-9y+5z=15}}$.
(iii)	Normal vector of the base is parallel to $\overrightarrow{OE}$. Hence, normal vector of the base = $\begin{pmatrix}0\\0\\1\end{pmatrix}$ Acute angle between plane *ABC* and the base $=\cos^{-1}\frac{\left\lvert\begin{pmatrix}3\\-9\\5\end{pmatrix}\bullet\begin{pmatrix}0\\0\\1\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}3\\-9\\5\end{pmatrix}\right\rvert\left\lvert\begin{pmatrix}0\\0\\1\end{pmatrix}\right\rvert}$ $=\cos^{-1}\frac{\lvert 5\rvert}{\sqrt{3^2+9^2+5^2}\sqrt{1}}$ $=\cos^{-1}\frac{5}{\sqrt{115}}$ $=\underline{\underline{62.2^\circ}}$ (1 dp)

(iv)	Since $AF : AE = 1:5$, $\overrightarrow{AF}=\frac{1}{5}\overrightarrow{AE}=\frac{1}{5}\left[\begin{pmatrix}0\\0\\10\end{pmatrix}-\begin{pmatrix}5\\0\\0\end{pmatrix}\right]=\begin{pmatrix}-1\\0\\2\end{pmatrix}$ Length of projection of AF onto plane ABC $=\frac{\left\lvert\overrightarrow{AF}\times\underset{\sim}{n}\right\rvert}{\left\lvert\underset{\sim}{n}\right\rvert}$ $=\frac{\left\lvert\begin{pmatrix}-1\\0\\2\end{pmatrix}\times\begin{pmatrix}3\\-9\\5\end{pmatrix}\right\rvert}{\left\lvert\begin{pmatrix}3\\-9\\5\end{pmatrix}\right\rvert}$ $=\frac{\left\lvert\begin{pmatrix}18\\11\\9\end{pmatrix}\right\rvert}{\sqrt{3^2+(-9)^2+5^2}}$ $=\frac{\sqrt{526}}{\sqrt{115}}$ $=2.1387$ ≈ 2.14

6(a)(i)	Since there is only one way the letters are in alphabetical order, Total number of ways = $\frac{11!}{3!2!2!}-1$ $= 1\ 663\ 199$
(a)(ii)	 Total number of ways = $\frac{4!}{2!}\times\frac{5!}{2!}\times{}^5P_2$ = 14 400
(b)	<u>Method 1:</u> Probability = $\frac{6}{11}\times\frac{5}{10}\times\frac{4}{9}\times\frac{3!}{2!}+\frac{5}{11}\times\frac{4}{10}\times\frac{3}{9}=\frac{14}{33}$ (or 0.424) <u>Method 2:</u> Probability = $\frac{{}^6C_1\,{}^5C_2+{}^5C_3}{{}^{11}C_3}$ <u>Method 3</u>: Probability = $1-\frac{{}^6C_3}{{}^{11}C_3}-\frac{{}^5C_1\,{}^6C_2}{{}^{11}C_3}=\frac{14}{33}$

7(i)	
(ii)	**(a)** Correlation coefficient between x and y is $r = 0.9483$ (4 d.p.) **(b)** Correlation coefficient between $\ln x$ and y is $r = 0.9849$ (4 d.p.)
(iii)	From the scatter diagram, as x increases, y increases at a decreasing rate. In addition, the product moment correlation coefficient between $\ln x$ and y, 0.9849, is closer to +1 as compared to that between x and y, 0.9483. Hence $y = c \ln x + d$ is the better model.
(iv)	Since x is the independent variable, neither the regression line of x on y nor the regression line of $\ln x$ on y should be used to estimate the value of x when $y = 200$.
(v)	Equation of regression line of y on $\ln x$ is $y = 106.5611 \ln x - 31.2643$ $y = 107 \ln x - 31.3$ when $y = 200$, $200 = 106.5611 \ln x - 31.2643$ $x = 8.76$ (3 s.f.) Since $y = 200$ is within the given range of data, which is an interpolation, and r is close to +1, indicating a strong positive linear correlation, the estimate is reliable.

8(i)	P(packet is unsatisfactory) $= 1 - (0.99)(0.98)^2(0.96)$ $= 0.087236 \approx 0.0872$ (3 s.f.)
(ii)	$X \sim \mathrm{B}(180, 0.087236)$ $\mathrm{P}(5 \le X < 10) = \mathrm{P}(X \le 9) - \mathrm{P}(X \le 4)$ $= 0.042669 \approx 0.0427$
(iii)	$\mathrm{P}(X > r) \le 0.12$ $1 - \mathrm{P}(X \le r) \le 0.12$ $\mathrm{P}(X \le r) \ge 0.88$ From GC, $\mathrm{P}(X \le 19) = 0.8429$ (< 0.88) $\mathrm{P}(X \le 20) = 0.8947$ (> 0.88) $\mathrm{P}(X \le 21) = 0.9323$ (> 0.88) $\therefore$ least $r = 20$
(iv)	Each packet has an equal chance of being selected and the selection of the packets is independent of one another. This method of selection is done so that a random sample will be obtained which is free from bias and will be representative of the population.

(v)

Required probability

$$= \left(0.087236\right)^2 \times \left(0.912764\right)^6 \times \frac{8!}{2!6!} \times \left(0.087236\right)$$

$= 0.010749 \approx 0.0107$

<u>Alternative Method</u>

Let Y be the the number of packets (out of 8) that are unsatisfactory.

$Y \sim \mathrm{B}(8, 0.087236)$

Required probability = $\mathrm{P}(Y = 2) \times (0.087236)$

$= 0.010749 \approx 0.0107$

9(i)

Box A \ Box B	(B) 4 $\left(\frac{1}{3}\right)$	(R) 5 $\left(\frac{2}{3}\right)$
(B) 1 $\left(\frac{1}{5}\right)$	5	5
(R) 2 $\left(\frac{2}{5}\right)$	8	7
(R) 3 $\left(\frac{2}{5}\right)$	12	8

$\mathrm{P}(X = 8)$ = P(2 from Box A and 4 from Box B)

+ P(3 from Box A and 5 from Box B)

$$= \left(\frac{2}{5}\right)\left(\frac{1}{3}\right) + \left(\frac{2}{5}\right)\left(\frac{2}{3}\right)$$

$$= \frac{2}{5}$$

(ii)

$$\mathrm{P}(X = 5) = \left(\frac{1}{5}\right)\left(\frac{1}{3}\right) + \left(\frac{1}{5}\right)\left(\frac{2}{3}\right) = \frac{1}{5}$$

$$\mathrm{P}(X = 7) = \left(\frac{2}{5}\right)\left(\frac{2}{3}\right) = \frac{4}{15}$$

$$\mathrm{P}(X = 12) = \left(\frac{2}{5}\right)\left(\frac{1}{3}\right) = \frac{2}{15}$$

The probability distribution of X is

x	5	7	8	12
$\mathrm{P}(X = x)$	$\frac{1}{5}$	$\frac{4}{15}$	$\frac{2}{5}$	$\frac{2}{15}$

(iii)

$$\mathrm{E}(X) = 5\left(\frac{1}{5}\right) + 7\left(\frac{4}{15}\right) + 8\left(\frac{2}{5}\right) + 12\left(\frac{2}{15}\right) = \frac{23}{3}$$

$$\mathrm{E}(X^2) = 5^2\left(\frac{1}{5}\right) + 7^2\left(\frac{4}{15}\right) + 8^2\left(\frac{2}{5}\right) + 12^2\left(\frac{2}{15}\right) = \frac{943}{15}$$

$$\mathrm{Var}(X) = \frac{943}{15} - \left(\frac{23}{3}\right)^2 = \frac{184}{45}$$

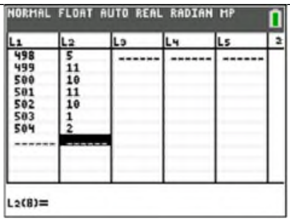
(iv)

Since sample size n = 50 is large, by Central Limit Theorem,

$$\overline{X} \sim \mathrm{N}\left(\frac{23}{3},\ \frac{184}{45(50)}\right) = \mathrm{N}\left(\frac{23}{3},\ \frac{92}{1125}\right) \text{ approximately.}$$

$\mathrm{P}\left(7.5 < \overline{X} < 8.5\right) = 0.71821 \approx 0.718$ (3 s.f.)

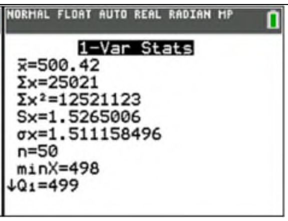
10(i)



```
NORMAL FLOAT AUTO REAL RADIAN MP
L1     L2     L3      L4      L5      2
498    5      ------  ------  ------
499    11
500    10
501    11
502    10
503    1
504    2
------ ------

L2(8)=
```

```
NORMAL FLOAT AUTO REAL RADIAN MP
1-Var Stats
List:L1
FreqList:L2
Calculate
```



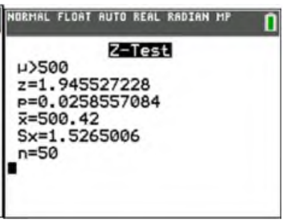
```
NORMAL FLOAT AUTO REAL RADIAN MP
1-Var Stats
x̄=500.42
Σx=25021
Σx²=12521123
Sx=1.5265006
σx=1.511158496
n=50
minX=498
↓Q1=499
```

Using GC,

$\overline{x} = 500.42$ (exact)

$s = 1.5265$ (to 5 s.f.)

```
NORMAL FLOAT AUTO REAL RADIAN MP
PRESS [<] OR [>] TO SELECT AN OPTION
Z-Test
Inpt:Data Stats
μ0:500
σ:1.5265005999451
List:L1
Freq:L2
μ:≠μ0 <μ0 >μ0
Color: BLUE <>
Calculate Draw
```



```
NORMAL FLOAT AUTO REAL RADIAN MP
Z-Test
μ>500
z=1.945527228
p=0.0258557084
x̄=500.42
Sx=1.5265006
n=50
```

Let X = vol. of green tea dispensed in a bottle and

μ = population mean green tea dispensed in a bottle

$\mathrm{H}_0 : \mu = 500$

$\mathrm{H}_1 : \mu > 500$

Under H_0 ,since n = 50 is large, by Central Limit Theorem,

$$\overline{X} \sim \mathrm{N}\left(500\ ,\ \frac{1.5265^2}{50}\right) \text{ approximately}$$

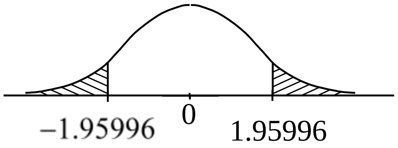
Test statistic: $z = \dfrac{\overline{x} - 500}{\left(\dfrac{1.5265}{\sqrt{50}}\right)}$

$$= \frac{500.42 - 500}{\left(\dfrac{1.5265}{\sqrt{50}}\right)} = 1.9455 \approx 1.95 \text{ (to 3 s.f)}$$

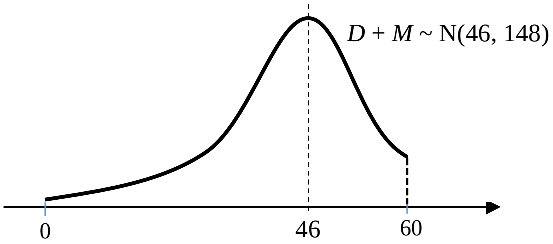
From GC, p-value = $0.025856 \approx 0.0259$ (to 3 s.f)

Since p-value = 0.0259 > 0.02, we do not reject Ho at 2% level of significance and conclude that there is insufficient evidence that the production manager's suspicion is valid.

The p-value is 0.0259 and it means there is a probability of 0.0259 of observing a test statistic, $z \geq 1.95$, given that the population mean green tea dispensed in a bottle is 500ml.

(ii)	For the production manager's suspicion ($\mu > 500$) to be valid, H_0 is rejected. Hence, p-value = 0.025856 < $\frac{\alpha}{100}$ $\therefore 2.59 < \alpha < 100$ (3 s.f)
(iii)	Since the sample size = 50 is large enough, Central Limit Theorem can be applied for sample means to follow a normal distribution approximately.
(b)	$s^2 = \frac{n}{n-1}\sigma_x{}^2 = \frac{50}{49}k^2$ $H_0 : \mu = 500$ $H_1 : \mu \neq 500$ Under H_0 ,since n = 50 is large, by Central Limit Theorem, $\overline{X} \sim \mathrm{N}\left(500\ ,\ \frac{\left(\frac{50k^2}{49}\right)}{50}\right) = \mathrm{N}\left(500\ ,\ \frac{k^2}{49}\right)$ approximately Test statistic: $z = \frac{\overline{x}-500}{\sqrt{\frac{k^2}{49}}} = \frac{502-500}{\frac{k}{7}} = \frac{14}{k}$ For α = 0.05,  If recalibration is done accurately ($\mu = 500$), H_0 is not rejected. Hence, $-1.95996 < z < 1.95996$ $-1.95996 < \frac{14}{k} < 1.95996$ $\frac{k}{14} < -0.51021$ (rejected) or $\frac{k}{14} > 0.51021$ $k > 7.1430$ $\therefore k > 7.14$ (to 3 s.f)

11(i)	Let D be the time in minutes past 7 pm that Benedict finishes his dinner. Let M be the time in minutes he spends on social media. $D \sim \mathrm{N}(10, 2^2)$ and $M \sim \mathrm{N}(k, 12^2)$, Then $D + M \sim \mathrm{N}(10 + k, 148)$ Given $\mathrm{P}(D + M > 60) = 0.125$ $\mathrm{P}\left(Z > \frac{60-(10+k)}{\sqrt{148}}\right) = 0.125$

	$\frac{50-k}{\sqrt{148}}=1.15035$ $k=50-1.15035\sqrt{148}$ = 36.005 ≈ 36.0
(ii)	
(iii)	$X = D + M$ ~ N(46, 148) $Y = D + G$ ~ N(55, 104) Then $Y - X$ ~ N(9, 252) $\mathrm{P}\left(\lvert Y-X\rvert \le 5\right)=\mathrm{P}\left(-5\le Y-X \le 5\right)$ = 0.21162 ≈ 0.212
(iv)	P(started revision late) = 0.8 P($D + M > 60$) + 0.2 P($D + G > 60$) = 0.16232 P(played online games | started revision late) = $\frac{0.2\ \mathrm{P}(D+G>60)}{0.16232}$
	= 0.38438 ≈ 0.384
(v)	*D*, *M* and *G* are independent normal variables.