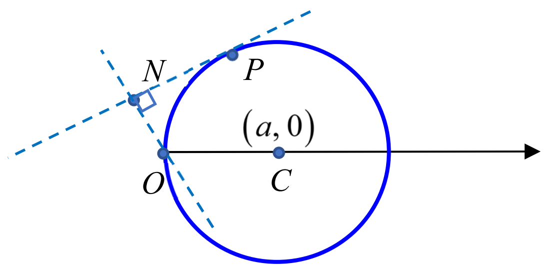
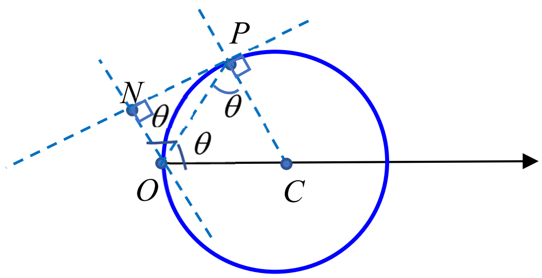
1	Let P_n be the statement $\left(\frac{a+b}{2}\right)^n \le \frac{a^n+b^n}{2}$ for $n \in \mathbb{Z}^+$ When $n=1$, LHS = $\frac{a+b}{2}$ RHS = $\frac{a+b}{2}$ $\therefore P_1$ is true. Assume P_k is true for for some $k \in \mathbb{Z}^+$, i.e. $\left(\frac{a+b}{2}\right)^k \le \frac{a^k+b^k}{2}$ Want to prove that P_{k+1} is true, i.e. $\left(\frac{a+b}{2}\right)^{k+1} \le \frac{a^{k+1}+b^{k+1}}{2}$ $\left(\frac{a+b}{2}\right)^k \le \frac{a^k+b^k}{2}$ $\left(\frac{a+b}{2}\right)^k\left(\frac{a+b}{2}\right) \le \left(\frac{a^k+b^k}{2}\right)\left(\frac{a+b}{2}\right)$ --- (*) $= \frac{a^{k+1}+a^kb+ab^k+b^{k+1}}{4}$ $= \frac{a^{k+1}+b^{k+1}}{2} - \frac{a^{k+1}+b^{k+1}}{4} + \frac{a^kb+ab^k}{4}$ (**) $= \frac{a^{k+1}+b^{k+1}}{2} - \left(\frac{a^{k+1}+b^{k+1}}{4} - \frac{a^kb+ab^k}{4}\right)$ $= \frac{a^{k+1}+b^{k+1}}{2} - \left(\frac{\left(a^k-b^k\right)\left(a-b\right)}{4}\right)$ $\le \frac{a^{k+1}+b^{k+1}}{2} \quad \because \frac{\left(a^k-b^k\right)\left(a-b\right)}{4} \ge 0$ Hence P_k is true $\Rightarrow P_{k+1}$ is true. Since P_1 is true and P_k is true $\Rightarrow P_{k+1}$ is true, by mathematical induction P_n is true for all $n \in \mathbb{Z}^+$.
2(a)	



2(b)

Let the polar coordinates of N be $\left(r_1, \theta_1\right)$ and the polar coordinates of P be $\left(2a\cos\theta, \theta\right)$



$\angle OPC = \theta \quad \left(\because OC = PC\right)$

$\angle PON = \angle OPC = \theta$

$\angle CON = \theta + \theta$

$\theta_1 = 2\theta$

$ON = OP\cos\theta = \left(2a\cos\theta\right)\cos\theta$

$r_1 = 2a\cos^2\theta$

$-\frac{\pi}{2} < \theta \leq \frac{\pi}{2}$

$-\pi < \theta_1 \leq \pi$

$\therefore r_1 = 2a\cos^2\theta = a\left(\cos 2\theta + 1\right) = a\left(1 + \cos\theta_1\right)$ (Shown)

3(a)

Characteristic equation:

$m^2 - 5m + 6 = 0$

$\left(m-3\right)\left(m-2\right) = 0$

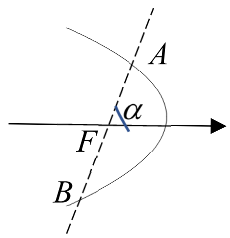
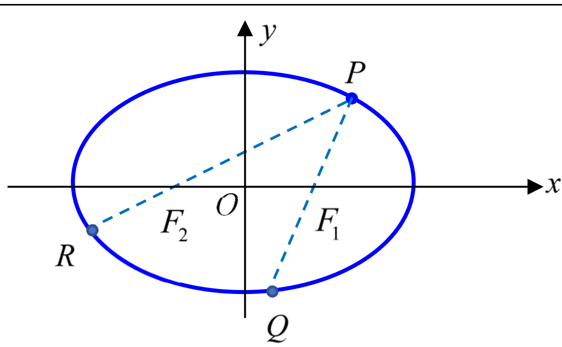
$m = 3 \quad \text{or} \quad m = 2$

$\therefore u_n = A\left(3\right)^n + B\left(2\right)^n$

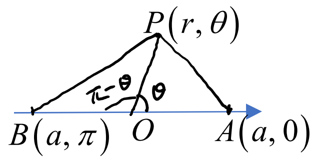
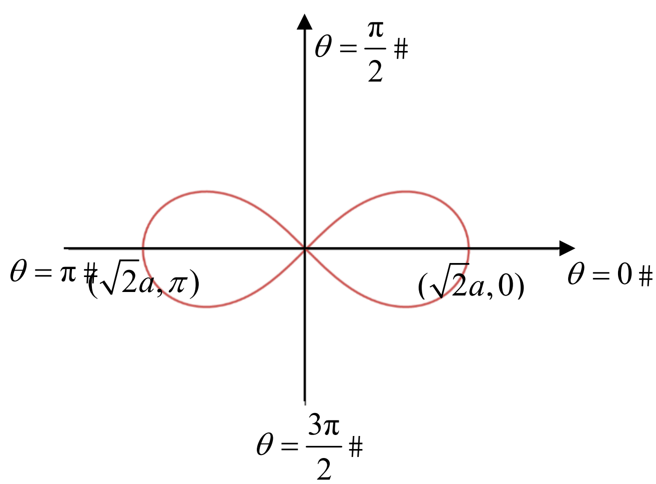
$u_1 = a, \quad a = A\left(3\right) + B\left(2\right)$

$u_2 = b, \quad b = A\left(9\right) + B\left(4\right)$

	$2B = 3a - b$ $B = \frac{3a-b}{2}$ $a = A(3) + B(2)$ $A = \frac{a - 2\left(\frac{3a-b}{2}\right)}{3} = \frac{b-2a}{3}$ $A = \frac{b-2a}{3}, \qquad B = \frac{3a-b}{2}$ $u_n = \left(\frac{b-2a}{3}\right)(3)^n + \left(\frac{3a-b}{2}\right)2^n$ $\therefore u_n = (b-2a)(3)^{n-1} + (3a-b)2^{n-1}$
3(b)	$\frac{u_n}{u_{n-1}} = \left(\frac{(b-2a)(3)^{n-1} + (3a-b)2^{n-1}}{(b-2a)(3)^{n-2} + (3a-b)2^{n-2}}\right)$ $\frac{u_n}{u_{n-1}} = \left(\frac{(b-2a) + (3a-b)\left(\frac{2}{3}\right)^{n-1}}{(b-2a)\left(\frac{1}{3}\right) + (3a-b)\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)^{n-1}}\right)$ If $b \neq 2a$, as $n \to \infty$, $\left(\frac{2}{3}\right)^{n-1} \to 0$, $\lim_{n\to\infty}\left(\frac{u_n}{u_{n-1}}\right) = 3$ If $b = 2a$, as $n \to \infty$, $\lim_{n\to\infty}\left(\frac{u_n}{u_{n-1}}\right) = 2$ Or Let the limit be L $L^2 - 5L + 6 = 0$ $(L-3)(L-2) = 0$ $L = 3$ or $L = 2$

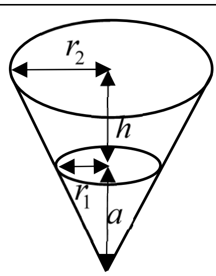
4(a)	The polar equation form of a conic is $r=\dfrac{k}{1+e\cos\theta}$.  Let F be the pole and A,B have polar coordinates $\left(AF,\alpha\right)$ and $\left(BF,\alpha+\pi\right)$ respectively Since A,B lies on the conic, $AF=\dfrac{k}{1+e\cos\alpha}$ and $BF=\dfrac{k}{1+e\cos\left(\alpha+\pi\right)}$ $1+e\cos\alpha=\dfrac{k}{AF}$ and $1+e\cos\left(\alpha+\pi\right)=\dfrac{k}{BF}$ $1-e\cos\alpha=\dfrac{k}{BF}$ Summing, $\dfrac{k}{AF}+\dfrac{k}{BF}=1+e\cos\alpha+1-e\cos\alpha=2$ $\dfrac{1}{AF}+\dfrac{1}{BF}=\dfrac{2}{k}$
4(b)	 By (i),

	$\frac{1}{PF_1}+\frac{1}{F_1Q}=\frac{2}{k}$ $1+\frac{PF_1}{F_1Q}=\frac{2}{k}PF_1$ ----------------(1) Similarly $\frac{1}{PF_2}+\frac{1}{F_2R}=\frac{2}{k}$ $1+\frac{PF_2}{F_2R}=\frac{2}{k}PF_2$ -------------(2) Summing (1) and (2) , $\left(1+\frac{PF_1}{F_1Q}\right)+\left(1+\frac{PF_2}{F_2R}\right)=\left(\frac{2}{k}PF_1\right)+\left(\frac{2}{k}PF_2\right)$ $2+\frac{PF_1}{F_1Q}+\frac{PF_2}{F_2R}=\frac{2}{k}\left(PF_1+PF_2\right)$ Since this is an ellipse, PF_1+PF_2 is a constant, let $PF_1+PF_2=C$, $\frac{PF_1}{F_1Q}+\frac{PF_2}{F_2R}=\frac{2C}{k}-2=$ constant (Shown)
5 (a)	$A:(a,0)$ $B:(-a,0)$ $P:(x,y)$ $AP\cdot BP=\sqrt{(x-a)^2+y^2}\sqrt{(x+a)^2+y^2}$ $AP\cdot BP=a^2$ $\left((x-a)^2+y^2\right)\left((x+a)^2+y^2\right)=a^4$ $\left(x^2-2ax+a^2+y^2\right)\left(x^2+2ax+a^2+y^2\right)=a^4$ $\left(x^2+y^2\right)^2+\left(x^2+y^2\right)\left(2ax+a^2\right)+$ $\left(x^2+y^2\right)\left(-2ax+a^2\right)+\left(2ax+a^2\right)\left(-2ax+a^2\right)=a^4$ $\left(x^2+y^2\right)^2+2\left(x^2+y^2\right)a^2+a^4-4a^2x^2=a^4$ $\left(x^2+y^2\right)^2+2a^2\left(y^2-x^2\right)=0$

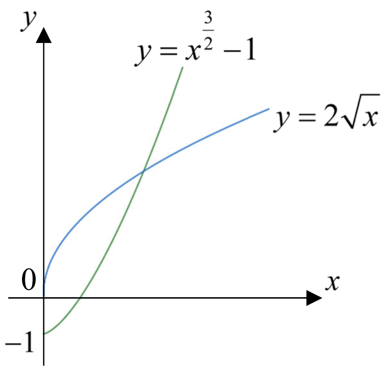
	$\left(r^2\right)^2 + 2a^2\left(r^2\sin^2\theta - r^2\cos^2\theta\right) = 0$ $r^2 = 2a^2\left(\cos^2\theta - \sin^2\theta\right) = 2a^2\cos 2\theta$ [Shown] Alternative Solution, by cosine rule  $AP^2 = a^2 + r^2 - 2ar\cos\theta$ $BP^2 = a^2 + r^2 - 2ar\cos(\pi-\theta)$ $= a^2 + r^2 + 2ar\cos\theta$ $(AP \cdot BP)^2$ $=\left(a^2 + r^2 - 2ar\cos\theta\right)\left(a^2 + r^2 + 2ar\cos\theta\right)$ $=\left(a^2 + r^2\right)^2 - (2ar\cos\theta)^2$ $a^4 = a^4 + 2a^2r^2 + r^4 - 4a^2r^2\cos^2\theta$ $2a^2r^2 + r^4 - 4a^2r^2\cos^2\theta = 0$ $r^4 = 4a^2r^2\cos^2\theta - 2a^2r^2$ $=2a^2r^2\left(2\cos^2\theta - 1\right)$ $=2a^2r^2\cos 2\theta$ $r^2 = 2a^2\cos 2\theta$
5(b)	

5(c)	For $\cos 2\theta \geq 0,\ 0 \leq \theta \leq \frac{\pi}{4}$. Area $= 4\left(\frac{1}{2}\right)\int_0^{\frac{\pi}{4}} 2a^2 \cos 2\theta \ \mathrm{d}\theta$ $= 4a^2 \int_0^{\frac{\pi}{4}} \cos 2\theta \ \mathrm{d}\theta$ $= 4a^2 \left[\frac{\sin 2\theta}{2}\right]_0^{\frac{\pi}{4}}$ $= 4a^2 \left(\frac{1}{2} - 0\right)$ $= 2a^2$ units^2
6(a)	Using Shell method, Volume of S rotating about x-axis $= 2\pi \int_0^1 xy \ \mathrm{d}y$ $= 2\pi \int_0^1 y\mathrm{e}^{2y-1} \ \mathrm{d}y$ (Shown) where $\mathrm{F}(y) = y\mathrm{e}^{2y-1}$.
6(b)	$\int_0^1 y\mathrm{e}^{2y-1} \ \mathrm{d}y = \left[\frac{1}{2} y\mathrm{e}^{2y-1}\right]_0^1 - \int_0^1 \frac{1}{2}\mathrm{e}^{2y-1} \ \mathrm{d}y$ $= \frac{1}{2}\mathrm{e} - \left[\frac{1}{4}\mathrm{e}^{2y-1}\right]_0^1$ $= \frac{1}{2}\mathrm{e} - \left(\frac{1}{4}\mathrm{e} - \frac{1}{4}\mathrm{e}^{-1}\right)$ $= \frac{1}{4}\mathrm{e} + \frac{1}{4}\mathrm{e}^{-1}$

6(c)	When $y = 0$, x-intercept = e^{-1} Using Disk method, Volume of T rotating about the x-axis $= \pi \int_{\mathrm{e}^{-1}}^{\mathrm{e}} y^2 \ \mathrm{d}x$ $= \pi \int_{\mathrm{e}^{-1}}^{\mathrm{e}} \left(\frac{1+\ln x}{2}\right)^2 \ \mathrm{d}x$ $= \frac{\pi}{4} \int_{\mathrm{e}^{-1}}^{\mathrm{e}} (\ln x + 1)^2 \ \mathrm{d}x$ Volume of cylinder $= \pi (1)^2 \mathrm{e}$ $= \pi \mathrm{e}$ Volume of S + Volume of T = Volume of cylinder $2\pi \int_0^1 y\mathrm{e}^{2y-1} \ \mathrm{d}y + \frac{\pi}{4} \int_{\mathrm{e}^{-1}}^{\mathrm{e}} (\ln x + 1)^2 \ \mathrm{d}x = \pi \mathrm{e}$ $\int_{\mathrm{e}^{-1}}^{\mathrm{e}} (\ln x + 1)^2 \ \mathrm{d}x = 4\left[\mathrm{e} - 2\int_0^1 y\mathrm{e}^{2y-1} \ \mathrm{d}y\right]$ $= 4\left[\mathrm{e} - \frac{2}{4}(\mathrm{e} + \mathrm{e}^{-1})\right]$ $= 4\left(\frac{1}{2}\mathrm{e} - \frac{1}{2}\mathrm{e}^{-1}\right)$ $= 2\left(\mathrm{e} - \frac{1}{\mathrm{e}}\right)$
6(d)	$y = \frac{1+\ln x}{2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2x}$ Surface area when rotated about the x-axis $= 2\pi \int_{\mathrm{e}^{-1}}^{\mathrm{e}} \left(\frac{1+\ln x}{2}\right)\sqrt{1 + \left(\frac{1}{2x}\right)^2} \ \mathrm{d}x$ $= 10.287666 \text{ units}^2$ $= 10.3 \text{ units}^2$

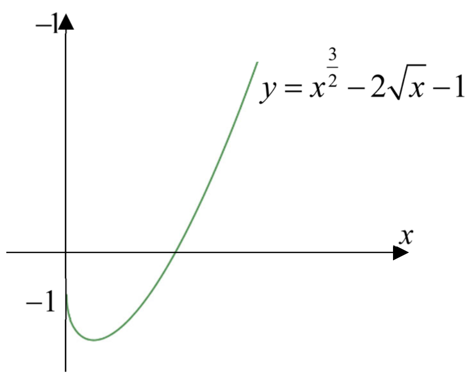
	The area of the surface generated when the arc of a curve with equation $y = \mathrm{e}^{2x-1}$ between the y-intercept and $y = \mathrm{e}$ is rotated through 2π radians about the y-axis is also 10.3 unit^2. $y = \mathrm{e}^{2x-1}$ is the inverse function of $2y = 1 + \ln x$. Hence the area of the surface generated when the arc of the curve with equation $y = \mathrm{e}^{2x-1}$ between the y-intercept and $y = \mathrm{e}$ is rotated through 2π radians about the y-axis is the same as the area of the surface generated when the arc of the curve with equation $2y = 1 + \ln x$ between the x-intercept and $x = \mathrm{e}$ is rotated through 2π radians about the x-axis.
7(a)	The number of cross-sectional areas given must be odd
7(b)	Using Simpson's rule on the first 5 surface areas $V_1 = \frac{4}{3}\left[190 + 4(127) + 100 + 100 + 4(76) + 20\right] \times 10^4$ $= 16.293 \times 10^6 \, \mathrm{m}^3$ Using Trapezoidal Rule on the last 2 surface areas $V_2 = \frac{3}{2}(20 + 14) \times 10^4$ $= 0.51 \times 10^6 \ \mathrm{m}^3$ Total Volume $= (16.293 + 0.51) \times 10^4$ $= 16.803 \times 10^6 \, \mathrm{m}^3$ $= 16.8 \times 10^6 \, \mathrm{m}^3$ (3 s.f.)
	 $\frac{a}{a+h} = \frac{r_1}{r_2}$ $ar_2 = (a+h)r_1$ $a(r_2 - r_1) = hr_1$ $a = \frac{hr_1}{r_2 - r_1}$

	$V=\frac{1}{3}\pi r_2^{\,2}\left(a+h\right)-\frac{1}{3}\pi r_1^{\,2}a$ $=\frac{1}{3}\pi a\left(r_2^{\,2}-r_1^{\,2}\right)+\frac{1}{3}\pi r_2^{\,2}h$ $=\frac{1}{3}\pi\frac{hr_1}{r_2-r_1}\left(r_2-r_1\right)\left(r_2+r_1\right)+\frac{1}{3}A_2h$ $=\frac{1}{3}\pi r_1r_2h+\frac{1}{3}\pi r_1^{\,2}h+\frac{1}{3}A_2h$ $=\frac{1}{3}h\left(\pi r_1r_2h+\pi r_1^{\,2}+A_2\right)$ $=\frac{1}{3}h\left(\sqrt{A_1A_2}+A_1+A_2\right)$
7(c)	$V_1=\frac{4}{3}\Big[190+\sqrt{190\left(127\right)}+127+127+\sqrt{100\left(127\right)}+100+$ $+100+\sqrt{100\left(76\right)}+76+76+\sqrt{76\left(20\right)}+20\Big]\times10^4$ $=16.136\times10^6\ \text{m}^3$ $V_2=\frac{3}{3}\left(20+\sqrt{20\left(14\right)}+14\right)\times10^4=0.39831\times10^6\ \text{m}^3$ $V=\left(16.136+0.39831\right)\times10^6$ $=16.53431\times10^6\ \text{m}^3$ $=16.5\times10^6\ \text{m}^3$ (3 s.f.)
7(d)	Difference $=\left(16.803-16.534\right)\ \times\ 10^6$ $=0.269\times10^6$
7(e)	80% of water = $0.8\times16.803\times10^6=13.442\times10^6$ Number of days the reservoir can last $\frac{13.442\times10^6}{0.7\times10^6}=19.2\approx19$ days Since the difference between answer in part (b) and (c) is less than half of 0.7×10^6, there will be no change in the estimated number of days.
8	<u>Method 1</u>



Since both graphs are strictly increasing and $y = x^{\frac{3}{2}} - 1$ is concaving upwards and $y = 2\sqrt{x}$ is concaving downwards, they will intersect at only one point. Hence $x^{\frac{3}{2}} - 2\sqrt{x} - 1 = 0$ has only one root.

<u>Method 2</u>



$y = x^{\frac{3}{2}} - 2\sqrt{x} - 1$

$\frac{\mathrm{d}y}{\mathrm{d}x} = x^{-\frac{1}{2}}\left(\frac{3}{2}x - 1\right) > 0$ when $x > \frac{2}{3}$

when $x > \frac{2}{3}$, the graph is strictly increasing.

$n = 2$

8(a)(i) (1)

$$f(x)=\frac{1}{4}\left(x^{\frac{3}{2}}-1\right)^2$$

$$f'(x)=\frac{1}{2}\left(x^{\frac{3}{2}}-1\right)\left(\frac{3}{2}x^{\frac{1}{2}}\right)$$

$$=\frac{3}{4}x^{\frac{1}{2}}\left(x^{\frac{3}{2}}-1\right)$$

In the interval $(2,3)$, $1.94<f'(x)<5.45$

Since $|f'(x)|>1$, the iteration may not be convergent.

(2)

$$f(x)=2+\frac{1}{\sqrt{x}}$$

$$f'(x)=-\frac{1}{2}x^{-\frac{3}{2}}$$

In the interval $(2,3)$, $-0.177<f'(x)<-0.0962$

Since $|f'(x)|<1$, the iteration converges.

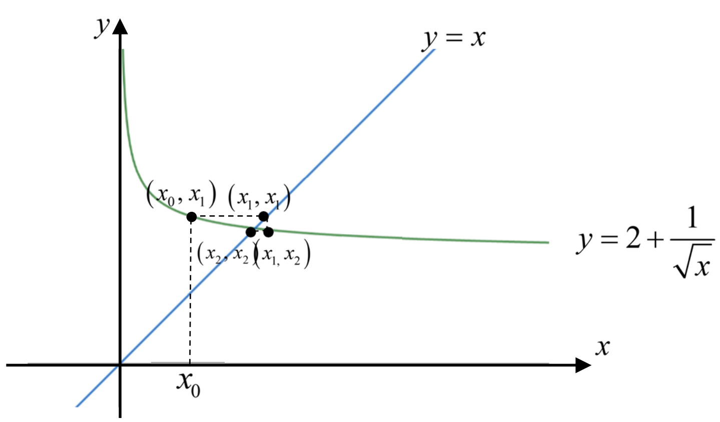
(3)

$$f(x)=x^{\frac{5}{2}}-x^{\frac{3}{2}}$$

$$f'(x)=\frac{5}{2}x^{\frac{3}{2}}-\frac{3}{2}x^{\frac{1}{2}}$$

In the interval $(2,3)$, $4.94<f'(x)<10.4$

Since $|f'(x)|>1$, the iteration may not converge.



8(a)(ii)	Using $\mathrm{f}(x_n) = 2 + \frac{1}{\sqrt{x_n}}$

n	$\mathrm{f}(x_n)$
1	2
2	2.7071
3	2.6078
4	2.6192
5	2.6179
6	2.6181
7	2.6180
8	2.6180

$\alpha = 2.6180$

Let $\mathrm{g}(x) = x^{\frac{3}{2}} - 2\sqrt{x} - 1$

g is continuous in $[2.61795, 2.61805]$

$\mathrm{g}(2.61795)\mathrm{g}(2.61805) = (-1.519 \times 10^{-4})(2.8965 \times 10^{-5}) < 0$

$\therefore 2.61795 < \alpha < 2.61805$

$\therefore \alpha = 2.6180$

8(bi)

$\mathrm{f}(x) = x^{\frac{3}{2}} - 2\sqrt{x} - 1$

$\mathrm{f}'(x) = \frac{3}{2}x^{\frac{1}{2}} - x^{-\frac{1}{2}}$

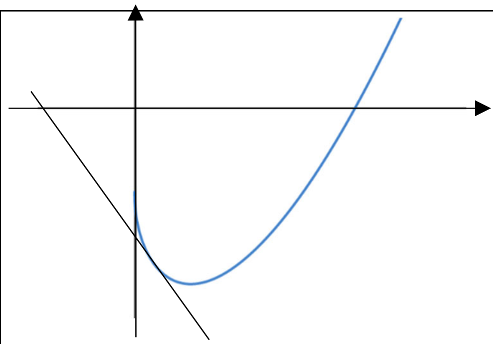
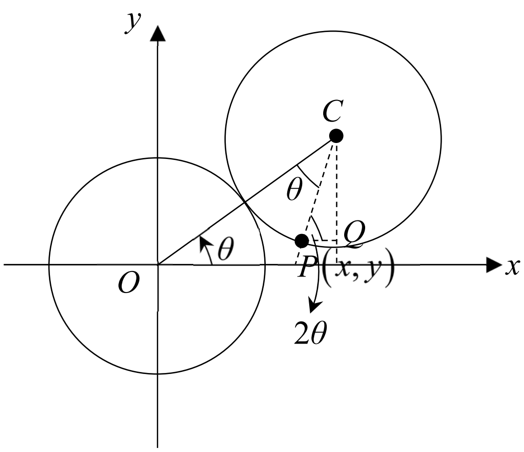
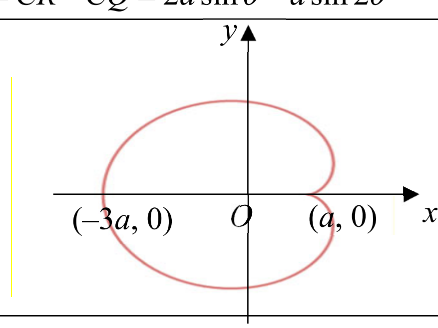
$$x_{n+1} = x_n - \frac{x_n^{\frac{3}{2}} - 2\sqrt{x_n} - 1}{\frac{3}{2}x_n^{\frac{1}{2}} - x_n^{-\frac{1}{2}}}$$

$$= x_n - \frac{2x_n^{\frac{3}{2}} - 4\sqrt{x_n} - 2}{3x_n^{\frac{1}{2}} - 2x_n^{-\frac{1}{2}}}$$

$$= \frac{3x_n^{\frac{3}{2}} - 2x_n^{\frac{1}{2}} - 2x_n^{\frac{3}{2}} + 4\sqrt{x_n} + 2}{3x_n^{\frac{1}{2}} - 2x_n^{-\frac{1}{2}}}$$

$$= \frac{x_n^{\frac{3}{2}} + 2x_n^{\frac{1}{2}} + 2}{3x_n^{\frac{1}{2}} - 2x_n^{-\frac{1}{2}}}$$

(bii)

	 When $x = \frac{2}{3}$, the gradient is zero and $\frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$ will be undefined. When $x < \frac{2}{3}$, the tangent will intersect the *x*-axis at the negative value. Therefore it will not tends towards the roots.
9(a)	 As the circular gear rotates by an angle of θ, $\angle OCP = \theta$ $\Rightarrow \angle CPQ = 2\theta$ (Ext. angle = 2 x int. opp. angles in a Δ) $CQ = a\sin 2\theta$ $PQ = a\cos 2\theta$ $\therefore x = OR - PQ = 2a\cos\theta - a\cos 2\theta$ $y = CR - CQ = 2a\sin\theta - a\sin 2\theta$
9(b)	

9(c)	$\dfrac{\mathrm{d}x}{\mathrm{d}\theta}=-2a\sin\theta+2a\sin 2\theta$ $\dfrac{\mathrm{d}y}{\mathrm{d}\theta}=2a\cos\theta-2a\cos 2\theta$ $\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2=\left(-2a\sin\theta+2a\sin 2\theta\right)^2$ $=4a^2\left(\sin 2\theta-\sin\theta\right)^2$ $=4a^2\left(2\cos\dfrac{3}{2}\theta\sin\dfrac{1}{2}\theta\right)^2$ $=16a^2\left(\cos\dfrac{3}{2}\theta\sin\dfrac{1}{2}\theta\right)^2$ $\left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2=\left(2a\cos\theta-2a\cos 2\theta\right)^2$ $=4a^2\left(\cos\theta-\cos 2\theta\right)^2$ $=4a^2\left(-2\sin\dfrac{3}{2}\theta\sin\left(-\dfrac{1}{2}\theta\right)\right)^2$ $=16a^2\left(\sin\dfrac{3}{2}\theta\sin\dfrac{1}{2}\theta\right)^2$ $\therefore\sqrt{\left(\dfrac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\dfrac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}$ $=\sqrt{16a^2\left(\cos\dfrac{3}{2}\theta\sin\dfrac{1}{2}\theta\right)^2+16a^2\left(\sin\dfrac{3}{2}\theta\sin\dfrac{1}{2}\theta\right)^2}$ $=4a\sqrt{\left(\cos\dfrac{3}{2}\theta\sin\dfrac{1}{2}\theta\right)^2+\left(\sin\dfrac{3}{2}\theta\sin\dfrac{1}{2}\theta\right)^2}$ $=4a\sqrt{\left(\sin\dfrac{1}{2}\theta\right)^2\left[\left(\cos\dfrac{3}{2}\theta\right)^2+\left(\sin\dfrac{3}{2}\theta\right)^2\right]}$ $=4a\sin\dfrac{\theta}{2}\qquad\left(0\le\dfrac{\theta}{2}\le\pi,\ \sin\dfrac{\theta}{2}\ge 0\right)$

Alternative Method:

$$\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}$$

$$=\sqrt{\left(-2a\sin\theta+2a\sin 2\theta\right)^2+\left(2a\cos\theta-2a\cos 2\theta\right)^2}$$

$$=\sqrt{4a^2\begin{pmatrix}\sin^2\theta-2\sin\theta\sin 2\theta+\sin^2 2\theta\\+\cos^2\theta-2\cos\theta\cos 2\theta+\cos^2 2\theta\end{pmatrix}}$$

$$=\sqrt{4a^2\left(2-2\sin\theta\sin 2\theta-2\cos\theta\cos 2\theta\right)}$$

$$=\sqrt{4a^2\left(2-2\left(\sin\theta\sin 2\theta+\cos\theta\cos 2\theta\right)\right)}$$

$$=\sqrt{4a^2\left(2-2\cos\theta\right)}$$

$$=2a\sqrt{2\left(1-\cos\theta\right)}$$

$$=2a\sqrt{2\left(2\sin^2\frac{\theta}{2}\right)}$$

$$=4a\sin\frac{\theta}{2}\qquad\left(0\le\frac{\theta}{2}\le\pi,\ \sin\frac{\theta}{2}\ge 0\right)$$

Length of path

$$=\int_0^{2\pi}\sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}\theta}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}\theta}\right)^2}\ \mathrm{d}\theta$$

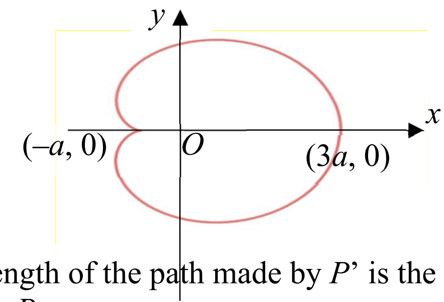
$$=4a\int_0^{2\pi}\left(\sin\frac{1}{2}\theta\right)\mathrm{d}\theta$$

$$=4a\left[-2\cos\frac{1}{2}\theta\right]_0^{2\pi}$$

$$=-8a\left[\cos\pi-\cos 0\right]$$

$$=-8a\left(-1-1\right)$$

$$=16a$$

9(d)	The comment is incorrect. When $\theta=\pi$, the position of point *P* will be $(-3a,0)$, and the position of point *P'* will be $(-a,0)$. Hence the path of *P'* is the path of *P* rotating by π radians as shown below.  Hence, the length of the path made by *P'* is the same as the length of the path made by *P*.
10(a)	$u_2=20+\frac{1}{7}(m-u_1-20)$ $u_k=10k+\frac{1}{7}(m-u_1-u_2-u_3-...-u_{k-1}-10k)$
10(b)	$u_{k+1}=10(k+1)+\frac{1}{7}\left(m-u_1-u_2-u_3-...-u_{k-1}-u_k-10(k+1)\right)$ $u_{k+1}-u_k=10+\frac{1}{7}(-u_k-10)$ $u_{k+1}=\frac{6}{7}u_k+\frac{60}{7}$
10(c)	$u_k=\frac{6}{7}u_{k-1}+\frac{60}{7}$ $u_k=\frac{6}{7}\left(\frac{6}{7}u_{k-2}+\frac{60}{7}\right)+\frac{60}{7}$ $=\left(\frac{6}{7}\right)^2u_{k-2}+\left(\frac{6}{7}\right)\frac{60}{7}+\frac{60}{7}$ $=\left(\frac{6}{7}\right)^3u_{k-3}+\left(\frac{6}{7}\right)^2\frac{60}{7}+\left(\frac{6}{7}\right)\frac{60}{7}+\frac{60}{7}$ $\vdots$ $=\left(\frac{6}{7}\right)^{k-1}u_1+\left(\frac{6}{7}\right)^{k-2}\frac{60}{7}+...+\left(\frac{6}{7}\right)^2\frac{60}{7}+\left(\frac{6}{7}\right)\frac{60}{7}+\frac{60}{7}$

	$u_k=\left(\frac{6}{7}\right)^{k-1}u_1+\dfrac{\frac{60}{7}\left[1-\left(\frac{6}{7}\right)^{k-1}\right]}{1-\frac{6}{7}}$ $=\left(\frac{6}{7}\right)^{k-1}\left(\frac{60}{7}+\frac{m}{7}\right)+60\left[1-\left(\frac{6}{7}\right)^{k-1}\right]$ $=\left(\frac{6}{7}\right)^{k-1}\left(\frac{60}{7}+\frac{m}{7}-60\right)+60$ $=\frac{1}{6}\left(\frac{6}{7}\right)^{k}(m-360)+60$
10(d)	Let T_n be the amount of cash prize given in *n* days. $T_n=\sum_{k=1}^{n}\left[\frac{1}{6}\left(\frac{6}{7}\right)^{k}(4000-360)+60\right]$ $=\frac{3640}{6}\sum_{k=1}^{n}\left(\frac{6}{7}\right)^{k}+60n$ $=\left(\frac{3640}{6}\right)\dfrac{\frac{6}{7}\left(1-\left(\frac{6}{7}\right)^{n}\right)}{1-\frac{6}{7}}+60n$ $=3640\left(1-\left(\frac{6}{7}\right)^{n}\right)+60n$ $3640\left(1-\left(\frac{6}{7}\right)^{n}\right)+60n\le 4000$ From GC, $n=13,\quad T_n=3929.5$ $n=14,\quad T_n=4059.4$ $\therefore n\le 13$ Hence it is not possible to host the event for 2 weeks.
10(e)	$\frac{1}{6}\left(\frac{6}{7}\right)^{k}(4000-360)+60<\frac{5}{100}(4000)$ $\left(\frac{6}{7}\right)^{k}<\frac{3}{13}$ $k>9.51$ $\therefore\left\{k\in\mathbb{Z}^{+}\mid 10\le k\le 13\right\}$

10(f)	$(m-360)\left[1-\left(\frac{6}{7}\right)^{n}\right]+60n \le m$ Let $n=14$ $(m-360)\left[1-\left(\frac{6}{7}\right)^{14}\right]+60(14) \le m$ $m\left[1-\left(\frac{6}{7}\right)^{14}\right]-m \le -60(14)+360\left[1-\left(\frac{6}{7}\right)^{14}\right]$ $-m\left(\frac{6}{7}\right)^{14} \le -480-360\left(\frac{6}{7}\right)^{14}$ $m \ge \dfrac{480+360\left(\frac{6}{7}\right)^{14}}{\left(\frac{6}{7}\right)^{14}}$ $m \ge 4514.285045$ $m \ge 4514.29$