1 On the same axes, sketch the graphs of $y=|x-a|$ and $y=|x-b|$, where a and b are constants such that $0<a<b$. You should show clearly the axial intercepts of both graphs. Hence solve the inequality $|x-a|<|x-b|$. [5]

2 **(a)** Without using a calculator, solve the inequality $\dfrac{17-5x}{x^2+5x-14}\geq -1$. [3]

(b) Hence solve the inequality $\dfrac{17x-5}{\frac{1}{x}+5-14x}\geq -1$. [3]

3 The non-zero vectors **a**, **b** and **c**, where **a**, **b** are non-parallel vectors, satisfy the equation $\mathbf{c}\times 3\mathbf{b}=5\mathbf{a}\times\mathbf{c}$.

(a) Determine, with clear reasons, the relationship between the vectors **c** and $5\mathbf{a}+3\mathbf{b}$. [4]

Referred to the origin *O*, it is given further that the vectors **a**, **b** and **c** are the position vectors of the points *A*, *B* and *C* respectively. Point *D* lies on the line segment *AB* such that it divides *AB* in the ratio $\lambda:1-\lambda$, where $0<\lambda<1$.

(b) Write down an expression for **d**, the position vector of point *D* in terms of **a** and **b**. [1]

The point *D* also lies on the line segment *OC*.

(c) Determine the exact value of λ. [3]

4 **(a)** The function f is defined by $\mathrm{f}:x\mapsto(x+3)^2-1$ for $x\in\mathbb{R},\ x\geq -3$.

Find $\mathrm{f}^{-1}(x)$, stating the domain of f^{-1}. [3]

(b) The function g is defined by

$$\mathrm{g}(x)=\begin{cases} x-1 & \text{for } 0<x<1, \\ x^2-1 & \text{for } 1\leq x\leq 2. \end{cases}$$

and $\mathrm{g}(x)=\mathrm{g}(x+2)$ for all values of *x*.

Sketch the graph of g for $-1\leq x<3$. Hence state the range of g. [3]

(c) The domain of g is now restricted to $-1\leq x<1$.

(i) Explain why the composite function $\mathrm{f}^{-1}\mathrm{g}$ exists. [1]

(ii) Find $\mathrm{f}^{-1}\mathrm{g}(x)$ in the form

$$\mathrm{f}^{-1}\mathrm{g}(x)=\begin{cases} \mathrm{p}(x) & \text{for } -1<x\leq 0, \\ \mathrm{q}(x) & \text{for } 0<x<1. \end{cases}$$

where $\mathrm{p}(x)$ and $\mathrm{q}(x)$ are expressions in terms of *x* to be determined. [3]

5 The curve C has equation

$$y=\frac{2x-6}{x^2+2x-3}.$$

(a) State the equations of the asymptotes of C. [2]

(b) Without using a calculator, find the range of values that y can take. [4]

(c) Sketch the graph of C, stating the equations of any asymptotes, the coordinates of the points where the curve crosses the axes and the stationary point(s). [4]

(d) Describe one transformation that will transform the curve C onto the curve $y=\frac{2x-8}{x^2-4}$. [1]

6 A curve C has parametric equations

$$x=\sqrt{23}\cos\left(t+\frac{\pi}{6}\right),\qquad y=2\sin t,\quad \text{where } 0\le t\le\pi .$$

(a) Find the exact Cartesian equation of l, the normal to C at the point $P\left(0,\sqrt{3}\right)$. [4]

(b) Let Q be a point on C such that the x-coordinate of Q is the minimum x-coordinate of C. The tangent to C at Q intersects l at point R. Find the exact coordinates of R. [3]

(c) Without using a graphing calculator, show that C has only one stationary point, and determine the nature of this stationary point. [4]

7 The line l_1 has equation $\mathbf{r}=11\mathbf{i}-3\mathbf{j}-\mathbf{k}+\lambda(\mathbf{i}-\mathbf{j}+2\mathbf{k})$, where λ is a parameter. The point A has coordinates $(2,0,-1)$.

(a) The plane p contains the line l_1 and the point A. Find a cartesian equation of the plane p. [3]

(b) Find the position vector of the point A', the reflection of the point A in the line l_1. [4]

The line l_2 passes through the point A, and is perpendicular to the line l_1. Planes p and q are perpendicular planes such that p and q meet at line l_2.

(c) Find a vector equation of the line l_2. [2]

(d) The plane Π is such that Π contains the point $(11,-3,-1)$ and is parallel to q. Find the perpendicular distance between Π and q. [2]

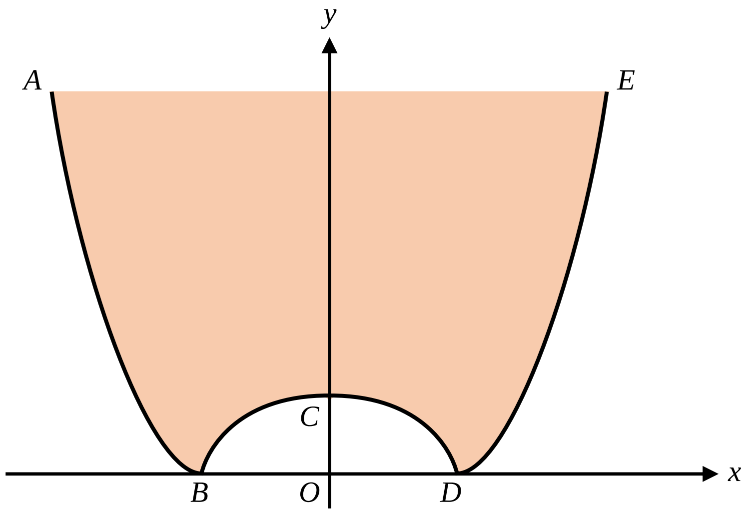
8 **(a)** A sequence $u_1, u_2, u_3, \ldots$ is such $u_{n+1} = 3u_n + An + B$, where A, B are constants and $n \geq 1$. Given that $u_1 = 2,\ u_2 = 5.5$ and $u_3 = 17.5$, find A and B, and find u_4. [4]

(b) It is given that $\mathrm{f}(r) = 2r^3 + 3r^2 + 4r + 5$.

(i) By considering $\mathrm{f}(r) - \mathrm{f}(r-1)$, show that $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$. [5]

(ii) Given that $\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$, show that $\sum_{r=1}^{n} \mathrm{f}(r) = \frac{n(n+1)(n^2+3n+5)}{2} + Cn$, where C is a constant to be found. [3]

9 The diagram below, not drawn to scale, shows the vertical cross-section of a circular coffee cup with an indented base, passing through the axis of symmetry $x = 0$. The cup is filled with espresso to its brim. The curves *AB* and *DE*, at the body, form parts of the curve with equation $y = x^2 - 4$, while the indentation at the base *BCD*, forms part of the curve with equation $y = 2\cos\left(\frac{\pi x^2}{8}\right)$. The units of x and y are centimetres. The diameter of the rim is 7 cm and the cup has negligible thickness.

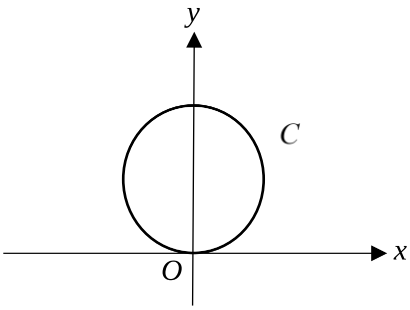


(a) Find the area of the cross-section of the cup shown in the diagram. [3]

(b) When the cup is empty, the indentation at the base can be viewed from the top. In order to conceal the indentation at the base, it is found that at least k cm^3 of espresso should be poured into the cup. Find the exact value of k. [6]

(c) When k cm^3 of espresso is already present in the cup, 14π cm^3 of hot water is further added to make a cup of Americano. Find the radius of the top surface of the Americano. [4]

10 A spherical container of radius 5 m is formed by rotating the following circle C about the y-axis.



The container has negligible thickness, and the circle C passes through the origin O.

(a) State a cartesian equation of C. [1]

Initially the spherical container is completely filled with water. Two engineers are calculating the time needed for the container to be completely drained from a small circular hole at the bottom. The volume of water in the container at time t seconds is denoted by V m^3.

[The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.]

(b) The first engineer proposes that the rate of change of V with respect to t is a constant k.

(i) Write down a differential equation relating V, t and k. [1]

(ii) Determine, with justification, the sign of k. [1]

(iii) Find V in terms of t and k, leaving your answer in exact form. [2]

(c) The second engineer argues that the rate at which water flows out from the hole will be at its greatest in the beginning, and decreases as the depth of water in the container decreases. He suggests using Torricelli's law, which says that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\alpha\sqrt{20h},$$

where α m^2 is the area of the circular hole at the bottom of the container, and h m is the depth of water in the container at time t seconds. The radius of the hole at the bottom of the container is found to be constant at 1 cm.

(i) Show that $\left(10h^{\frac{1}{2}} - h^{\frac{3}{2}}\right)\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{\sqrt{5}}{5000}$. [4]

(ii) Hence find the numerical value of t when the container is completely drained. [4]