<u>**Check your Understanding (Graphing)**</u>

<u>**Section 1:Asymptotes**</u>

<u>**MUTIPLE CHOICE.**</u>
<u>**Choose the one alternative that best completes the statement or answers the question.**</u>

1. What is the vertical asymptote(s) of $h(x)=\dfrac{1}{x-4}$?

(a) None
(b) $x=4$
(c) $x=-4$
(d) $x=1$

(b)

2. What is the horizontal asymptote(s) of $h(x)=2+\dfrac{1}{x-4}$?

(a) None
(b) $y=4$
(c) $y=2$
(d) $x=4$ (c)

3. What is the vertical asymptote(s) of $h(x)=\dfrac{x}{x-4}$?

(a) None
(b) $x=4$
(c) $x=-4$
(d) $x=1$ (b)

4. What is the horizontal asymptote(s) of $h(x)=\dfrac{x}{x-4}$?

(a) None
(b) $y=4$
(c) $y=2$
(d) $y=1$ (d)

5. What is the horizontal asymptote(s) of $\mathrm{h}(x)=\dfrac{32x^2}{8x^2-5}$?

(a) None
(b) $y=4$
(c) $y=5$
(d) $y=\sqrt{5}$ (b)

6. What is the vertical asymptote(s) of $\mathrm{h}(x)=\dfrac{x-5}{(x-9)(x+8)}$?

(a) $x=9, x=-8$
(b) $x=5$
(c) $x=-9, x=8$
(d) $x=-5$ (a)

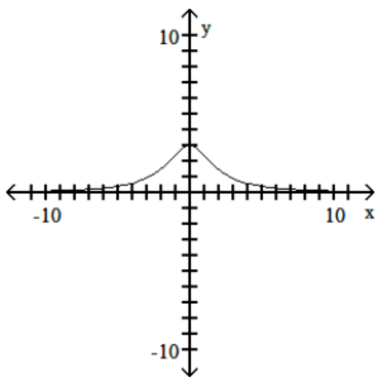
7. What is the horizontal asymptote(s) of $\mathrm{h}(x)=\dfrac{x^2+3x-4}{x-4}$?

(a) None
(b) $y=4$
(c) $y=2$
(d) $y=-3$ (d)

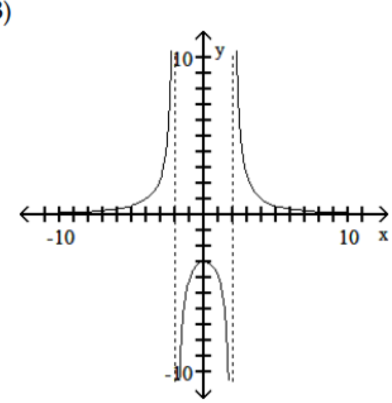
Match the equation with the appropriate graph?

8. $f(x) = \dfrac{2x^2}{x^2 - 4}$

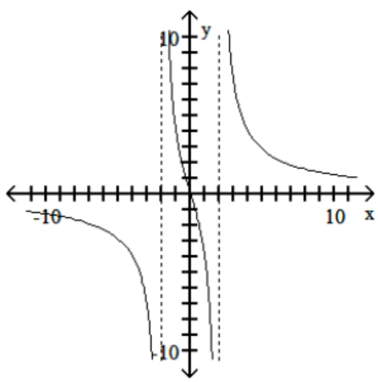
A)



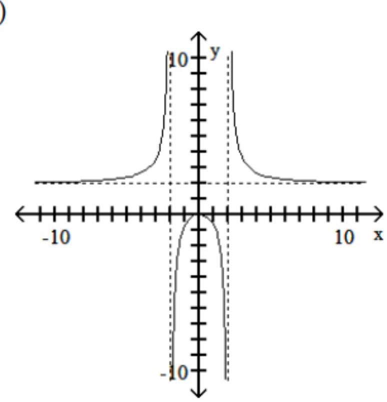
B)



C)



D)



(D)

Section 2:Stationary points

9. **RI Promo 8865/2018/Q6 (part)**

The curve C has equation $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = 6x - 3x^2 - 4x^3$.

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. Hence find the coordinates of the stationary points on the curve.

(ii) Use a non-calculator method to determine the nature of each of the stationary points.

Solution:

(i)

$\frac{\mathrm{d}y}{\mathrm{d}x} = 6 - 6x - 12x^2$

For stationary points, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

$\Rightarrow 6 - 6x - 12x^2 = 0 \Rightarrow x = -1$ or $\frac{1}{2}$

When $x = -1$, $y = -5$

When $x = \frac{1}{2}$, $y = \frac{7}{4}$

$\therefore$ Coordinates of the stationary points are $(-1, -5)$ and $\left(\frac{1}{2}, \frac{7}{4}\right)$.

(ii)

$\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -6 - 24x$

When $x = -1$, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 18 > 0$ (minimum point)

When $x = \frac{1}{2}$, $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = -18 < 0$ (maximum point)

$\therefore$ $(-1, -5)$ is a minimum point and $\left(\frac{1}{2}, \frac{7}{4}\right)$ is a maximum point.

Section 3: Graphing Problems

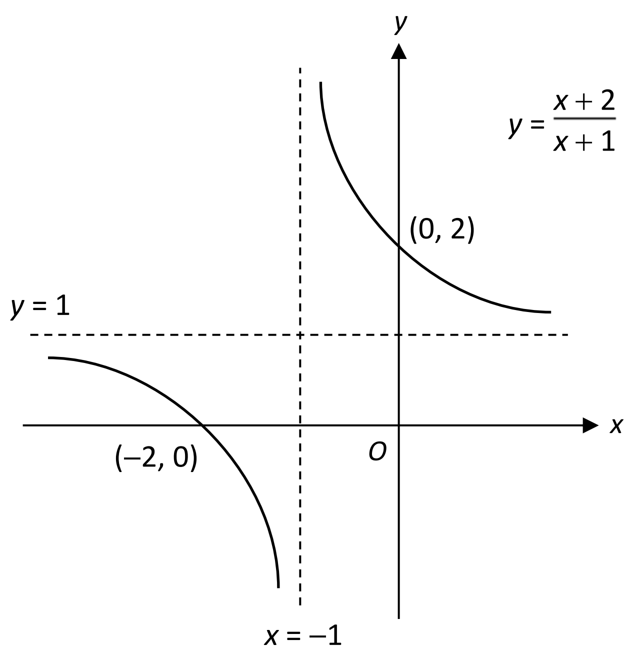
10. **RIJC Promo 8865/2015/Q2**

Sketch the curve with equation

$$y = \frac{x+2}{x+1},$$

stating the equations of any asymptotes and the coordinates of the points where the curve crosses the axes.

Solution:



11. **MIJC Promo 8865/2015/Q5**

(i) Sketch the graph of $y = \dfrac{1}{x-4}$. [2]

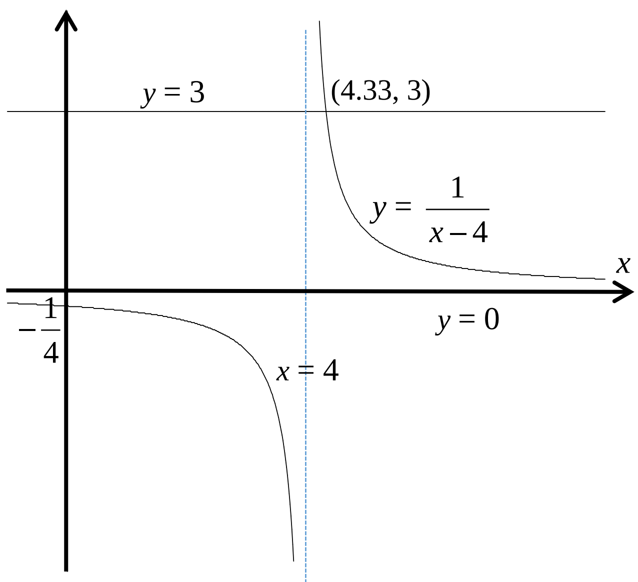
(ii) By drawing an additional graph in (i), solve the inequality $\dfrac{1}{x-4} - 3 \le 0$. [2]

(iii) Hence solve the inequality $\dfrac{1}{e^x - 4} \le 3$. [3]

Answer: (ii) $x < 4$ or $x \ge 4.33$. (3 s.f.)

(iii) $x < \ln 4$ or $x \ge 1.47$. (3 s.f.)

Solution:

11(i)	
(ii)	$\dfrac{1}{x-4} - 3 \le 0 \Rightarrow \dfrac{1}{x-4} \le 3$ From graph, $x < 4$ or $x \ge 4.33$. (3 s.f.)
(iii)	$\dfrac{1}{e^x-4} \le 3 \Rightarrow \dfrac{1}{e^x-4} - 3 \le 0$ $\Rightarrow e^x < 4$ or $e^x \ge 4.3333$ $\Rightarrow x < \ln 4$ or $x \ge 1.47$. (3 s.f.)

12. **MIJC Promo 8865/2015/6**

The curve C has equation $y = \dfrac{bx+3}{x+a}$, where a and b are constants. It is given that x = 1 and y = 2 are vertical and horizontal asymptotes of C respectively.

(i) State the values of a and b. [2]

(ii) Given further that C intersects the y-axis at (0, –3), find the coordinates of the point of intersection with the x-axis. [2]

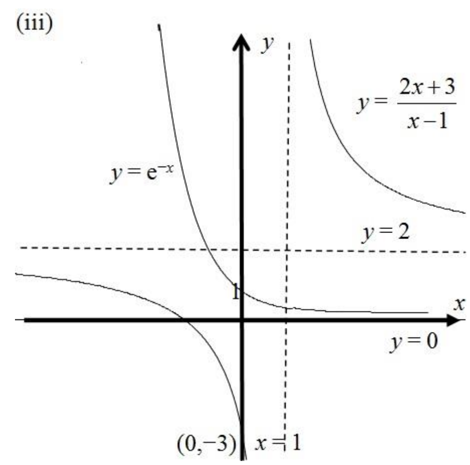
(iii) Sketch the graph of $y = \dfrac{bx+3}{x+a}$, showing clearly all the asymptotes and intersections with both axes. [3]

(iv) By adding a suitable graph to the sketch in (iii), determine the number of solutions of the equation

$$\ln\left(\frac{bx+3}{x+a}\right) = -x$$ [3]

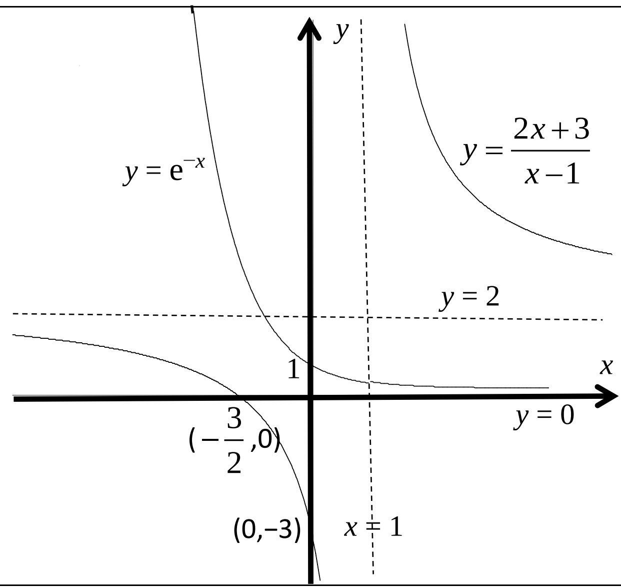
Answer: (i) a = –1, b = 2

(ii) $(-\frac{3}{2}, 0)$



(iv) 0

Solution:

12(i)	$a = -1$, $b = 2$.
12(ii)	When $y = 0$, $0 = \frac{2x+3}{x-1} \Rightarrow x = -\frac{3}{2}$. Hence, required point is $(-\frac{3}{2}, 0)$.
12(iii)	
12(iv)	$\ln\left(\frac{bx+3}{x+a}\right) = -x \Rightarrow \frac{bx+3}{x+a} = e^{-x}$ From the graph in **(iii)**, since the graph of $y = e^{-x}$ and the graph of $y = \frac{bx+3}{x+a}$ do not intersect, the required number of solutions = 0.

13. **RVHS Promo 8865/2018/Q4**

(i) Sketch the graph of $y = \dfrac{3x+5}{x-4}$, labelling clearly the equations of any asymptotes and the exact coordinates of any points where the curve crosses the axes.

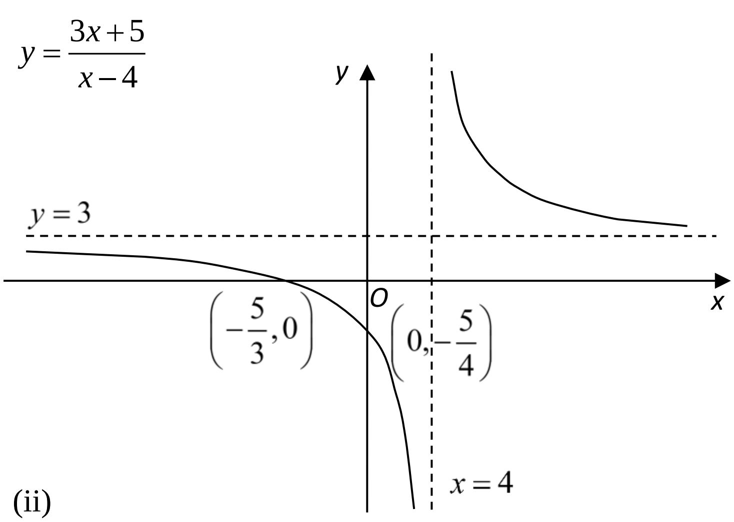
(ii) Hence, find the exact solution(s) of the inequality $\dfrac{3x^2+5}{x^2-4} > 0$.

(iii) By adding a suitable curve to the sketch in part **(i)**, solve $e^{\frac{3x+5}{x-4}} \le x+3$.

Answer: $x < -2$ or $x > 2$; $-1.88 \le x < 4$ or $x \ge 33.1$

Solution:

(i)



(ii)

Solving $\dfrac{3x+5}{x-4} > 0$ from the graph in (i), we have

$x < -\dfrac{5}{3}$ or $x > 4$

Replacing x with x^2, we have

$x^2 < -\dfrac{5}{3}$ (rej) or $x^2 > 4$

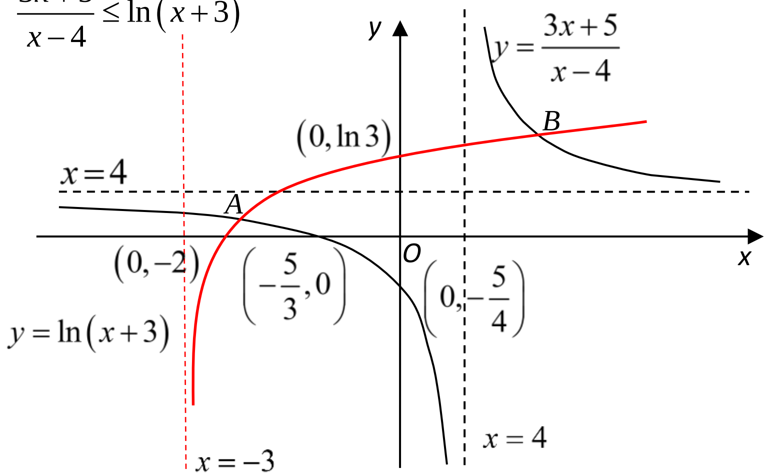
$x < -2$ or $x > 2$

(iii)

$$e^{\frac{3x+5}{x-4}} \le x+3$$

$$\ln e^{\frac{3x+5}{x-4}} \le \ln(x+3)$$

$$\frac{3x+5}{x-4} \le \ln(x+3)$$



Intersections : A(-1.88, 0.110) and B(33.1, 3.59)

$-1.88 \le x < 4 \quad \text{or} \quad x \ge 33.1$

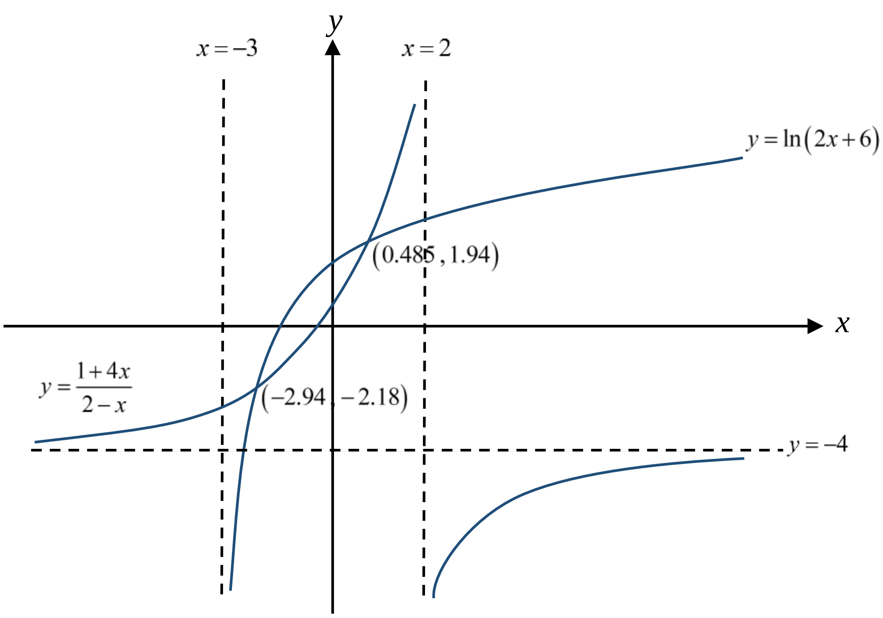
14. **TJC Promo 8865/2018/Q5**

Sketch the curves $y=\dfrac{1+4x}{2-x}$ and $y=\ln(2x+6)$ on the same diagram, stating clearly the equation of any asymptotes and coordinates of any intersection points.

Hence solve the inequality $\dfrac{1+4x}{2-x}\geq \ln(2x+6)$.

Answer: $-3<x\leq -2.94$ or $0.485\leq x<2$

Solution:



$$\frac{1+4x}{2-x}\geq \ln(2x+6)$$

$-3<x\leq -2.94$ or $0.485\leq x<2$