TANJONG KATONG GIRLS' SCHOOL
PRELIMINARY EXAMINATION
SECONDARY FOUR EXPRESS

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

MATHEMATICS 4052/02

Paper 2 **7 August 2024**

2 hour 15 minutes

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE ON ANY BARCODES.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total of the marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer to three significant figures. Gives answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

For Examiner's use

Mathematical Formulae

Compound interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi rl$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

TURN OVER FOR QUESTION 1

1 **(a)** Solve the equation $\frac{3-2x}{7} = \frac{1}{3}(2x-1)$.

Answer $x =$ ……... [2]

(b) Simplify

(i) $\frac{25a^2 - 9b^2}{15a - 10ab + 6b^2 - 9b}$

Answer …… [3]

(ii) $\frac{3c}{4ab} \div \frac{c^3}{12a^2b}$.

Answer …… [2]

(c) **(i)** Solve the equation $-x^2 + 9x - \frac{7}{2} = 0$ by **completing the square**. Give your solutions correct to two decimal places.

Answer $x =$ ……. or …… [3]

(ii) Explain why $-x^2 + 9x - \frac{7}{2} = 18$ has no real solution.

……..

……..

……..

…….. [2]

2 (a) In a particular month, 21 people took a driving test.

One of the 21 people is selected at random.

The probability that it is a man who passed the test is $\frac{1}{7}$.

Two of the 21 people are selected at random.

The probability that they are both women who failed the test is $\frac{1}{10}$.

Complete the table of information about the 21 people who took the test on that particular month.

	Passed the test	**Failed the test**
Men		**9**
Women		

[4]

(b) $\varepsilon = \{x : x \text{ is a student in a class}\}$
$A = \{x : x \text{ is a student who listens to pop music}\}$
$B = \{x : x \text{ is a student who listens to classical music}\}$

It is given that $\text{n}(\varepsilon) = 35$, $\text{n}(A) = 22$, $\text{n}(B) = 14$ and $\text{n}(A \cap B') = 12$.

(i) Describe in words the set $A \cap B'$.

……...

……... [1]

(ii) Find the number of students who listen to both pop and classical music.

Answer ……..... [1]

(iii) Find the number of students who listen to classical music only.

Answer ……... [1]

(iv) Find the number of students who listen to neither pop nor classical music.

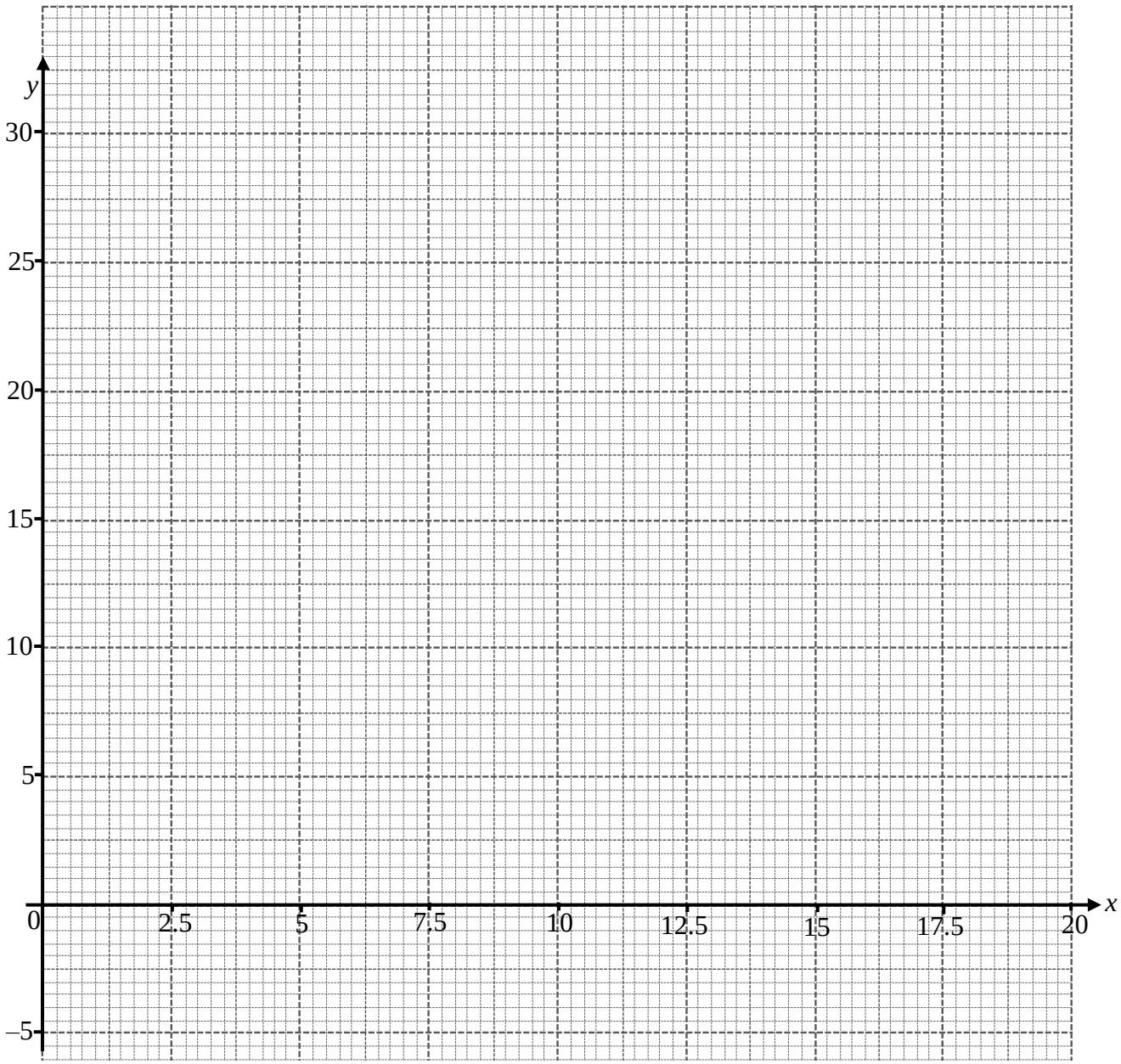
Answer ……... [1]

3 **(a)** Complete the table of values for $y = 2x + \frac{20}{x} - 15$.

x	1	2.5	5	10	12.5	15	20
y	7		–1	7	11.6	16.3	26

[1]

(b) On the grid, draw the graph of $y = 2x + \frac{20}{x} - 15$ for $0 < x \leq 20$.



[3]

(c) Using the graph, state the range of values of x for which $2x + \frac{20}{x} < 22$.

Answer …… [2]

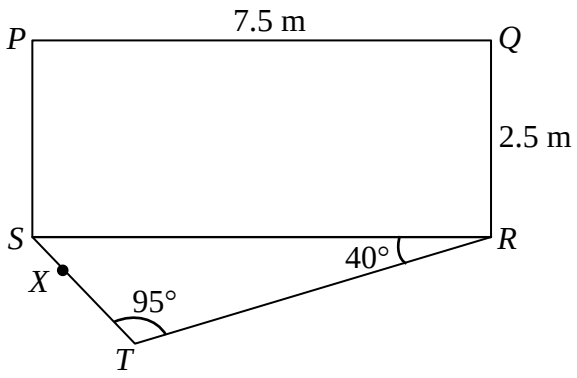
(d) By drawing a tangent, find the gradient of the curve at the point where $x = 4$.

Answer …… [2]

(e) By drawing a suitable straight line on the grid, find the solutions of the equation $x^2 - 13x + 20 = 0$.

Answer $x =$ …….. or ……. [2]

4 In the diagram, *PQRS* is a vertical rectangular wall of height 2.5 m and length 7.5 m.
T is a point on level ground from the base of the wall *RS*.
$\angle STR = 95°$ and $\angle SRT = 40°$.
X is a point on *ST* such that *SX* : *XT* is 1 : 3.



(a) Calculate the length of *SX*.

Answer ……..... m [3]

(b) Calculate the length of *XR*.

Answer …… m [3]

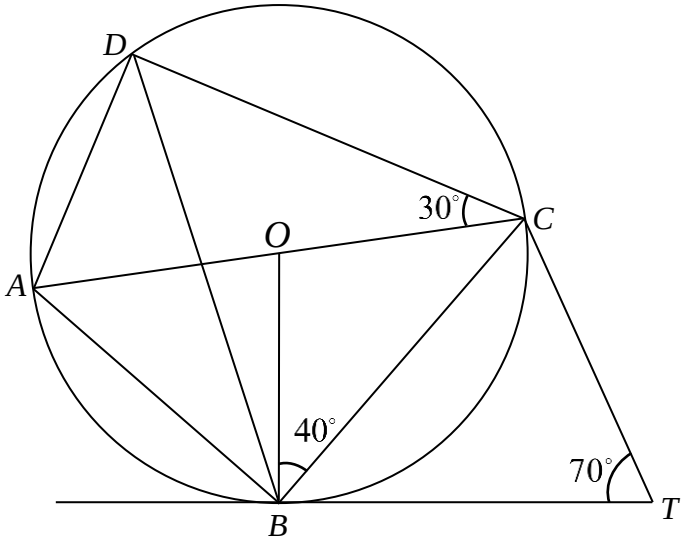
(c) Calculate the length of PT.

Answer …….. m [1]

(d) A bird sat on the ledge of the wall along PQ such that its angle of elevation θ from X is the largest. Find θ.

Answer ……..……. ^{o} [4]

5 In the diagram, O is the centre of the circle through A, B, C, D and TB is the tangent at B. AC is the diameter of the circle, and the length of the minor arc BAD is 11 cm.
Given that $\angle OBC = 40^\circ$, $\angle ACD = 30^\circ$ and $\angle BTC = 70^\circ$.



(a) Find, giving a reason for each step of your working,

(i) $\angle ABD$,

Answer ……..…… [1]

(ii) $\angle BAD$.

Answer ……..…… [2]

(iii) Taking $\pi = \frac{22}{7}$, calculate the radius of the circle.

Answer ……..…… cm [2]

(b) **(i)** Show that triangle *BCD* is similar to triangle *CTB*. Give a reason for each statement you make.

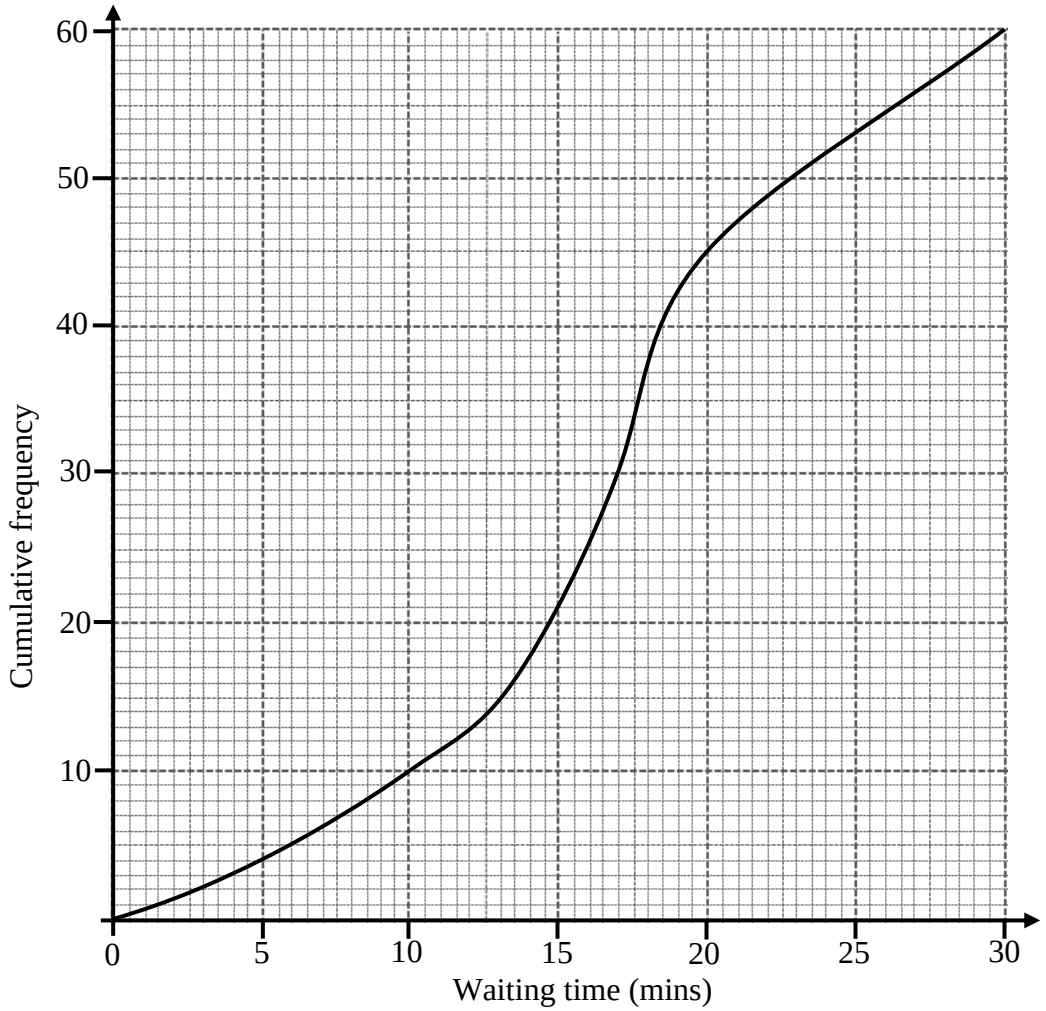
Answer

[4]

(ii) **Hence**, express CT in terms of x and y given that $BC = x$ cm and $BD = y$ cm.

Answer ……..…….. [2]

6 The cumulative frequency graph shows distribution of the waiting times before being served, in minutes, of 60 customers at restaurant *A* on a particular day.



(a) Use the curve to estimate

(i) the median waiting time,

Answer ……..…… min [1]

(ii) the interquartile range,

Answer ……..…… min [1]

(iii) the 35th percentile of the waiting time.

Answer ……..……. min [1]

(b) Given that 20% of the customers waited more than x minutes to be served, find the value of x.

Answer $x =$ ……..…… [1]

(c) The data below represents the waiting times of 10 customers from restaurant *B* on the same day.

10, 12, 14, 14, 15, 17, 17, 18, 22, 28

Calculate

(i) the median waiting time,

Answer ……..……. min [1]

(ii) the interquartile range.

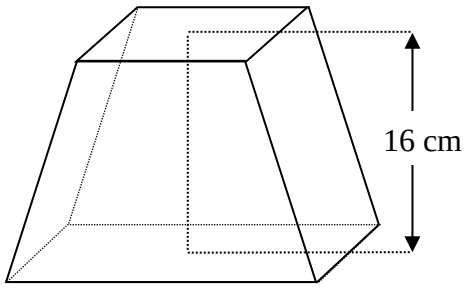
Answer ……..……. min [1]

(d) The owner of restaurant *B* claims that his restaurant is more efficient and consistent in their service compared to restaurant *A* based on the data given. Give a reason why this is not a fair comparison.

……

…… [1]

7 (a)



The figure shows a solid frustum of height 16 cm cut from right pyramid with square base. The horizontal top surface has an area of 5 cm^2 while the base area is 125 cm^2. The portion that is removed is h cm high.

(i) Show that $h = 4$.

Answer

[2]

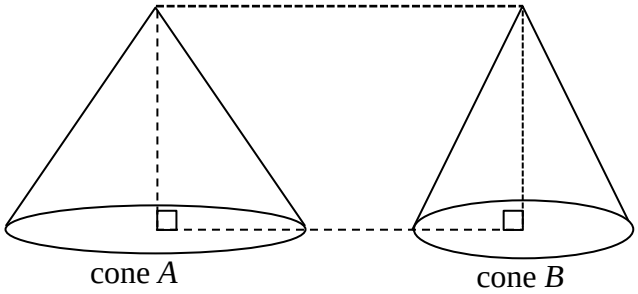
(ii) Calculate the volume of the frustum.

Answer …… cm^3 [3]

(iii) Find the percentage of the right pyramid that has been removed.

Answer …….. % [2]

(b) Two solid cones have the same height but the radius of cone *A* is 1.5 times of cone *B*. Given that the volume of cone *B* is 240 cm^3.



Find the volume of cone *A*.

Answer ……... cm^3 [3]

8 **(a)** $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 9 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$, $\overrightarrow{CD} = \begin{pmatrix} d \\ 4 \end{pmatrix}$.

(i) Given that $\overrightarrow{CD}$ is parallel to $\overrightarrow{AB}$, find the value of d.

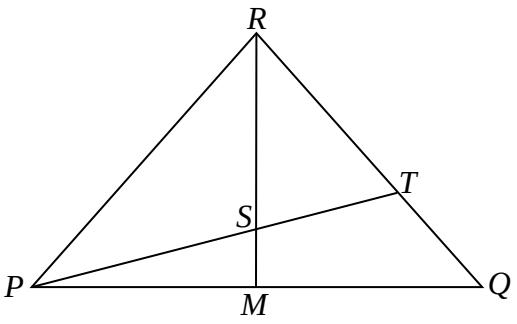
Answer $d =$ ……... [2]

(ii) Find $\left|\overrightarrow{AC}\right|$.

Answer …….. units [2]

TURN OVER FOR QUESTION 8(b)

(b)



PQR is a triangle and M is the mid-point of PQ. It is given that $\overrightarrow{PQ} = \mathbf{a}$, $\overrightarrow{PR} = \mathbf{b}$, $\overrightarrow{QT} = h\overrightarrow{QR}$ and $\overrightarrow{RS} = k\overrightarrow{RM}$, where k and h are constants.

(i) Express $\overrightarrow{PS}$ in terms of k, **a** and **b**.

Answer ……..……... [2]

(ii) Express $\overrightarrow{PT}$ in terms of h, **a** and **b**.

Answer ……..……... [2]

(iii) **Hence** show that $hk + 2 = 2(h + k)$ if P, S and T are collinear.

Answer

[1]

(iv) Given that $h = \frac{1}{3}$ and $k = \frac{4}{5}$, find the ratio of $PS : ST$.

Answer ……. : ……. [2]

9 A factory prints posters for corporate events. The prices of printing posters of different sizes are listed below:

Type of banners	Size in inches	Prices (per copy)	Additional cost
Bulletin poster	11×17	\$2.00	5% of the total cost if printing a total area of more than 120 m^2
Mini poster	12×18	\$2.50	
Medium poster	18×24	\$3.50	
Large poster	24×36	\$5.00	

The factory has the following guidelines for operating the printers.

Guidelines
• A printer operates from 9.00 am to 4.00 pm every day. • A printer needs to be cooled for a period of 30 minutes after every 2 hours of printing. • A printer needs to be serviced if its rate of printing has slowed down to less than 60 m^2 a day.

(a) Given that 1 inch = 2.54 cm, find the area of a bulletin poster in m^2.

Answer ……. m^2 [1]

(b) Mr Tan wants to print 1200 bulletin posters for an event. Calculate how much does it cost.

Answer \$ ……. [2]

(c) The factory uses 2 printers to print Mr Tan's order for an entire day. The older printer takes 15 seconds more to print one bulletin poster compared to the newer printer and the rates of printing of the 2 printers are constant throughout the day.

An operator of the printers claims that the older printer needs servicing. Do you agree? Justify your answers with clear working.

Answer

……

…… [7]

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Answer Key

Question			Answer		Question			Answer
1	**(a)**		$x = 0.8 / \frac{4}{5}$		**6**	**(a)**	**(i)**	17 min
	(b)	**(i)**	$\frac{5a+3b}{3-2b}$				**(ii)**	7 min
		(ii)	$\frac{9a}{c^2}$				**(iii)**	15 min
	(c)	**(i)**	$x = 8.59$ or 0.41			**(b)**		21.5 min
		(ii)	Since the maximum value of $y = -x^2 + 9x - \frac{7}{2}$ is $16.75 / \frac{67}{4} <$ 18. There is no solution when $y = 18$.			**(c)**	**(i)**	16 min
2	**(a)**		<table><tr><th></th><th>Passed</th><th>Failed</th></tr><tr><td>Men</td><td>3</td><td>9</td></tr><tr><td>Women</td><td>2</td><td>7</td></tr></table>				**(ii)**	4 min
	(b)	**(i)**	A set of students who only listen to pop music but not classical music.				**(iii)**	The sample sizes are not the same.
		(ii)	10		**7**	**(a)**	**(i)**	Use similar areas to show
		(iii)	4				**(ii)**	827 cm^3
		(iv)	9				**(iii)**	$\frac{4}{5}\%$ / 0.8%
3	**(a)**		–2			**(b)**		540 cm^3
	(b)		See Graph		**8**	**(a)**	**(i)**	$d = -\frac{4}{3}$
	(c)		$1 < x < 10$				**(ii)**	14.0 unit
	(d)		0.753			**(b)**	**(i)**	$\frac{1}{2}k\mathbf{a} + (1-k)\mathbf{b}$
	(e)		$x = 1.783$ or 11.217				**(ii)**	$(1-h)\mathbf{a} + h\mathbf{b}$
4	**(a)**		1.21 m				**(iii)**	Use parallel vectors and equal scalar multiple to show.
	(b)		6.70 m				**(iv)**	$PS : ST = 3 : 2$
	(c)		5.45 m		**9**	**(a)**		0.121 m^2
	(d)		71.1° (1 d.p)			**(b)**		$2520
5	**(a)**	**(i)**	30°			**(c)**		Since the older printer only printed 57.9 m^2 < 60 m^2. It will need to be serviced.
		(ii)	110°					
		(iii)	4.5					
	(b)	**(i)**	AA similarity test					
		(ii)	$\frac{x^2}{y}$					