Name	Index Number	Class

WOODGROVE SECONDARY SCHOOL

A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

O-LEVEL PRELIMINARY EXAMINATION 2024

LEVEL & STREAM : SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

SUBJECT (CODE) : MATHEMATICS (4052)

PAPER : 01

DATE (DAY) : 27 AUGUST 2024 (TUESDAY)

DURATION : 2 HOURS 15 MINUTES

Mark Scheme

READ THESE INSTRUCTIONS FIRST

Write your class, index number and name on the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** questions.

The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question and if the answer is not exact, give the answer correct to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value of 3.142.

DO NOT TURN OVER THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.

Student's Signature		**Parent's Signature**	
Date		**Date**	

90

This document consists of **20** printed pages including this cover page.
Setter : Mr Eric Bay

Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1+\frac{r}{100}\right)^n$$

Mensuration

Curved surface area of a cone = πrl

Surface area of a sphere = $4\pi r^2$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of triangle } ABC = \frac{1}{2}ab\sin C$$

Arc length = $r\theta$, where θ is in radians

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\sum fx}{\sum f}$$

$$\text{Standard Deviation} = \sqrt{\frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f}\right)^2}$$

Answer **all** the questions.

1 **(a)** Calculate $\dfrac{-11+\sqrt{(-11)^2-\dfrac{20}{11}}}{8\times 0.6}$.

Write your answer correct to 4 significant figures.

-0.01728 ----------B1

Answer …….. [1]

(b) Write your answer to **part (a)** in standard form.

-1.728×10^{-2} ----------B1

Answer …….. [1]

2 Expand and simplify $(2x+5)(3+4x)$.

$(2x+5)(3+4x)$

$=6x+8x^2+15+20x$ ----------M1

$=8x^2+26x+15$ ----------A1

Answer …….. [2]

3 Solve

(a) $3-4x=21$.

$3-4x=21$

$-4x=18$

$x=-\dfrac{9}{2}$ or $-4\dfrac{1}{2}$ or -4.5 ----------B1

Answer …….. [1]

(b) $2x<3x+1\le 13$,

$2x<3x+1\le 13$

$2x<3x+1$ $\quad$ $3x+1\le 13$

$-x<1$ $\quad$ $3x\le 12$

$x>-1$ $\quad$ $x\le 4$ ---------M1

$-1<x\le 4$---------A1

Answer …….. [2]

4 **(a)** Express 4312 as a product of its prime factors.

$2^3 \times 7^2 \times 11$ ----------B1

Answer …….. [1]

(b) Given $588 = 2^2 \times 3 \times 7^2$.

Find

(i) The largest integer which is a factor of both 588 and 4312.

196 ----------B1

Answer …….. [1]

(ii) The smallest integer which is the multiple of both 588 and 4312.

12936 ----------B1

Answer …….. [1]

(iii) The smallest integer value of *m* such that 4312*m* is a perfect square.

22 ----------B1

Answer …….. [1]

5 Given that the coordinates of point A is (–2,11) and point B is (5, –11).

Find

(a) length AB,

length $AB = \sqrt{(-11-11)^2 + (5-(-2))^2}$ ----------M1

$= \sqrt{533}$

$= 23.086$

$= 23.1$ (3sf) ----------A1

Answer …….. [2]

(b) equation of the straight-line AB.

gradient $m = \dfrac{-11-11}{5-(-2)}$

$= -\dfrac{22}{7}$ ----------M1

$11 = -\dfrac{22}{7}(-2) + c$

$11 = \dfrac{44}{7} + c$

$c = -\dfrac{33}{7}$ ----------M1

$y = -\dfrac{22}{7}x - \dfrac{33}{7}$ ----------A1

Answer …….. [3]

6 Simplify $\dfrac{9x^2-4}{12x^2-x-6}$.

$\dfrac{9x^2-4}{12x^2-x-6} = \dfrac{(3x-2)(3x+2)}{(3x+2)(4x-3)}$ ---------- M2

$= \dfrac{(3x-2)}{(4x-3)}$ ---------- A1

Answer …….. [3]

7 Solve $\frac{2}{x+1}-3=\frac{1}{2x-5}$.

$$\frac{2}{x+1}-3=\frac{1}{2x-5}$$
$$\frac{2-3(x+1)}{x+1}=\frac{1}{2x-5}$$
$$\frac{2-3x-3}{x+1}=\frac{1}{2x-5}$$
$$\frac{-3x-1}{x+1}=\frac{1}{2x-5}$$ ----------M1

$$(-3x-1)(2x-5)=x+1$$
$$-6x^2+15x-2x+5=x+1$$
$$-6x^2+13x-x+5-1=0$$
$$-6x^2+12x+4=0$$
$$3x^2-6x-2=0$$ ----------M1

$$x=\frac{6\pm\sqrt{(-6)^2-4(3)(-2)}}{2(3)}$$
$$=\frac{6\pm\sqrt{60}}{6}$$ ----------M1
$$=\frac{6+\sqrt{60}}{6} \text{ or } \frac{6-\sqrt{60}}{6}$$
$$=2.29 \text{ or } -0.291 \text{ (3sf)}$$ ----------A1

Answer …….. [4]

8 The table below shows a multiple-choice test Sam and Roger took.

	Correct	No attempt	Incorrect
Sam	14	5	1
Roger	15	0	5

(a) Represent this information in a 2×3 matrix, **S**.

$\begin{pmatrix}14 & 5 & 1\\ 15 & 0 & 5\end{pmatrix}$ ----------B1 $\mathbf{S}=\begin{pmatrix} & & \\ & & \end{pmatrix}$

Answer [1]

(b) The marks are awarded as follow:

Correct = 2 marks
No attempt = 0 mark
Incorrect = -1 mark

Represent the information in a 3×1 matrix, **T**.

Evaluate **ST**

$T=\begin{pmatrix}2\\0\\-1\end{pmatrix}$ ----------M1

$\begin{pmatrix}14 & 5 & 1\\ 15 & 0 & 5\end{pmatrix}\begin{pmatrix}2\\0\\-1\end{pmatrix}=\begin{pmatrix}27\\25\end{pmatrix}$ ----------A1

Answer ……..…… [2]

(c) Explain what your answer to **(b)** represents.

Total score Sam and Roger get respectively----------B1

Answer ……..……..

……

……

……..…… [1]

9 Factorise completely.

(a) $abc-3c-6+2ab$

$abc-3c-6+2ab$
$=abc-3c+2ab-6$
$=c(ab-3)+2(ab-3)$ ----------M1
$=(ab-3)(c+2)$ ----------A1

Answer …….. [2]

(b) $80x^4-5$

$80x^4-5$
$=5(16x^4-1)$ ----------M1
$=5(4x^2+1)(4x^2-1)$ ----------M1
$=5(4x^2+1)(2x+1)(2x-1)$ ----------A1

Answer …….. [3]

10 Ken divides his monthly salary into daily expenses, spending and saving in the ratio 3:4:5 respectively. Later he decides to spend \$1200 more daily the new ratio become 9:8:7. Calculate his monthly salary.

3 : 4 : 5
6 : 8 : 10 ----------M1
3 units represent \$1200 ----------M1
1 unit represent \$400
24 units represent 24 × \$400 = \$9600 ----------A1

Answer …….. [3]

11 John conducted a survey on the average time spent reading per week. This survey was carried out outside Woodlands Regional Library. The table below represents the survey results.

Time spends (t) in hours	$0 \le t < 2$	$2 \le t < 4$	$4 \le t < 6$	$6 \le t < 8$	$8 \le t < 10$	$10 \le t < 12$
Frequency	3	8	9	15	3	2

(a) Calculate the percentage of people spend between 6 to 8 hours per week in reading.

$\frac{15}{40} \times 100\%$

$= 37.5\%$ ----------B1

Answer ……..…….% [1]

(b) Calculate the average time spend in reading.

5.65 hours ----------B1

Answer ……..…….. hours [1]

(c) John concludes that the result in **part (b)** is the average of time spent in reading by a Singaporean. Do you agree, give a reason.

No, because the survey is done outside a library people who tends to read more[1]

therefore it is bias [1]

Answer ……..……..

……

……

……

……..…… [2]

12 The expression $x^2 - 4x + 7$ can be written in the form of $(x-a)^2 + b$

(a) Find the value of a and b.

$x^2 - 4x + 7$

$= (x-2)^2 - 4 + 7$

$= (x-2)^2 + 3$

$a = 2$

$b = 3$

Answer a= …..…….. [1]

b= …..…….. [1]

(b) Explain why the expression gives a minimum value.

Answer

$(x-2)^2 \geq 0$ ----------M1

the smallest it can have is 0.

therefore the expression gave a minimum values ---------A1

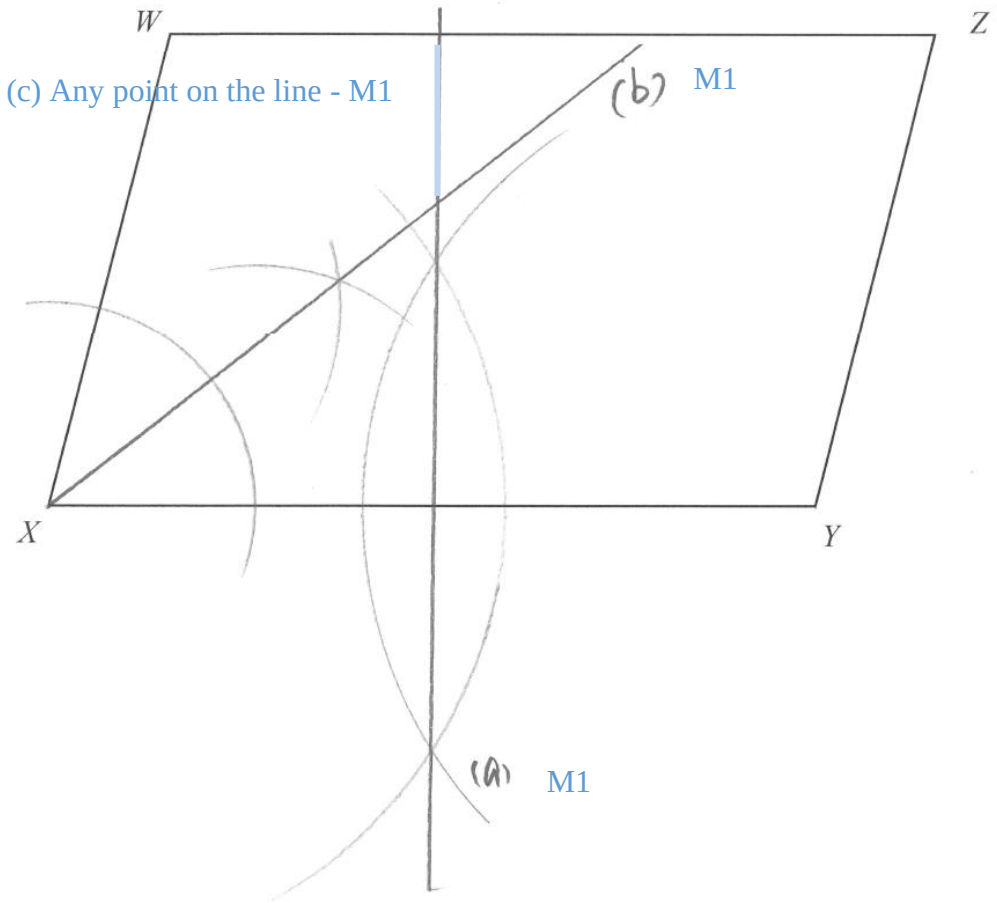
[2]

(c) Write down the minimum value.

3 ----------B1

Answer …….. [1]

13 The diagram shows a quadrilateral *WXYZ*.

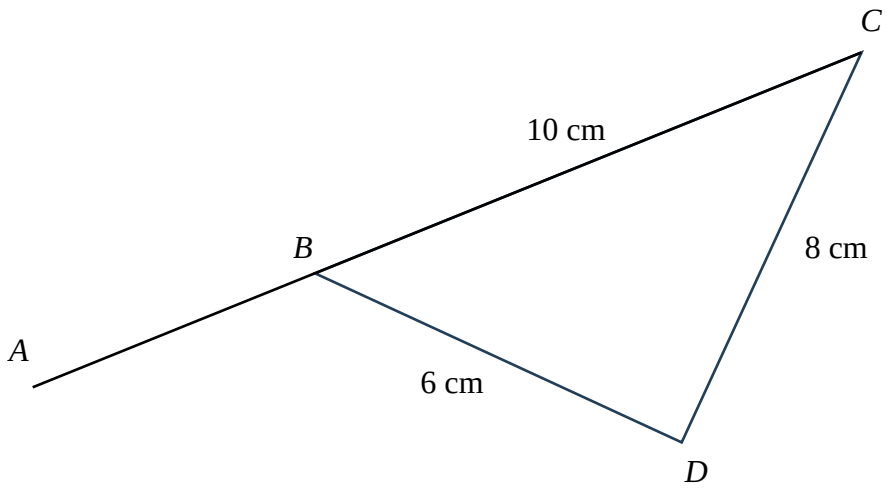


(a) Construct the perpendicular bisector of *XY*. [1]

(b) Construct the bisector of angle *WXY*. [1]

(c) Point *A* in the quadrilateral is equidistant from *X* and *Y* **and** is closer to the line *WX* then to line *XY*.
Mark and label a possible location for point *A* in the diagram above. [1]

14 In the diagram ABC is a straight line, BC = 10 cm, CD = 8 cm and BD = 6 cm.



(a) Explain why a circle that passes through B, C and D can be drawn in the above diagram.
Give reasons for each step of your working.

Answer

$BC^2 = 100$

$BD^2 + CD^2 = 64 + 36$

$= 100$

$BD^2 + CD^2 = BC^2$ ----------M1

by the converse of Pythagoras' theorem

angle BDC is 90°----------M1

a circle that passes through B, C and D can be drawn

(angle in semi circle)----------A1

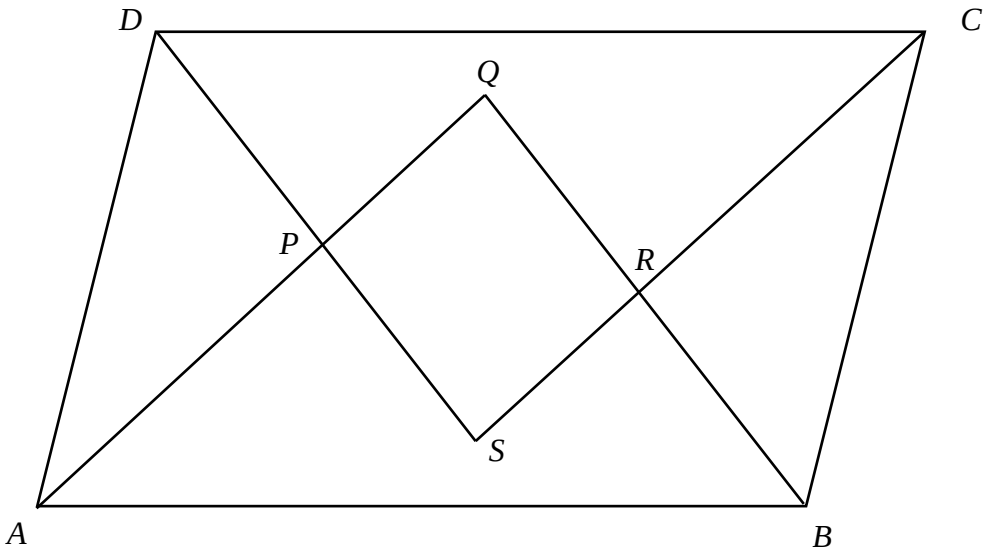
[3]

(b) Hence find the exact value of cos ABD

$-\frac{3}{5}$ ----------B1

Answer …….. [1]

15 The diagram shows a parallelogram *ABCD*. *APQ*, *BRQ*, *CRS* and *DPS* are straight lines which bisect angles *A*, *B*, *C* and *D* respectively.



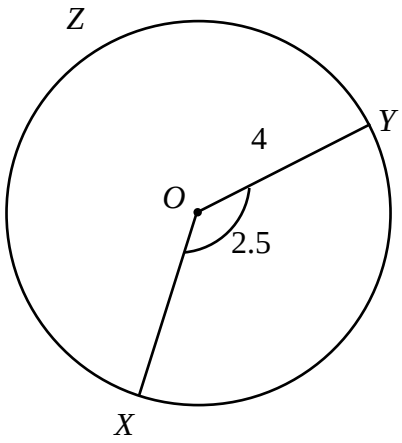
(a) Show that angle *PAD* = angle *RCB*. [2]

Answer:
$\angle DAB = \angle BCD$ (opposite angle of parallelogram) ----------M1
$\angle DAP = \angle PAB = \angle BCR = \angle RCD$ (bisect angle)
$\therefore \angle PAD = \angle RCB$ ----------A1

(b) Prove that triangles *ADP* and *CBR* are congruent. [3]

Answer:
in triangle *ADP* and triangle *CBR*,
$\angle PAD = \angle RCB$ (from (a))
$AD = CB$ opposite side of parallelogram ----------M1
$\angle ADP = \angle CBR$ ----------M1
$\therefore$ triangle *ADP* and triangle *CBR* (ASA) ----------A1

16 X, Y and Z lie on a circle with centre O and radius 4 cm.
Angle XOY = 2.5 radians.



(a) Find the area of minor sector XOY.

$$\text{Area of Sector} = \frac{1}{2}r^2\theta$$
$$= \frac{1}{2}(4)^2 \times 2.5$$
$$= 20 \text{ cm}^2 \text{ ----------B1}$$

Answer ……cm^2 [1]

(b) **(i)** Write down, in term of π, for reflex angle XOY.

$2\pi - 2.5$ ----------B1

Answer …….. [1]

(ii) Find the length of the arc XZY, in term of π.

$4(2\pi - 2.5)$
$= (8\pi - 10)$ ----------B1

Answer ……cm [1]

(iii) The major sector $XZYO$ is used to make a cone.
Calculate the base radius of the cone.

$4(2\pi - 2.5) = 2\pi r$ ----------M1
$$r = \frac{4(2\pi - 2.5)}{2\pi}$$
$r = 2.41$ ----------A1

Answer ……cm [2]

17 **(a)** Simplify.

(i) $\dfrac{a^3}{3a^2} \times 27a^6$

$\dfrac{a^3}{3a^2} \times 27a^6$

$= \dfrac{9a^3}{a^2} \times a^6$ ----------M1

$= 9a^7$ ----------A1

Answer …….. [2]

(ii) $\sqrt{16x^{\frac{4}{3}}}$

$\sqrt{16x^{\frac{4}{3}}} = 4\left(x^{\frac{4}{3}}\right)^{\frac{1}{2}}$ ----------M1

$= 4x^{\frac{2}{3}}$ ----------A1

Answer …….. [2]

(b) $\dfrac{49^{2a}}{7^b} = 343^{4c}$. Find an expression for b in terms of a and c.

$\dfrac{49^{2a}}{7^b} = 343^{4c}$

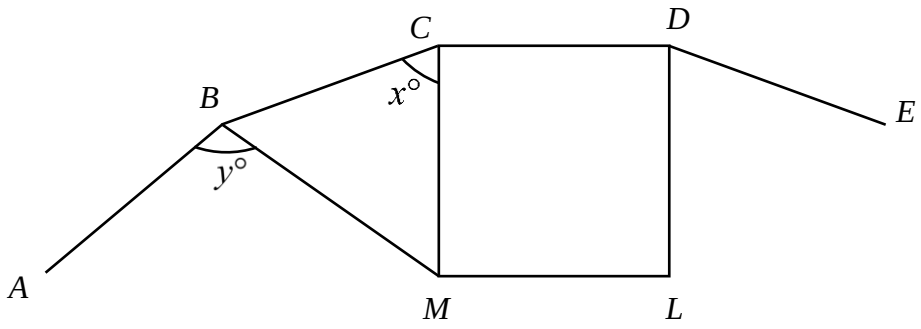
$\dfrac{7^{4a}}{7^b} = 7^{12c}$ ----------M1

$4a - b = 12c$ ----------M1

$b = 4a - 12c$ ----------A1

Answer …….. [3]

18 *ABCDE* is part of a regular polygon which has an exterior angle of $20°$. *CDLM* is a square.



Find

(a) the value of x,

$\angle BCD = 180 - 20$

$= 160$ ----------M1

$x = 160 - 90$

$= 70$ ----------A1

Answer …….. ° [2]

(b) the value of y.

$\angle MBC = \frac{180 - 70}{2}$

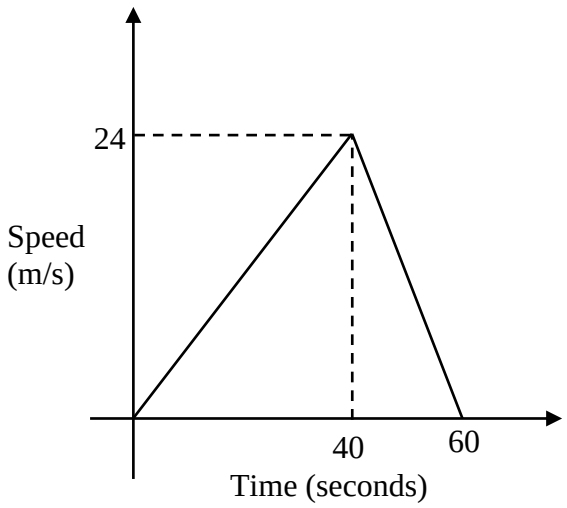
$= 55$ ----------M1

$y = 160 - 55$

$= 105$ ----------A1

Answer …….. ° [2]

19 The diagram below shows the speed-time graph of a car's journey.



For this journey, calculate

(a) the acceleration during the first 40 seconds,

$a = \frac{24}{40}$

$= 0.6$ ----------B1

Answer ……...……m/s^2 [1]

(a) The total distance travelled.

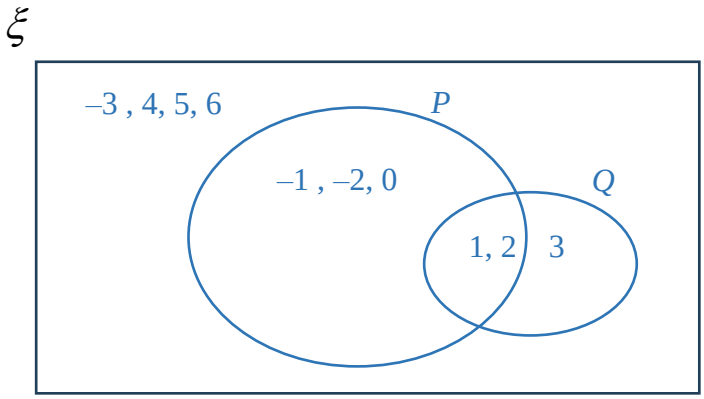
Total distance travelled $= \frac{1}{2} \times 60 \times 24$

$= 720$ ----------B1

Answer …..…….m [1]

20 $\xi = \{ x : x \text{ is an integer and } -3 \leq x < 7 \}$
$P = \{ x : -3 < x < 3 \}$
$Q = \{ x : 0 < x \leq 3 \}$

(a) Draw a Venn diagram below to illustrate this information. [1]



(b) List the elements in

(i) P',
{–3, 3, 4, 5, 6} ----------B1

Answer …….. [1]

(ii) $P \cap Q$.

{1, 2} ----------B1

Answer …….. [1]

(c) Write down $\mathrm{n}(P' \cup Q)$
7 ----------B1

Answer …….. [1]

21 The first four terms in a sequence of numbers, x_1, x_2, x_3, x_4, … are given below.

$$x_1=2(0)+1=1$$
$$x_2=2(1)+3=5$$
$$x_3=2(2)+5=9$$
$$x_4=2(3)+7=13$$

(a) Write down an expression for x_5.

17 ----------B1

Answer …….. [1]

(b) Find an expression, in term of n, for the n^{th} term, x_n, of the sequence.

$x_n = 2(n-1)+1+2(n-1)$ ----------M1

$= 2n-2+1+2n-2$

$= 4n-3$ ----------A1

Answer …….. [2]

(c) Evaluate x_{20}

$x_{20} = 4(20)-3$

$= 77$ ----------B1

Answer …….. [1]

(d) Explain why 203 is not a term of this sequence.

Answer

$x_n = 4n-3$

$203 = 4n-3$

$4n = 206$

$n = 51.5$

Since n is not a positive integer, [1]

therefore 203 is not a term of this sequence [1]

[2]

22 A survey was carried out to find out the number of emails received in a week by each of a group of students.

The table below represents the result of the survey.

Number of emails (n)	Frequency
$0 \le n < 10$	8
$10 \le n < 20$	13
$20 \le n < 30$	25
$30 \le n < 40$	30
$40 \le n < 50$	18
$50 \le n < 60$	6

(a) Find the probability that two students, chosen in random, both received at least 40 emails.

$\frac{24}{100} \times \frac{23}{99}$ ----------M1

$= \frac{46}{825}$ ----------A1

Answer …….. [2]

(b) Which interval contain the median number of emails received by the students.

$30 \le n < 40$ ----------B1

Answer …….. [1]

(c) Calculate an estimate of the mean number of emails received by the students.

30.5 ----------B1

Answer …….. [1]

(d) Calculate an estimate of the standard deviation.

$SD = \sqrt{\frac{110100}{100} - (30.5)^2}$ ----------M1

$=13.1$ ----------A1

Answer …….. [2]

END OF PAPER 1