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JURONG SECONDARY SCHOOL
2020 GRADUATION EXAMINATION
SECONDARY 4 EXPRESS

CANDIDATE NAME	

CLASS		INDEX NUMBER	

ADDITIONAL MATHEMATICS 4047/02

PAPER 2
Candidates answer on the Question Paper.
Additional Materials : Writing Paper (1 sheet)

31 August 2020
2 hours 30 minutes

READ THESE INSTRUCTIONS FIRST

Write your name, class and index number on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 100.

For Examiner's Use
100

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer, and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = 2\cos^2 A - 1 = 1 - 2\sin^2 A = \cos^2 A - \sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}bc\sin A$$

1 **(i)** Differentiate $x\ln(x+1)$ with respect to x. [2]

(ii) Hence, find $\int \ln(x+1)\ \mathrm{d}x$. [3]

2 **(i)** Prove that $(\sin^2 2x)(\cot^2 x-\tan^2 x)=4\cos 2x$. [4]

(ii) Hence, solve the equation $\sin^2 2x=\dfrac{2\sqrt{3}}{\cot^2 x-\tan^2 x}$ for $0°\le x\le 180°$. [3]

3 **(i)** Prove that $x-1$ is a factor of x^3+x^2-5x+3. [1]

(ii) Express $\dfrac{2x^2+3x-11}{x^3+x^2-5x+3}$ as the sum of partial fractions. [6]

4 **(a)** Solve the equation $2\log_5 x + \log_{25} 16 = \log_5 (9x - 2)$. [3]

(b) Given that $a > b > 1$ and $\frac{1}{\log_a b} - \frac{1}{\log_b a} = \sqrt{293}$, find the value of

$\frac{1}{\log_{ab} a} - \frac{1}{\log_{ab} b}$. [5]

5 A curve has the equation $y = xe^{2x}$.

(i) Find the x-coordinates of the stationary point on the curve. [3]

(ii) Find the value of k for which $\dfrac{d^2y}{dx^2} = ke^{2x}(1+x)$. [3]

(iii) Hence, determine the nature of the stationary point. [2]

6 $A\ (-1,3)$, $B\ (3,3)$ and $C\ (0,6)$ are points that lie on the circumference of circle C_1.

(i) Find the equation of the perpendicular bisector of BC. [2]

(ii) Hence, find the equation of circle C_1. [4]

(iii) Given that BP is the diameter of the circle, find the coordinates of P. [2]

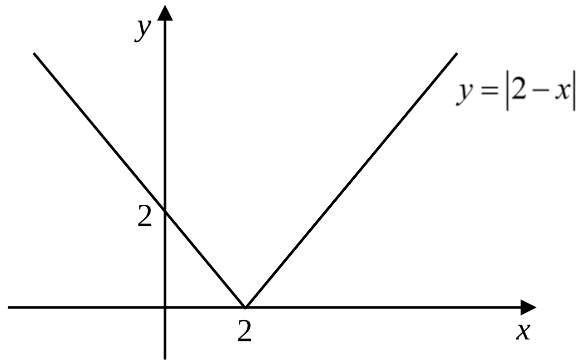
(iv) A second circle C_2 has equation $(x-2)^2+(y+6)^2=19$. Explain why circle C_1 will not intersect circle C_2. [2]

7 **(a)** **(i)** Solve the equation $\left|x^2+x-6\right|=6$. [3]

(ii) Sketch the graph of $y=\left|x^2+x-6\right|$, indicating clearly the coordinates of the intercepts & turning points. [3]

(iii) Hence, state the range of values of x for which $\left|x^2+x-6\right|<6$. [1]

(b) The diagram shows part of the graph $y=|2-x|$.



A line $y=mx+c$ is drawn on the same axes to determine the number of solutions to the equation $|2-x|=mx+c$.

(i) If $m=1$, state the range of values of c such that there is 1 solution to the equation. [1]

(ii) If c = 0, state the range of values of m such that the equation has 2 distinct solutions. [2]

8 The table shows experimental values of two variable, x and y, which are connected by an equation of the form $y = \dfrac{px}{q+x}$.

x	1	3	5	7	9
y	0.25	0.5	0.625	0.7	0.75

(i) Plot y against $\dfrac{y}{x}$ for the given data and draw a straight line graph. [3]

(ii) Use your graph to estimate

(a) the values of p and q, [3]

(b) the value of y when $2x = 7y$. [2]

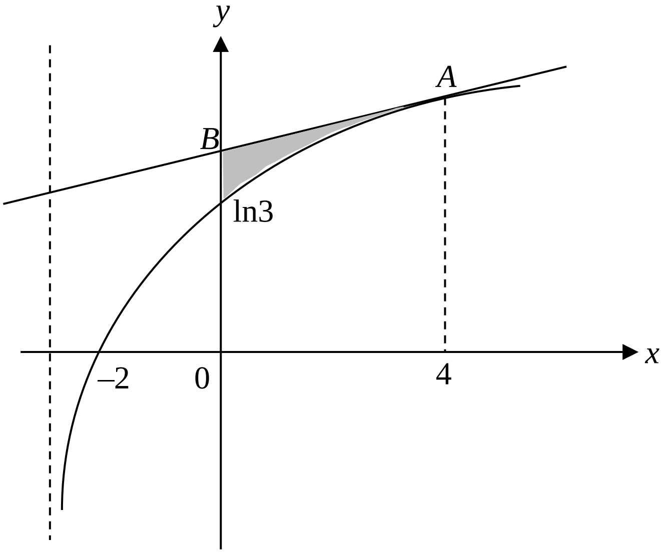
(iii) By drawing a suitable line on your graph, solve the following simultaneous equations.

$$y = \frac{px}{q+x}$$

$$y = xy$$

[3]

9 The diagram below shows part of the graph of $y = a\ln(x-k)$, $x > k$.
The curve intersects the y-axis at $(0, \ln 3)$ and intersects the x-axis at $(-2, 0)$.



(i) Find the value of a and k. [3]

(ii) The tangent to the curve at point A cuts the y - axis at B. The x-coordinate of point A is 4.
Calculate the area of the shaded region. [8]

10 **(i)** It is given that the range of values of x that satisfies the inequality $2x^2 + px + q < 0$ is $-\frac{1}{2} < x < 3$, where p and q are integers.
Prove that $p = -5$ and $q = -3$. [2]

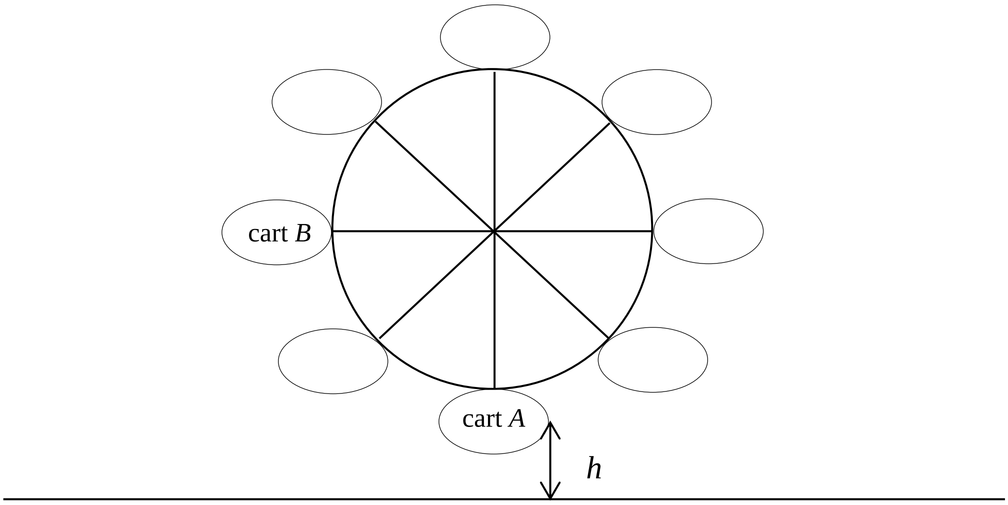
(ii) It is also given that the roots of the equation $x^2 + qx - p = 0$ are α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. [2]

(b) Find the value of $\alpha^3 + \beta^3$. [3]

(c) Hence, or otherwise, find an equation whose roots are $\frac{\alpha^2}{\beta}$ and $\frac{\beta^2}{\alpha}$. [4]

11



The diagram above shows a Ferris Wheel in an amusement park.
The height above ground level, h m, of cart A on the Ferris Wheel is modelled by the equation $h = 35 - 30\cos(kt)$, where k is a constant and t is the time in minutes after cart A started the ride at the lowest position as shown in the diagram.
The total time taken to complete one revolution is 20 minutes. Assume that the size of the cart is negligible.

(i) State the diameter of the Ferris Wheel. [1]

(ii) Show that the value of k is $\frac{\pi}{10}$ radians per minute. [2]

(iii) When cart A was at the lowest position, cart B was at the left-most position as shown in the diagram.
Using the equation of h, deduce a similar expression for the height above ground level, g m, of cart B, in terms of t. [2]

(iv) Hence, using your answers in **(ii)** and **(iii)**, show that the vertical distance, d m, between the two carts is given by $d = |30\sin(kt) + 30\cos(kt)|$. [2]

(v) Express d in the form of $|R\sin(kt+\alpha)|$, where $R > 0$ and $\alpha \leq \frac{\pi}{2}$. [2]

(vi) State the maximum value of d. [1]

(vii) Find the minimum value of t for d to be the maximum. [2]

--END OF PAPER--