WOODGROVE SECONDARY SCHOOL
A COMMUNITY OF FUTURE-READY LEARNERS AND THOUGHTFUL LEADERS

O LEVEL PRELIMINARY EXAMINATION 2024

LEVEL & STREAM : SECONDARY 4 EXPRESS / 5 NORMAL ACADEMIC

SUBJECT (CODE) : MATHEMATICS (4052)

PAPER : 02

DATE (DAY) : 22 AUGUST 2024 (THURSDAY)

DURATION : 2 HOURS 15 MINUTES

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class on all the work you hand in.
Write in dark blue or black pen.
You may use a HB pencil for any diagrams, graphs, tables or rough working.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed for any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks in this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

DO NOT TURN OVER THE QUESTION PAPER UNTIL YOU ARE TOLD TO DO SO.
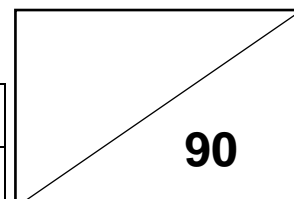
Mathematical Formulae

Compound Interest

$$\text{Total amount} = P\left(1 + \frac{r}{100}\right)^n$$

Mensuration

$$\text{Curved surface area of a cone} = \pi r l$$

$$\text{Surface area of a sphere} = 4\pi r^2$$

$$\text{Volume of a cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Volume of a sphere} = \frac{4}{3}\pi r^3$$

$$\text{Area of a triangle } ABC = \frac{1}{2}ab\sin C$$

$$\text{Arc length} = r\theta, \text{ where } \theta \text{ is in radians}$$

$$\text{Sector area} = \frac{1}{2}r^2\theta, \text{ where } \theta \text{ is in radians}$$

Trigonometry

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

Statistics

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f}$$

$$\text{Standard deviation} = \sqrt{\frac{\Sigma fx^2}{\Sigma f} - \left(\frac{\Sigma fx}{\Sigma f}\right)^2}$$

Answer **all** the questions.

1 (a)

$$b = \frac{a}{a-1} + \frac{2}{c}$$

(i) Find the value of b when $a = 3$ and $c = 4$.

Answer $b =$ ………………….….. [1]

(ii) Rearrange the formula to make a the subject.

Answer ……..……………….….. [3]

(b) Solve these simultaneous equations.

$$4x - y = -11$$
$$5x + 3y = -1$$

Answer $x =$ …………………..…..

$y =$ …………………..….. [3]

(c) Solve the equation $\dfrac{x}{3x-1} - \dfrac{5}{2x+3} = 1$.

Give your solutions correct to three decimal places.

Answer $x =$ …………… or …………… [4]

2 (a) Timmy wants to invest a sum of money in one of the following investment plans.

Plan A	Pays 2% simple interest per year
Plan B	Pays 2% compound interest per year

Which investment plan is the better choice for Timmy?
Explain your answer.

Answer : ……………………………………………………………………………

………………………………………………………………………………………

……………………………………………………………………………………… [1]

(b) Mary deposits $8000 in a savings account at a simple interest rate of 3.5% per year. Calculate the total value of her savings at the end of 12 years.

Answer $ …….……………….….. [2]

(c) Peter invested $50 000 in an investment plan from 2010 to 2024.
The value of his investment increased by r% at the end of every year.
At the end of 2024, the total value of Peter's investment is $65 320.

Calculate the value of r.

Answer $r =$ ……………….….. [3]

(d) The current exchange rate between Singapore dollars (SGD) and Thailand baht (THB) is given as SGD 1 = THB 27.16.

Susan bought a bag while shopping in Thailand. The original price of the bag was THB 56 000 and Susan bought it at a 15% discount.

Susan paid for the bag using her credit card which charged her an additional 2% fee for overseas transaction.

Calculate the total amount, including credit card fee, that Susan had to pay in Singapore dollars (SGD). Correct your answer to the nearest cent.
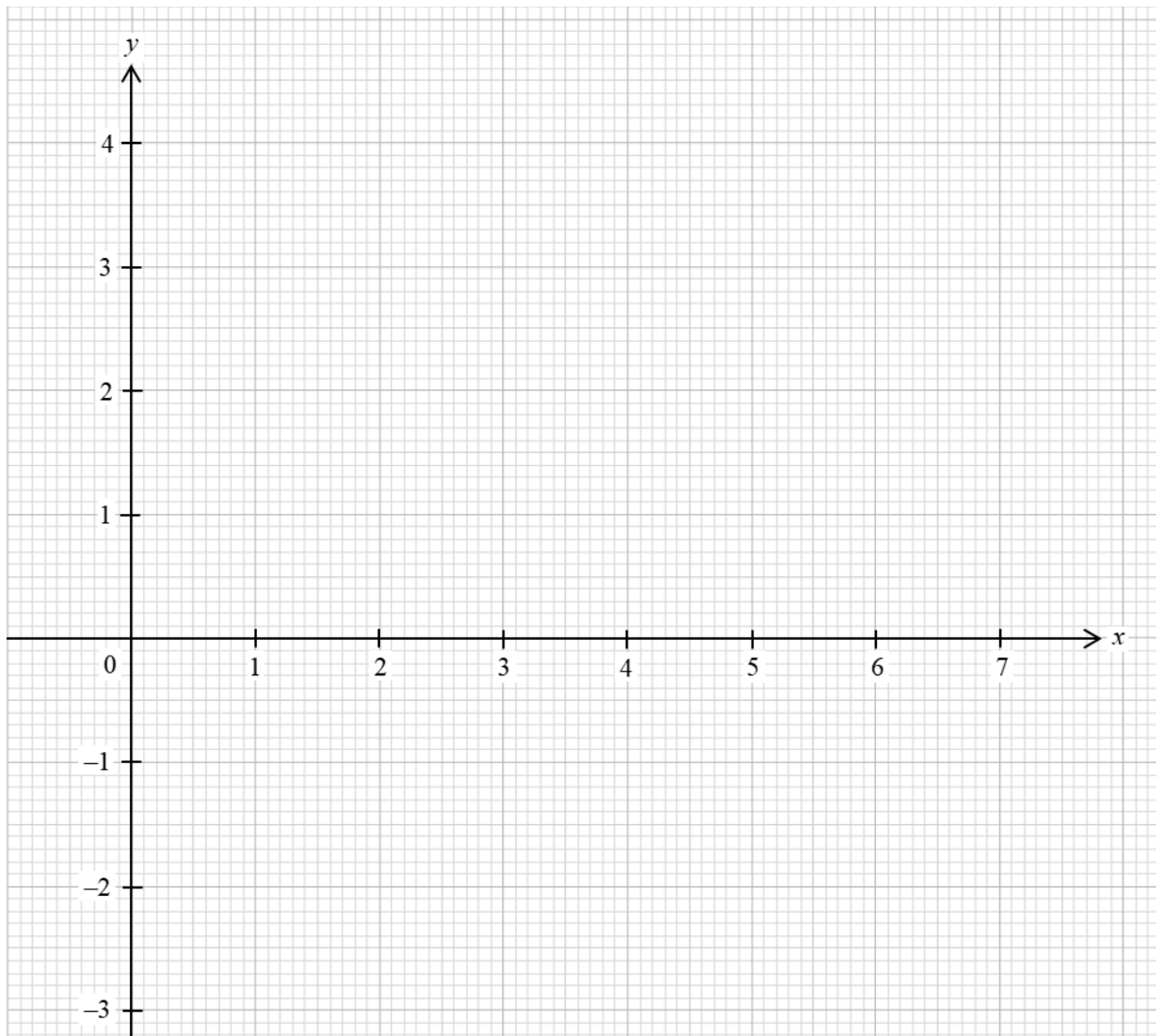
Answer $ …….……………….….. [3]

3 **(a)** The table of values for $y = x + \dfrac{5}{x} - 7$ is given below.

x	0.5	1	1.5	2	3	4	5	6	7
y	3.5	−1.0	−2.2	−2.5	−2.3	−1.8	k	−0.2	0.7

Find the value of k.

Answer $k = $ ……………….…… [1]

(b) On the grid below, draw the graph of $y = x + \dfrac{5}{x} - 7$ for $0.5 \le x \le 7$ [3]

(c) Use your graph to find the solutions to $x + \dfrac{5}{x} = 5$ in the range of $0.5 \le x \le 7$.

Answer $x =$ ………….… and …………… [3]

**(d)** By drawing a tangent, find the gradient of the curve at $(4, -1.8)$.

Answer Gradient = ……………….….. [2]

(e) The solutions of $Px^2 + Qx + 5 = 0$ are the same as the x-coordinates of the intersections between the graph $y = x + \dfrac{5}{x} - 7$ and the line $y = -x + 3$.

Find the values of P and Q.

Answer $P =$ ……………….…..

$Q =$ ……………….….. [3]

4 (a) The diagram below shows two circles, with the same centre at *O*.
Points *A*, *B* and *C* lie on the bigger circle and point *D* lies on the smaller circle.
AC is a straight line that passes through the centre of both circles.
AB and *BC* are both tangents to the smaller circle.
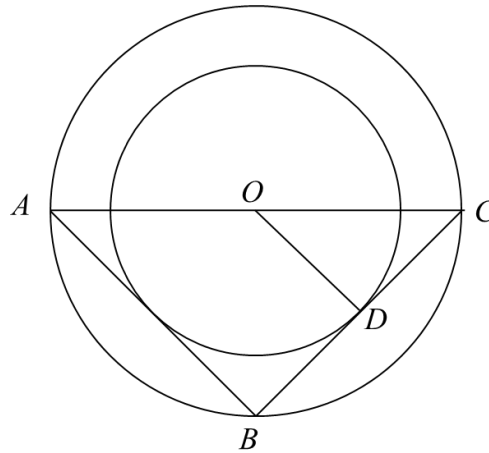
(i) Show that triangles *ABC* and *ODC* are similar.
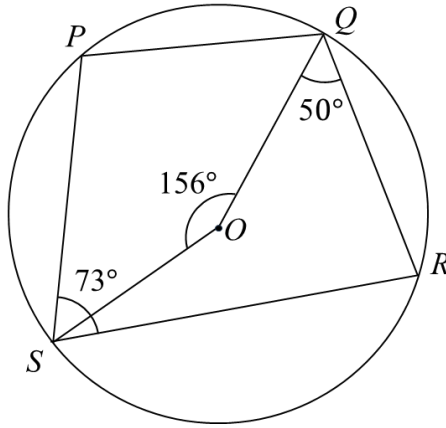Give a reason for each statement you made.

Answer [2]

(ii) The area of triangle *ODC* is given as 15cm^2.
Find the area of trapezium *ABDO*.

Answer …….………………..….. cm^2 [2]

(b)

PQRS are points on the circumference of a circle, centre *O*.
Angle *OQR* = 50°, angle *PSR* = 73° and obtuse angle *SOQ* = 156°.

(i) Find angle *QRS*.
Give a reason for each step of your working.

Answer …….……………….…..° [1]

(ii) Find angle *PSO*.
Give a reason for each step of your working.

Answer …….……………….…..° [2]

(iii) Find angle *PQO* .
Give a reason for each step of your working.

Answer ……..……………..…..° [2]

(iv) Explain whether line *PQ* parallel to line *SR*.
Support your answer with workings.

Answer : …………………………………………………………………

………………………………………………………………………………...

[2]

5 The masses of two batches of 400 steel bars produced by factory *A* and *B* were measured and recorded. The cumulative frequency graph below shows the masses.
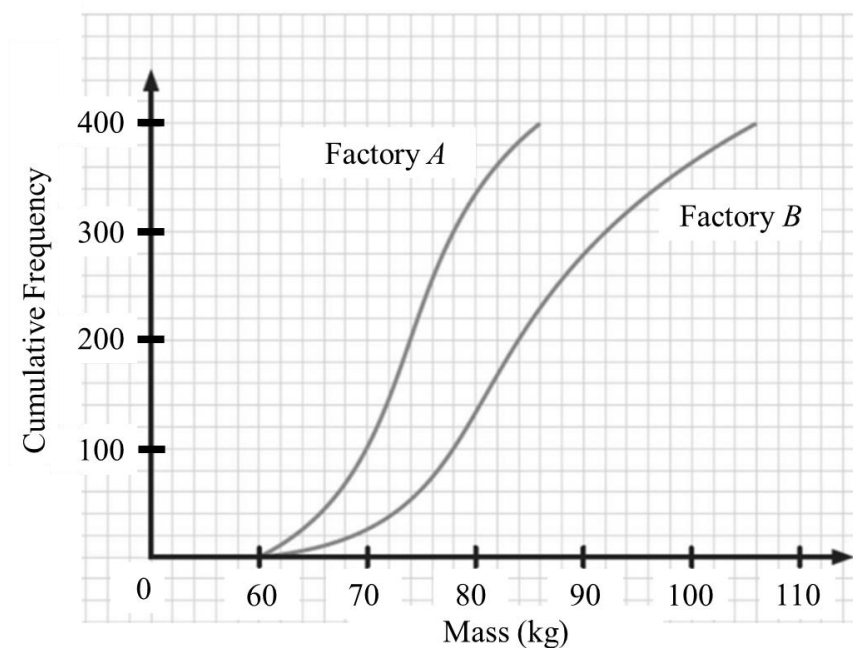
(a) For the steel bars produced by factory *A*, find

(i) the median,

Answer ……..………………..…. kg [1]

(ii) the interquartile range.

Answer ……..………………..…. kg [2]

(b) The steel bars produced by factory *B* are heavier than those produced by factory *A*. Do you agree? Explain your answer.

Answer :

…………………………………………12………………………………………………..
[1]
…………………………………………………………………………………………..

(c) Which factory is more consistent in the production of steel bars?
Support your answer with workings.

Answer :

……………………………………………………………….…………………..

………………………………………………………………………………….. [2]

(d) One steel bar is chosen at random from factory *A* and another steel bar is chosen at random from factory *B*.

Find, as a fraction in its simplest form, the probability that both steel bars chosen are more than 80kg.

Answer …….………………….….. [3]

6 **(a)** Point A is translated to point B by the vector $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$.

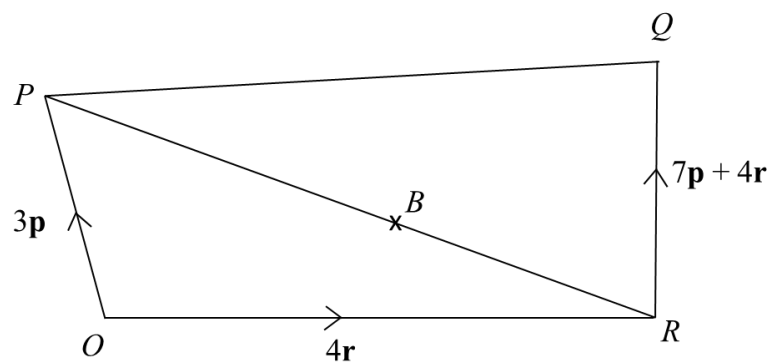The position vector of point B is $\begin{pmatrix} 3 \\ 8 \end{pmatrix}$.

(i) Find $|\overrightarrow{OA}|$.

Answer …….…………….….. [2]

(ii) P is a point that lies on AB and has coordinates $(k, 10)$.
Find the value of k.

Answer $k =$ ………………..…… [2]

(b)

OPQR is a quadrilateral.

$\overrightarrow{OP} = 3\mathbf{p}$, $\overrightarrow{OR} = 4\mathbf{r}$ and $\overrightarrow{RQ} = 7\mathbf{p} + 4\mathbf{r}$.

B is a point on PR such that $PB : BR = 3 : 2$.

Write each of the following in terms of **p** and **r** .
Give your answer in their simplest form.

(i) $\overrightarrow{OQ}$,

Answer …….……………….….. [1]

(ii) $\overrightarrow{PQ}$,

Answer …….……………….….. [1]

(iii) $\overrightarrow{OB}$.

Answer …….……………….….. [3]

(c) Explain why points *O*, *B* and *Q* are **not** collinear.

Answer :

…………………………………………………………………………………………..

………………………………………………………………………………………….. [1]
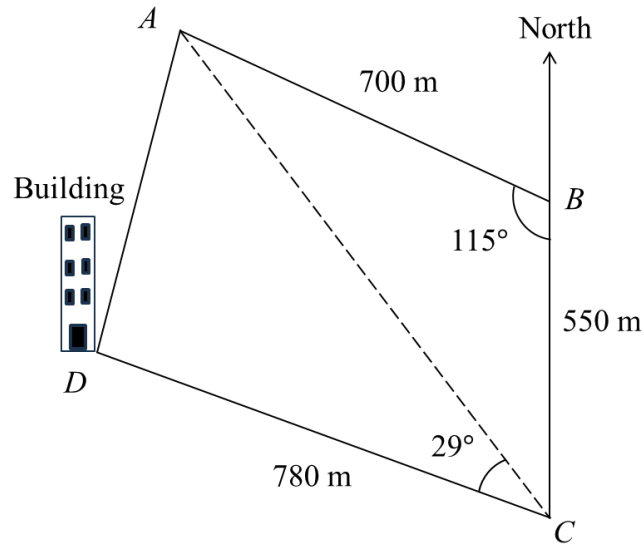
7

The diagram above shows a park *ABCD* that is on horizontal ground, crossed by a straight path *AC*. Point *B* is due north of *C*.

$AB = 700$ m , $BC = 550$ m and $CD = 780$ m .
Angle $ABC = 115°$ and angle $ACD = 29°$.

(a) Show that length $AC = 1057.32$ m, correct to two decimal places.

Answer   [2]
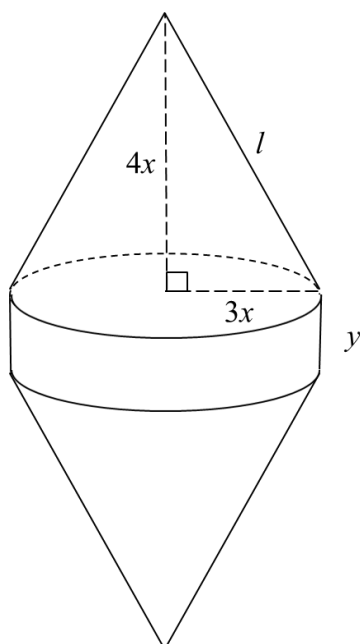
(b) Calculate the bearing of *D* from *C*.

Answer …….………………….….° [3]

(c) Find the total area of the park *ABCD*.
Correct your answer to the nearest whole number.

Answer ……..……………….….. m^2 [2]

(d) A vertical building is located at point *D*.
The angle of elevation of the top of the building from point C is 10.9° .
Find the height of the building.

Answer ……..……………….….. m   [2]

8

The diagram above shows a solid structure that is formed from two identical cones and a cylinder. The cylinder is attached to the bases of the two cones.

The cone has radius $3x$ cm, height $4x$ cm and slant height l cm.
The cylinder has radius $3x$ cm and height y cm.

(a) Express l in terms of x.

Answer $l =$ ……………………..….. [1]

(b) The volume of the cylinder is equal to the volume of one cone.

Show that $y = \dfrac{4}{3}x$.

Answer [2]

(c) The total surface area of the solid structure is $(200-x)\pi \text{ cm}^2$.
Write an expression, in terms of x, for the total surface area of the solid and show
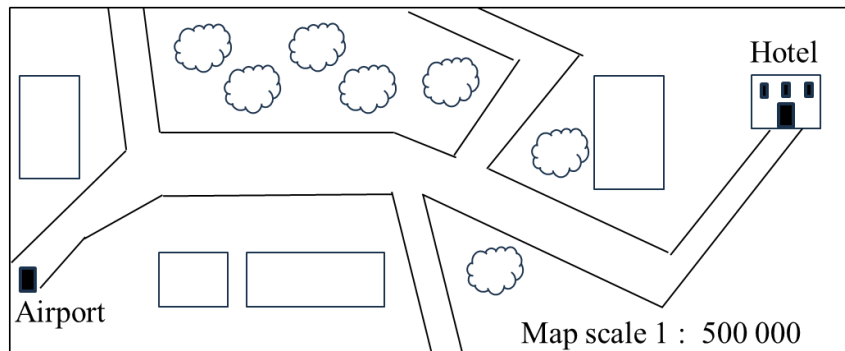that it reduces to $38x^2 + x - 200 = 0$.

Answer [3]

(d) Solve the equation $38x^2 + x - 200 = 0$, giving your solutions correct to 2 decimal
places.

Answer $x =$ ……………………… [3]

9 David will travel on his own for a holiday day trip to Australia. He is planning to rent a car for a total of 9 days for the whole trip.

(a) David plans to pick up his rental car from the airport and drive from the airport to his hotel. He uses the above map, drawn to scale, to plan his way to the hotel. The above map has a scale of 1 : 500 000.

(i) Measure and estimate the total distance that David needs to drive from the airport to the hotel.

Answer …….……………….….. km   [2]

(ii) Assuming that David will drive at an average speed of 65 km/h from the airport to the hotel. Calculate the time he will take to drive from airport to the hotel. Give your answer correct to the nearest minute.

Answer …….……………….….. minutes   [2]

(b) The information below contains the different costs of renting a car in Australia.

Rental charges for car.

Type of vehicle	Daily Rental Cost (Excluding petrol and charging costs)			Mileage (Distance travelled based on vehicle type
	Less than 7 days	Less than 30 days	More than 30 days	
Petrol Car (5 seater)	$125	$112.50	$93.75	10.3 km per litre of petrol
Petrol Car (7 seater)	$170	$153	$136	8.7 km per litre of petrol
Electric Car (5 seater)	$198	$178.20	$158.40	5.25 km per kWh of charge
Electric Car (7 seater)	$220	$198	$165	4.54 km per kWh of charge

Taxes

- 10% Goods and Service Tax (GST) on all purchases of goods and service in Australia.

Petrol and charging cost

Price of petrol: $3.10 per litre of petrol (excluding GST)

Charging cost: $1.25 per kWh of charge (including GST)

(b) David estimates that his total driving distance for the whole trip will be 850 km, including the journey to and fro from the airport to the hotel.

After some calculations, David concludes that renting the electric car (5 seater) is the cheapest option for his whole trip. Do you agree with David?

Justify your answer with workings.

Answer [6]

END OF PAPER 2