Short summary of Kant

Kant wishes to address the problems faced by the R+E – knowledge, if it is to be absolutely certain, is minimal and restricted to Analytic A Priori. But AAPs are not meaningful knowledge since they are tautologies. Empiricism cannot give us knowledge because of Hume's attack on Causation which discredited Newtonian science. (Not to mention the problem with the corrigibility of our beliefs regarding SD) So if there is to be meaningful knowledge, then it is to be found in Synthetic A Priori because a priori gives us necessity and universality whereas synthetic gives us new knowledge by the combining of 2 non-necessary concepts. Kant then offers us one body of knowledge that we all do think is knowledge – math – AND argues that math is SAP. So if math is SAP, then SAP, and thus meaningful knowledge, is possible. This is especially important in the realm of metaphysics and science which Hume had thrown doubt on with the Problem of Induction and Constant Conjunction. For Kant, metaphysical knowledge and science had to be knowledge that is necessary and universal (hence his need to ensure the a priori-ness of such claims).

[Note: for Kant, it is the AXIOMS themselves that are SAP, not the move from axioms to theorem – that happens via deduction and thus is analytic -> nothing new is gotten from the move from axioms to theorems. Instead, the new knowledge comes from the axioms themselves when 2 hitherto unrelated concepts are put together like "shortest distance" and "straight line".]

Since SAP has been shown to be possible, Kant can then proceed to show how we obtain knowledge – it is via his Copernican turn in philosophy. Here, instead of proving conclusively that his epistemology is correct, Kant makes an IBE by ASSUMING that the mind is an active constituter of knowledge and then proceeding to show that if we were to assume so, we are able to recover Newtonian science from Humean scepticism by making causality (among other things) a necessary part of our experience. Thus, we can arrive at meaningful metaphysical knowledge that is certain because now, we can once again use causality to predict how things will turn out, for example.

Yet how is Math both the result of R+E AND still be SAP? Well, even for AAPs like "a bachelor is an unmarried man", one can only arrive at knowledge of this proposition via knowing what the object/subject is – in this case, a bachelor. But we can only know what the term, bachelor, means by 'marrying' the concept with experience – be it by learning about it in a dictionary or from someone else or encountering a real bachelor; otherwise, it will be, as Kant says, "empty" (concepts without intuitions are empty) or "noise" (intuitions without concepts are noise). Yet the proposition/definition of a bachelor is STILL a priori because the JUSTIFICATION for the proposition is a priori and not a posteriori – we do not need to prove that all bachelors are unmarried men by checking, empirically, with every single bachelor we meet (nor is this, of course, possible). In this way, the SOURCE of our knowledge is distinct from how we JUSTIFY our knowledge. For Kant, the SOURCE of knowledge is BOTH reason and experience. But for true knowledge, i.e. that which is necessary and universal, the justification is from reason, i.e. a priori, because only a priori can ensure necessity and universality.

So in the case of Math, the JUSTIFICATION is also A Priori – we do not need to refer to physical triangles to prove that the sum of angles of a triangle is 180 degrees; experience can never prove any claim necessarily nor universally. But we still require experience in math because in order for us to know what a triangle is, we need to 'marry' the concept of triangle with the necessary experience for us to truly understand "triangle". Otherwise, the term "triangle" means nothing to us; it becomes gobbledygook. For example, a triangle is defined as a planar figure bounded by 3 lines. But in order to understand this definition, we need to know the meaning of the various terms, like "planar" and "line". Yet this would not be possible, Kant says, without the raw data from the noumenal world coming together with our mental filters to give us the meaning of such terms. So even in supposedly strictly a priori enterprises like math, there is still a role for experience.

The interesting thing for Kant is that in postulating Math as SAP, and that the geometry of the phenomenal world is Euclidean space, he opened himself up to the problem of non-Euclidean geometry. Because the point (from "Kant and Mathematical Knowledge" by Thomas McFarlane under J1 Math folder) is that an SAP proposition is that which is necessary and universal for the **phenomenal** world that we live in. So this means, for Kant, that the phenomenal world is Euclidean in space, i.e. we HAVE to experience things on an Euclidean plane and not a non-Euclidean space . This was not a problem for a long time because most people thought until recently that only Euclidean geometry was possible. Until mathematicians like Riemann came along who decided to try playing around with Euclid's axioms, specifically contradicting them, like the axiom of parallel lines never meeting, to see if a consistent but separate geometry was possible. And so it proved – Riemannian geometry was born. This showed that Euclidean geometry was *not* the only geometry possible. This didn't prove a problem for Kantians yet because Kant's point is that *this phenomenal world*, not all logical possibilities, is Euclidean in space – that's the point of Kant's Critique of Pure Reason - > that we can only ever have knowledge of this phenomenal world and not beyond it. The bigger problem for Kant came when Einstein proved that actually, our world is *not* Euclidean in space with his theory of general relativity. This showed therefore that Kant's postulation was wrong.

But is this a fatal blow to Kant's project? No. Because he could simply concede the point and say that his postulation was wrong.

Nonetheless, McFarlane's point goes further – that actually, in our experience of this phenomenal world, we experience the world in 2 geometries as it were: Euclidean and non-Euclidean, such that now, geometry is *not* SAP. Think for example how astronauts would experience the planet when they're in outer space (non-Euclidean) and when they're on Earth (Euclidean). Instead, it is merely AAP because it cannot speak of the necessity of our spatial experience. So Math as SAP (for geometry) is debunked. Ayer also countered the same for Arithmetic – that 7+5=12 is indeed AAP because we will find that actually, analysing 7 and 5 together does give us 12.

[Note: McFarlane's point is that SAP means that there's only one physical/empirical possibility that we experience but AAP is only of logical possibilities, i.e. it need not be describing anything in the phenomenal world though the meaning of the terms is still only possible if reason and experience come together to give us that knowledge AND that the terms are only understandable in this phenomenal world, saying nothing of the noumenal world which is unknowable. For example, a bachelor is an unmarried man – as mentioned earlier, we need both reason and experience to understand the terms. But knowing this AAP doesn't mean whether we know if a bachelor actually exists. It could well have been that this is an entirely made up word but with components that actually exist, like a unicorn being made up of horse and horn; here, a bachelor is an unmarried person who is a man. No real bachelor need ever existed for us to understand the term so long as we know what unmarried and man mean. Similarly, a unicorn is a horse with one horn is an AAP because if you negate the statement, it is a contradiction and we can know via reason whether the statement is true. In this sense, an AAP can be a mere logical possibility but still be the product of R+E giving us knowledge of those terms.
Also, I suppose the point is that for any rich enough set of axioms, you can always arrive at theorems. But these theorems are just logical possibilities and need say nothing about our experience of the real world because these axioms are just fictional constructs that we have made up which need have no relation to the real world.
So AAP can admit of many possibilities, even contradictory ones, such that if there's Euclidean and non-Euclidean geometry, it means that geometry is AAP and not SAP
SAP on the other hand is not just necessary but meaningful knowledge, i.e. that which extends our understanding of the phenomenal world. For example, "every event has a cause" is a SAP because

there is nothing in event that necessitates a cause such that knowing this to be true gives us something new that we didn't know before. But to say that "every event has a cause" is an SAP is to make a necessary statement about how the world functions. Similarly, by saying that geometry is SAP, Kant is saying that it is necessary that the space we inhabit and interact with is Euclidean because geometry is the study of the space we inhabit.]

Upshot? Math is NOT SAP as Kant claimed. BUT it doesn't mean that the primary claim about our experience – that it is spatial – is not SAP; it still is precisely because we HAVE to experience things in space. The only change for Kant is that this space need not be Euclidean.