General Certificate of Education Ordinary Level
JUYING SECONDARY SCHOOL, SINGAPORE
Secondary Four Express/Five Normal Academic Preliminary

CANDIDATE NAME											
CENTRE NUMBER	S					INDEX NUMBER					

ADDITIONAL MATHEMATICS

4049/01

Paper 1

22 August 2024
2 hours 15 minutes

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your Centre number, index number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
The number of marks is given in brackets [ ] at the end of each question or part question.

If working is needed in any question it must be shown with the answer.
Omission of essential working will result in loss of marks.
The total number of marks for this paper is 90.

The use of an approved scientific calculator is expected, where appropriate.
If the degree of accuracy is not specified in the question, and if the answer is not exact, give the answer to three significant figures. Give answers in degrees to one decimal place.
For π, use either your calculator value or 3.142.

This document consists of **18** printed pages.

Set by: Mr Albert Lui
Vetted by: Mdm Norhafiani Bte Abdul Majid

Mathematical Formulae

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\Delta = \frac{1}{2}ab\sin C$$

<u>Answer **ALL** the questions</u>

1 (a) The function f is defined, for all values of x, by $f(x) = (2x - x^2)e^x$.

Find the range of values of x such that $f(x)$ is a decreasing function. [4]

$f'(x) = (2 - 2x)e^x + (2x - x^2)e^x$

$= e^x(2 - x^2)$ B1

Decreasing Function:

$f'(x) < 0$

$e^x(2 - x^2) < 0$ M1

Since $e^x > 0$,

$2 - x^2 < 0$

$x^2 - 2 > 0$

$\left(x + \sqrt{2}\right)\left(x - \sqrt{2}\right) > 0$ M1

$x < -\sqrt{2}$ $x > \sqrt{2}$ A1

(b) The gradient function of the curve is $2(p + 1)x + 2$, where p is a constant.

Given that the tangent to the curve at $(2, -2)$ is parallel to $y + 2x - 5 = 0$, find the value of p. [3]

$\frac{dy}{dx} = 2(p + 1)x + 2$

$2(p + 1)x + 2 = -2$ M1

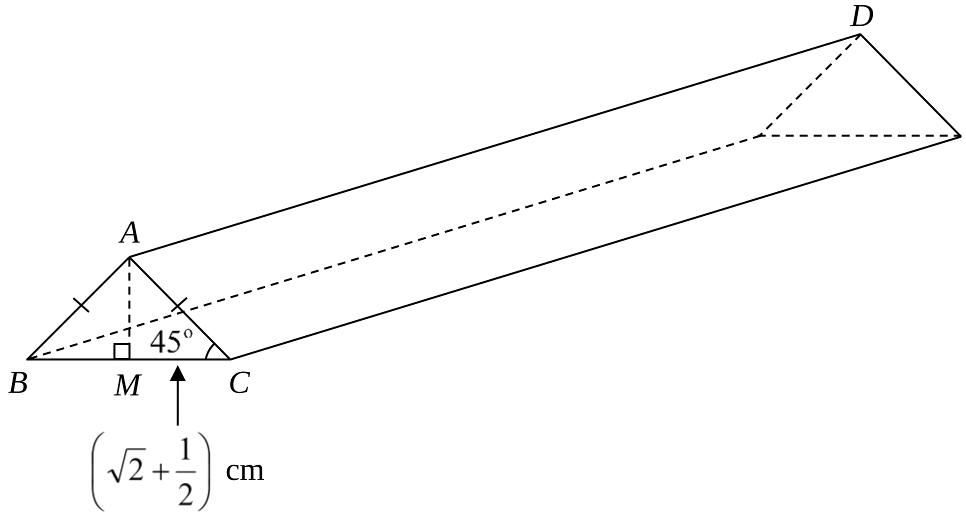
$(p + 1)x = -2$ M1

When $x = 2$,

$2p + 2 = -2$

$p = -2$ A1

2 The diagram shows a chocolate bar in the form of a triangular prism and the cross-section of the chocolate bar is an isosceles triangle with $AB = AC$.
$MC = \left(\sqrt{2} + \frac{1}{2}\right)$ cm and $\angle ACB = 45°$.



(a) Find the exact length of $\boldsymbol{AC}$. [3]

$\mathbf{cos\,45°} = \dfrac{\sqrt{2}+\frac{1}{2}}{AC}$ M1

$\boldsymbol{AC} = \dfrac{2\left(\sqrt{2}+\frac{1}{2}\right)}{\sqrt{2}}$

$= \dfrac{2\sqrt{2}+1}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$ M1

$= \mathbf{2} + \dfrac{\sqrt{2}}{2}$ or $\dfrac{4+\sqrt{2}}{2}$ A1

(b) Given that the volume of the chocolate bar is $\left(\mathbf{25} + \mathbf{22}\sqrt{\mathbf{2}}\right)\mathbf{cm^3}$, find the length of $\boldsymbol{AD}$ in the form $\left(\boldsymbol{a} + \boldsymbol{b}\sqrt{\mathbf{2}}\right)$ cm, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are integers. [4]

Vol $= \frac{1}{2} \times \left(\frac{4+\sqrt{2}}{2}\right) \times \left(\frac{4+\sqrt{2}}{2}\right) \times \boldsymbol{AD}$ M1

$\mathbf{25} + \mathbf{22}\sqrt{\mathbf{2}} = \dfrac{9+4\sqrt{2}}{4}\boldsymbol{AD}$

$\boldsymbol{AD} = \dfrac{25+22\sqrt{2}}{\frac{9+4\sqrt{2}}{4}}$

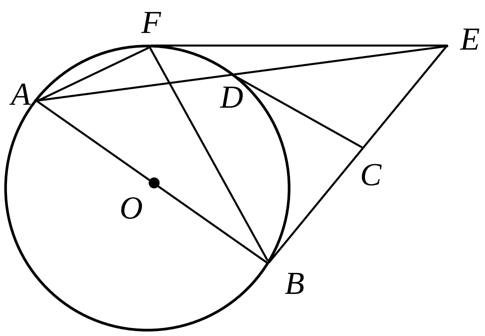
$= \dfrac{100+88\sqrt{2}}{9+4\sqrt{2}} \times \dfrac{9-4\sqrt{2}}{9-4\sqrt{2}}$ M1

$= \dfrac{900-400\sqrt{2}+792\sqrt{2}-704}{49}$

$= \dfrac{196+392\sqrt{2}}{49}$ M1

$= \mathbf{4} + \mathbf{8}\sqrt{\mathbf{2}}$ A1

3



The diagram shows a circle, centre *O*, with diameter *AB*. The points *D* and *F* lie on the circle. The point *E* is such that *EB* and *EF* are tangents to the circle.

(a) Given that the points C and D are midpoints of BE and AE respectively, prove that angle $DCE = 90°$. [3]

$\angle ABC = 90°$ (tangent perpendicular radius) M1
DC parallel AB (mid point theorem) M1
Angle $DCE = 90°$ (corresponding angles) A1

(b) Given that triangle *BEF* is equilateral, prove that $\angle BEF = \angle BAF$. [2]

$\angle EBF = \angle BAF$ (alternate segment theorem) M1

Since $\angle EBF = \angle BEF$,
$\angle BEF = \angle BAF$ (shown) A1

4 **(a)** Find the remainder when $6x^3 - 13x^2 + 17x - 6$ is divided by $2x - 1$. [2]

When $x = \frac{1}{2}$,

Remainder $= 6\left(\frac{1}{2}\right)^3 - 13\left(\frac{1}{2}\right)^2 + 17\left(\frac{1}{2}\right) - 6$ M1

$= 0$ A1

(b) Show that there is only one real root of the equation

$6x^3 - 13x^2 + 17x - 6 = 0.$ [3]

$(2x-1)(6x^2 - 10x + 12) = 0$ B1

$x = \frac{1}{2}$ $6x^2 - 10x + 12 = 0$

$3x^2 - 5x + 6 = 0$

Discriminant:

$b^2 - 4ac = 25 - 4(3)(6)$

$= -47$ B1

Since $-47 < 0$,

$3x^2 - 5x + 6 = 0$ has no real roots, hence equation has only 1 real root which is $x = \frac{1}{2}$. B1

5 Solve the following equations.

(a) $5^x - 5^{\frac{x}{2}+1} = 6,$ [3]

Let $y = 5^{\frac{x}{2}}$

$y^2 - 5y - 6 = 0$ M1

$y = 6$ $y = -1$ (reject) A1

$\frac{x}{2}\lg 5 = \lg 6$

$x = 2.23$ B1

(b) $2\lg(x-3)-\lg(x+7)=\frac{1}{\log_{100}10}$. [4]

$\lg\frac{(x-3)^2}{x+7}=\frac{\lg 100}{\lg 10}$ M2

$100=\frac{(x-3)^2}{x+7}$ M1

$x^2-106x-691=0$

$x=112$ or $x=-6.16$ (rej) A1

6 **(a)** State the values between which the principal value of $\sin^{-1}x$ must lie. [1]

$-90° \leq \sin^{-1}x \leq 90°$

$-\frac{\pi}{2} \leq \sin^{-1}x \leq \frac{\pi}{2}$

(b) Find the principal value of $\tan^{-1}1$ in radian in exact form. [1]

Principal value $=\frac{\pi}{4}$

7 Given that $\cot\theta=-\frac{3}{4}$ and that $\tan\theta$ and $\cos\theta$ have opposite signs, without evaluating θ, find the exact values of each of the following.

(a) $\cos(-\theta)$, [2]

$\tan\theta=-\frac{4}{3}$, lies in 4th quadrant M1

$\cos(-\theta)=\cos\theta$

$=\frac{3}{5}$ A1

(b) $\sin 2\theta$ [2]

$=2\sin\theta\cos\theta$

$=2\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)$ M1

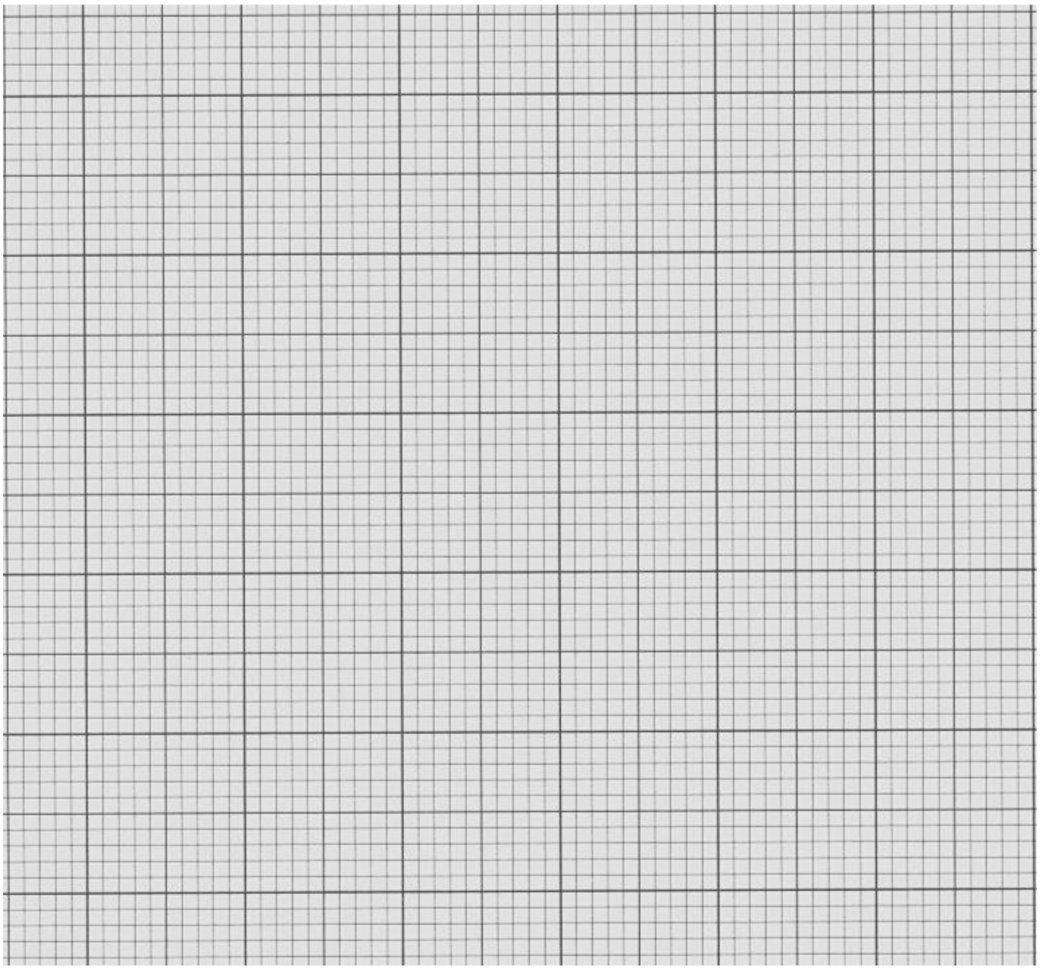
$=-\frac{24}{25}$ A1

8. The approximate mean distance ***x*** (in millions of kilometres) from the centre of the Sun and the period of the orbit ***T*** (in Earth years) are recorded in the table.

	Mercury	**Venus**	**Mars**	**Uranus**
x	58	108	228	2871
T	0.24	0.62	1.88	84.11

It is believed that the planets orbiting around the Sun obey a law of the form $T = kx^n$, where k and n are constants.

(a) Express the equation in a form suitable for drawing a straight line graph and draw the graph using appropriate scaling on both axes. [4]



$\lg T = lgk + nlgx$

(b) Use your graph to estimate the value of k and of n, to two significant figures. [3]

Lg k = -3.2

k = 0.00063 (0.00063 to 0.00079)

n = $\frac{2-9-3.2)}{3.5-0} = 1.49 = 1.5$ (1.4 to 1.6)

(c) Using the graph, find the orbital period of the Earth, if the distance between the Earth and the Sun is about $149.6 \times 10^6\, km$. Give your answer correct to the nearest integer. [2]

lg 149.6 = 2.17 = lg x

lg T = 0 ⇒ T = 1 (0.79 to 1.25)

(d) If the orbital period of the Jupiter is 11.86 Earth years, estimate the distance of the Jupiter from the Sun in km using your graph. [2]

lg 11.86 = 1.07

lg x = 2.9 = 794000000 km (631000000 to 1000000000 km)

9. **(a)** Express $\frac{2x^3+2x^2-7x+4}{x(x-1)^2}$ in partial fractions. [5]

$= 2 + \frac{6x^2-9x+4}{x(x-1)^2}$ B1

Let $\frac{6x^2-9x+4}{x(x-1)^2} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2}$

$6x^2 - 9x + 4 = A(x-1)^2 + Bx(x-1) + Cx$

When $x = 0$,

$4 = A$ B1

When $x = 1$,

$6 - 9 + 4 = C$

$C = 1$ B1

When $x = -1$,

$6 + 9 + 4 = 4(4) + 2B - 1$

$2B = 4$

$B = 2$ B1

$\frac{2x^3+2x^2-7x+4}{x(x-1)^2} = 2 + \frac{4}{x} + \frac{2}{x-1} + \frac{1}{(x-1)^2}$ B1

(b) Hence evaluate $\int_2^4 \frac{4x^3+4x^2-14x+8}{3x(x-1)^2}\, dx$. [4]

$$\int_2^4 \frac{4x^3+4x^2-14x+8}{3x(x-1)^2}\, dx = \frac{2}{3}\int_2^4 \frac{2x^3+2x^2-7x+4}{x(x-1)^2}\, dx \qquad \text{M1}$$

$$= \frac{2}{3}\int_2^4 \left[2 + \frac{4}{x} + \frac{2}{x-1} + \frac{1}{(x-1)^2}\right] dx$$

$$= \frac{2}{3}\left[2x + 4\ln x + 2\ln(x-1) - \frac{1}{x-1}\right]_2^4 \qquad \text{M1}$$

$$= \frac{2}{3}\left[8 + 4\ln 4 + 2\ln 3 - \frac{1}{3} - 4 - 4\ln 2 - 2\ln 1 + 1\right]$$

$$= \frac{2}{3}\left[\frac{14}{3} + 4\ln 2 + 2\ln 3\right] \qquad \text{M1}$$

$$= 6.42 \qquad \text{A1}$$

10. **(a)** Find the range of values of k for which the line $2x - y = 5$ intersects the curve $xy = kx - 2$ at two distinct points. [4]

$x(2x-5) = kx - 2$

$2x^2 - 5x - kx + 2 = 0$ B1

Intersects at 2 distinct points:

$(-5-k)^2 - 4(2)(2) > 0$ M1

$k^2 + 10k + +9 > 0$

$(k+1)(k+9) > 0$

$k < -9$ $\qquad$ $k > -1$ A2

(b) Find the smallest integer value of h for which the graph $y = 2x^2 - 4x + h$ lies entirely above the line $y = 3$ for all values of x. [3]

$2x^2 - 4x + h - 3 > 0$

Curve lies above line:

$b^2 - 4ac < 0$ M1

$(-4)^2 - 4(2)(h - 3) < 0$

$8h > 40$

$h > 5$ A1

smallest integer value of $h = 6$ B1

11. **(a)** Prove the identity $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = 2\operatorname{cosec}\theta$. [4]

LHS $= \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$

$= \frac{1+2\cos\theta+\cos^2\theta+\sin^2\theta}{\sin\theta(1+\cos\theta)}$ M2

$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$ M1

$= 2\operatorname{cosec}\theta$ A1

$= RHS$ (shown)

(b) Hence, find all the angles from $0° \leq \theta \leq 360°$ which satisfy the equation $\frac{1+\cos 2\theta}{\sin 2\theta} + \frac{\sin 2\theta}{1+\cos 2\theta} = \tan 75°$. [3]

$2\operatorname{cosec} 2\theta = \tan 75°$

$\sin 2\theta = \frac{2}{\tan 75°}$ M1

$basic\ angle = \sin^{-1}\frac{2}{\tan 75°}$

$= 32.404858°$ M1

$2\theta = 32.404858°, 180° - 32.404858°, 32.404858° + 360°, 540° - 32.404858°$

$\theta = 16.2°, 73.8°, 196.2°, 253.8°$ A1

12. Find the derivatives of each of the following, simplifying your answer.

(a) $y = 3\left(1 - \frac{x}{3}\right)^4$ [1]

$\frac{dy}{dx} = 12\left(-\frac{1}{3}\right)\left(1 - \frac{x}{3}\right)^3$

$= -4\left(1 - \frac{x}{3}\right)^3$

(b) $f(x) = (2 - 3x)\left(\sqrt{1 - 4x}\right)$ [3]

$f'(x) = -3\left(\sqrt{1 - 4x}\right) + \frac{1}{2}(-4)(2 - 3x)(1 - 4x)^{-\frac{1}{2}}$ M1

$= (1 - 4x)^{-\frac{1}{2}}\left[-3(1 - 4x) - 4 + 6x\right]$ M1

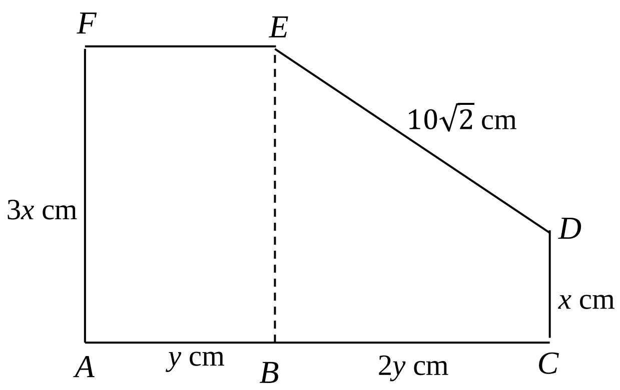
$= \frac{18x - 7}{\sqrt{1 - 4x}}$ A1

(c) $\frac{dy}{dx} = \frac{2(3x-2)}{4+x}$ [2]

$\frac{d^2y}{dx^2} = \frac{6(4+x)-(6x-4)}{(4+x)^2}$ M1

$= \frac{28}{(4+x)^2}$ A1

13.



The diagram shows a glass window $ABCDEF$, consisting of a rectangle $ABEF$ of height $3x$ cm and width y cm and a trapezium $BCDE$ in which $CD = x$ cm and $BC = 2y$ cm. ABC is a straight line and $DE = 10\sqrt{2}$ cm.

Given that x can vary,

(a) show that the area of the glass window $S = 7x\left(\sqrt{50 - x^2}\right)$, [3]

Looking at triangle,

$4y^2 + 4x^2 = 200$

$y^2 + x^2 = 50$ B1

Total area $A = 3xy + \frac{1}{2}(x + 3x)(2y)$ M1

$= 7x\left(\sqrt{50 - x^2}\right)$ A1

(b) find the value of x for which S has a stationary value and determine whether this value of A is a maximum or a minimum. [5]

$\frac{dS}{dx} = 7\left(\sqrt{50 - x^2}\right) + \frac{1}{2}(-2x)(7x)(50 - x^2)^{-\frac{1}{2}}$

$= (50 - x^2)^{-\frac{1}{2}}[7(50 - x^2) - 7x^2]$

$= \frac{350 - 14x^2}{\sqrt{50 - x^2}}$ B1

Stationary value of S:

$\frac{dS}{dx} = 0$ M1

$350 - 14x^2 = 0$

$x = 5$ $\qquad$ $x = -5$ (rej) $\qquad$ A1

	$x = 4.9$	$x = 5$	$x = 5.1$
$\frac{dS}{dx}$	2.72	0	-2.89
shape	/	—	\

Proof B1 (can be 1st or 2nd derivative)

When $x = 5$, S is a maximum $\qquad$ B1

14. It is given that $f(x)$ is such that $f'(x) = \cos 4x - \sin 2x$. Given also that $f\left(\frac{\pi}{2}\right) = \frac{1}{4}$, show that $f''(x) + 4f(x) = 3 - 3\sin 4x$. [5]

$f''(x) = -4\sin 4x - 2\cos 2x$ B1

$f(x) = \int(\cos 4x - \sin 2x)\ dx$

$= \frac{\sin 4x}{4} + \frac{\cos 2x}{2} + c$ B1

When $x = \frac{\pi}{2}$,

$\frac{1}{4} = -\frac{1}{2} + c$

$c = \frac{3}{4}$

$f(x) = \frac{\sin 4x}{4} + \frac{\cos 2x}{2} + \frac{3}{4}$ B1

$f''(x) + 4f(x) = -4\sin 4x - 2\cos 2x + 4\left[\frac{\sin 4x}{4} + \frac{\cos 2x}{2} + \frac{3}{4}\right]$ M1

$= -4\sin 4x - 2\cos 2x + \sin 4x + 2\cos 2x + 3$

$= 3 - 3\sin 4x$ (shown) A1