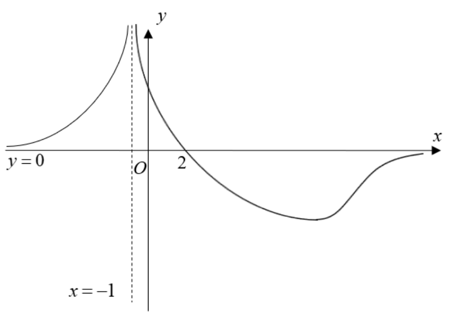
2024 Year 5 H2 Math Practice Paper 1 Suggested Solution

1	chocolate and walnut cakes respectively. $5b+3c+2w=52$ ⋯ (1) $3b+4c+5w=68$ ⋯ (2) $4b+8c+2w=75$ ⋯ (3) Using GC, $b=4.50$, $c=5.50$ and $w=6.50$

2	$y=\frac{x^2}{4}+4$ ↓ C': Replace y by $-y$ $-y=\frac{x^2}{4}+4$ ↓ B': Replace x by $2x$ $-y=\frac{(2x)^2}{4}+4$ $-y=x^2+4$ ↓ A': Replace y by $y-4$ $-(y-4)=x^2+4$ $-y+4=x^2+4$ $y=-x^2$ Equation of the original curve: $y=-x^2$

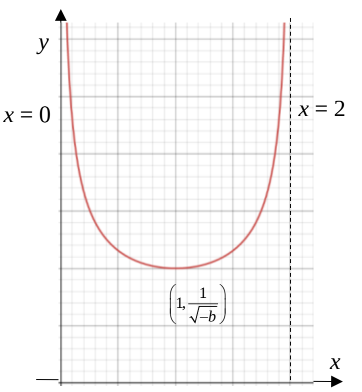
3(a)	For $y=\frac{1}{\mathrm{f}(x)}$ to be increasing, $y=\mathrm{f}(x)$ will be decreasing. Range of values: $x\ge 2$.
(b)	

Commented [OMFJ1]: SY - vertical asymptote, origin
Commented [KSY2R1]: Is this ok?

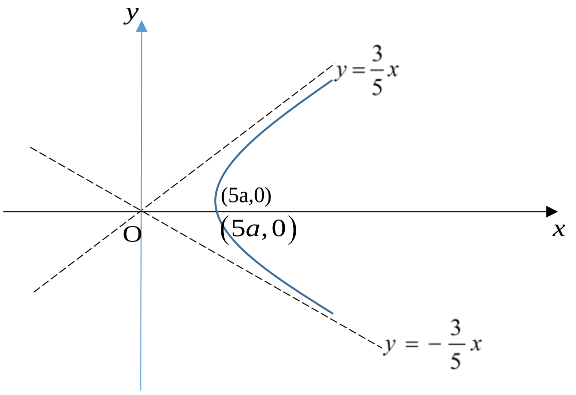
	

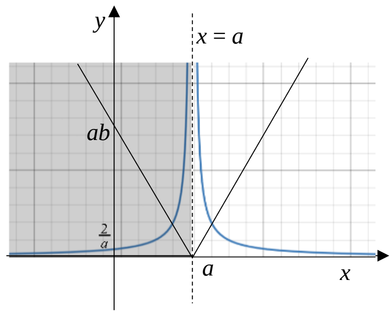
4(a)	Since f(1) = f(4) = 5, f is not one-one. Hence, f does not have an inverse.
(b)	Let $y=\mathrm{f}(x)=\dfrac{x^2+4}{x}, \quad 0<x\le 2$ $xy=x^2+4$ $x^2-yx+4=0$ $\left(x-\dfrac{y}{2}\right)^2-\dfrac{y^2}{4}+4=0$ $\left(x-\dfrac{y}{2}\right)^2-\dfrac{y^2-16}{4}=0$ $\left(x-\dfrac{y}{2}\right)^2=\dfrac{y^2-16}{4}$ $x-\dfrac{y}{2}=\pm\sqrt{\dfrac{y^2-16}{4}}$ $x=\dfrac{y}{2}\pm\sqrt{\dfrac{y^2-16}{4}}$ $x=\dfrac{y\pm\sqrt{y^2-16}}{2}$ Since $0<x\le 2$, $x=\dfrac{y-\sqrt{y^2-16}}{2}$ Hence, $\mathrm{f}^{-1}(x)=\dfrac{x-\sqrt{x^2-16}}{2}, \quad x\ge 4.$ $D_{\mathrm{f}^{-1}}=[4,\infty)$ **Alternative** (to check plus or minus): When $x=1$, $\mathrm{f}(x)=\dfrac{1^2+4}{1}=5$ When $y=5$,

	$x=\dfrac{5+\sqrt{5^2-16}}{2}=4$ (rej +ve), $x=\dfrac{5-\sqrt{5^2-16}}{2}=1$ Hence $f^{-1}(x)=\dfrac{x-\sqrt{x^2-16}}{2}$, $x\geq 4$.

5	bx^2-2bx $=b(x^2-2x)$ $=b\left((x-1)^2-1\right)$ $=b(x-1)^2-b$ Maximum value at $-b$ when x = 1. Hence turning (minimum) point is at $\left(1,\dfrac{1}{\sqrt{-b}}\right)$ $y=\dfrac{1}{\sqrt{bx^2-2bx}}$ $bx^2-2bx=0$ $bx(x-2)=0$ $x=0$ or $x=2$  Line of symmetry: x = 1.

6(i)	$x=5a\sec\theta, y=3a\tan\theta$ $\tan^2\theta+1=\sec^2\theta$ $(\frac{y}{3a})^2+1=(\frac{x}{5a})^2, x\geq 5a$
(ii)	

Qn	Suggested Solution
7(a)	
(b)	$\frac{2}{\lvert x-a\rvert}=b\lvert x-a\rvert$ $(x-a)^2=\frac{2}{b}$ $x-a=\pm\sqrt{\frac{2}{b}}$ $x=a\pm\sqrt{\frac{2}{b}}$ Hence, for $\frac{2}{\lvert x-a\rvert}>b\lvert x-a\rvert$. $a-\sqrt{\frac{2}{b}}<x<a+\sqrt{\frac{2}{b}},\quad x\neq a$

Commented [CMYE3]: With graph, to accept answer with symmetry deduction, to award full credit

<table>
<tr><th>Qn</th><th>Suggested Solution</th></tr>
<tr><td>8(i)</td><td>

$$\cos 3x = 1 - \frac{(3x)^2}{2!} + \frac{(3x)^4}{4!} + \ldots = 1 - \frac{9}{2}x^2 + \frac{27}{8}x^4 + \ldots$$

$$\begin{aligned}\ln(1+\cos 3x) &= \ln\left(1 + \left(1 - \frac{9}{2}x^2 + \frac{27}{8}x^4 + \ldots\right)\right)\\ &= \ln 2\left(1 - \frac{9}{4}x^2 + \frac{27}{16}x^4 + \ldots\right)\\ &= \ln 2 + \ln\left(1 - \frac{9}{4}x^2 + \frac{27}{16}x^4 + \ldots\right)\\ &= \ln 2 + \left(-\frac{9}{4}x^2 + \frac{27}{16}x^4 + \ldots\right) - \frac{\left(-\frac{9}{4}x^2 + \frac{27}{16}x^4 + \ldots\right)^2}{2} + \ldots\\ &= \ln 2 - \frac{9}{4}x^2 + \frac{27}{16}x^4 - \frac{81}{32}x^4 + \ldots\\ &= \ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4 + \ldots\end{aligned}$$

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<tr><td>(ii)</td><td>

$$\begin{aligned}\int_0^{0.5} x\ln(1+\cos 3x)\,\mathrm{d}x &= \int_0^{0.5} x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4 + \ldots\right)\mathrm{d}x\\ &= \int_0^{0.5}\left((\ln 2)x - \frac{9}{4}x^3 - \frac{27}{32}x^5 + \ldots\right)\mathrm{d}x\\ &= \left[(\ln 2)\frac{x^2}{2} - \frac{9}{16}x^4 - \frac{9}{64}x^6 + \ldots\right]_0^{0.5}\\ &\approx 0.04929 \text{ (5 d.p.)}\end{aligned}$$

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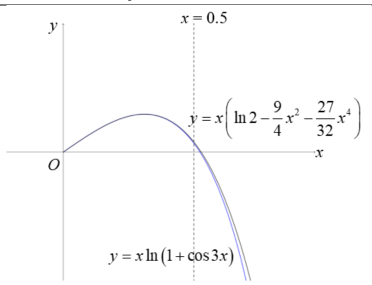
<tr><td>(iii)</td><td>Using GC, $\int_0^{0.5} x\ln(1+\cos 3x)\mathrm{d}x = 0.04900$ (5 d.p.)</td></tr>
<tr><td>(iv)</td><td>



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<tr><td></td><td>From the diagram, it can be seen that the graphs of $y = x\ln(1+\cos 3x)$ and $y = x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4\right)$ are close to each other mostly from $x = 0$ to $x = 0.5$. Hence, the approximated value of $\int_0^{0.5} x\left(\ln 2 - \frac{9}{4}x^2 - \frac{27}{32}x^4 + \ldots\right)\mathrm{d}x$ from (ii) is approximately equal to the actual value of $\int_0^{0.5} x\ln(1+\cos 3x)\mathrm{d}x = 0.04900$ (5 d.p.) in (iii).</td></tr>
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	Suggested Solution
9a (i)	$\sum_{r=0}^{n}\frac{(x+3)^r}{4^{r+1}}=\frac{1}{4}\sum_{r=0}^{n}\left(\frac{x+3}{4}\right)^r$ $=\frac{1}{4}\sum_{r=0}^{n}\left[\left(\frac{x+3}{4}\right)^0+\left(\frac{x+3}{4}\right)^1+\ldots+\left(\frac{x+3}{4}\right)^n\right]$ $=\frac{1}{4}\left[\frac{1-\left(\frac{x+3}{4}\right)^{n+1}}{1-\frac{x+3}{4}}\right]$ $=\frac{1}{4}\left[\frac{1-\left(\frac{x+3}{4}\right)^{n+1}}{\frac{1-x}{4}}\right]$ $=\frac{1}{1-x}\left[1-\left(\frac{x+3}{4}\right)^{n+1}\right]$
(ii)	Common ratio, r of G.P. = $\frac{x+3}{4}$ When $x=-5$, $r=\frac{-5+3}{4}=-\frac{1}{2}$. Since $\lvert r\rvert=\frac{1}{2}<1$, the G.P. converges. Hence, the series $\sum_{r=0}^{n}\frac{(x+3)^r}{4^{r+1}}$ converges. $\lim_{n\to\infty}\sum_{r=0}^{n}\frac{(-5+3)^r}{4^{r+1}}=\lim_{n\to\infty}\frac{1}{1-(-5)}\left[1-\left(\frac{-5+3}{4}\right)^{n+1}\right]$ $=\lim_{n\to\infty}\frac{1}{6}\left[1-\left(-\frac{1}{2}\right)^{n+1}\right]$ $=\frac{1}{6}$
(b)(i)	$\sum_{r=6}^{2k}r(3r-2)=\sum_{r=6}^{2k}(3r^2-2r)$ $=3\left[\sum_{r=1}^{2k}r^2-\sum_{r=1}^{5}r^2\right]-2\sum_{r=6}^{2k}r$ $=3\left[\frac{2k}{6}(2k+1)(4k+1)-\frac{5}{6}(6)(11)\right]$ $-2\left(\frac{2k-6+1}{2}(6+2k)\right)$ $=k(2k+1)(4k+1)-165-2(2k-5)(3+k)$ $=k(2k+1)(4k+1)-2(2k-5)(k+3)-165$

Qn	Suggested Solution
10(a)	$\int \frac{1-3x}{1+9x^2}\,\mathrm{d}x$ $= \int \frac{1}{1+9x^2}\,\mathrm{d}x - \int \frac{3x}{1+9x^2}\,\mathrm{d}x$ $= \frac{1}{9}\int \frac{1}{\left(\frac{1}{3}\right)^2 + x^2}\,\mathrm{d}x - \frac{1}{6}\int \frac{18x}{1+9x^2}\,\mathrm{d}x$ $= \frac{1}{3}\tan^{-1}(3x) - \frac{1}{6}\ln\left(1+9x^2\right) + c$
(b)	$x = 2\sin\theta \;\Rightarrow\; \frac{\mathrm{d}x}{\mathrm{d}\theta} = 2\cos\theta$ $\int \frac{(x-1)^2}{\sqrt{4-x^2}}\,\mathrm{d}x = \int \frac{(2\sin\theta-1)^2}{\sqrt{4-4\sin^2\theta}}(2\cos\theta)\;\mathrm{d}\theta$ $\int \frac{(2\sin\theta-1)^2}{2\cos\theta}(2\cos\theta)\;\mathrm{d}\theta$ $= \int 4\sin^2\theta - 4\sin\theta + 1\;\mathrm{d}\theta$ $= \int 4\left(\frac{1-\cos 2\theta}{2}\right) - 4\sin\theta + 1\;\mathrm{d}\theta$ $= \int 3 - 2\cos 2\theta - 4\sin\theta\;\mathrm{d}\theta$ $= 3\theta - \sin 2\theta + 4\cos\theta + c$ $= 3\sin^{-1}\left(\frac{x}{2}\right) - \frac{x\sqrt{4-x^2}}{2} + 2\sqrt{4-x^2} + c$
(c)	$\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1}(2x^2)\right) = \frac{4x}{\sqrt{1-\left(2x^2\right)^2}}$ $= \frac{4x}{\sqrt{1-4x^4}}$ $\int\left(2x\sin^{-1}(2x^2)\right)\mathrm{d}x = x^2\sin^{-1}(2x^2) - \int \frac{4x^3}{\sqrt{1-4x^4}}\,\mathrm{d}x$ $= x^2\sin^{-1}(2x^2) + \frac{1}{4}\int \frac{-16x^3}{\sqrt{1-4x^4}}\,\mathrm{d}x$ $= x^2\sin^{-1}(2x^2) + \frac{1}{4}\frac{\left(1-4x^4\right)^{\frac{1}{2}}}{\frac{1}{2}} + c$ $= x^2\sin^{-1}(2x^2) + \frac{1}{2}\left(1-4x^4\right)^{\frac{1}{2}} + c$

Qn	Suggested Solution
11 (i)	The amount of money in the investment account after 1 month is 111% of the investment at the beginning of the month, i.e. GP with a = 1500 and r = 1.11 Hence the projected profit at the end of 1 full year = $1500(1.11)^{12} - 1500$ = \$3747.68
11 (ii)	<table><tr><td>End of r^{th} month</td><td>Amt in account</td></tr><tr><td>1</td><td>200(1.11)</td></tr><tr><td>2</td><td>[200 + 200(1.11)](1.11) = 200(1.11) + 200(1.11)^2</td></tr><tr><td>3</td><td>[200 + 200(1.11) + 200(1.11)^2](1.11) = 200(1.11) + 200(1.11)^2 + 200(1.11)^3</td></tr></table> $\therefore$ the projected amount of money $= 200(1.11) + 100(1.11)^2 + \ldots + 100(1.11)^n$ $= \dfrac{200(1.11)\left(1.11^n - 1\right)}{1.11-1}$ $= \dfrac{22200}{11}\left[\left(\dfrac{111}{100}\right)^n - 1\right]$
11 (iii)	Projected amount of money in Carl's investment account at the end of one year $= \dfrac{22200}{11}\left[\left(\dfrac{111}{100}\right)^{12} - 1\right] = \$5042.33$ (2 d.p.)
11 (iv)	Let n be the number of months after Carl started his investment. $\therefore \dfrac{22200}{11}\left[\left(\dfrac{111^n}{100}\right) - 1\right] > 1500(1.11)^n$ Let $T_n = \dfrac{22200}{11}\left[\left(\dfrac{111^n}{100}\right) - 1\right] - 1500(1.11)^n$ From GC, $T_{13} = -5.937 < 0$ $T_{14} = 215.41 > 0$ $T_{15} = 461.11 > 0$ On 31 Jan 2023, Betty has \$5.94 more in her account than Carl. So, Carl would have more money in his account on the following day, ie. 1 Feb 2023. On 31 Jan 2023, Carl's projected amount in his account = $\dfrac{22200}{11}\left[\left(\dfrac{111^{13}}{100}\right) - 1\right]$ = \$5818.98 Betty's projected amount in her account = $1500(1.11^{13})$ = \$5824.92 Carl still has less money in his account than what Betty on 28 Feb 2023 On 1 Feb 2023, Carl's projected amount will increase to \$6018.98 > \$5824.92. Hence, Carl will have more money in his account than Betty has in hers on 1 Feb 2023.
(v)	The amount of money lost due to scam cases each year forms a GP with first term \$633.3 million and common ratio 0.95.

	$\therefore$ The estimated total amount of money lost due to scam cases $= \dfrac{633.3}{1-0.95}$ million = \$12666 million

Qn	Suggested Solutions
12(a) (i) (A)	$u_{n+1} = 2u_n - k, \quad u_1 = a = 4,$ $k = 1:$ The population is always on the uptrend. Population will grow to **infinity**
(B)	$u_1 = 4,\ k = 5$ $u_2 = 2(4) - 5 = 3$ $u_3 = 2(3) - 5 = 1$ $u_4 = 2(1) - 5 = -3 < 0$ The population will become **extinct** by 4th year
(ii)	$u_2 = 2(4) - k = 8 - k$ $u_3 = 2u_2 - k = 2(8-k) - k \ = 16 - 3k = 52$ $\therefore k = -12$
(b)	$v_n = u_n - k$ $\dfrac{v_{n+1}}{v_n} = \dfrac{u_{n+1} - k}{u_n - k}$ $= \dfrac{2u_n - k - k}{u_n - k} = 2$ (a constant) Thus v_n is a geometric progression $v_n = v_1(2)^{n-1} = (u_1 - k)2^{n-1} = (a-k)2^{n-1}$ $u_n = v_n + k = (a-k)2^{n-1} + k$ $u_{n+1} = (a-k)2^n + k$
(c)	$2^n \to \infty$ as $n \to \infty$ For population to stay stabilized as $n \to \infty$, $a - k = 0$, i.e. $a = k$
(d)	Rewriting $u_n = (a-k)2^{n-1} + k$ $S = (a-k)\left[2^4 + 2^9 + ... + 2^{79}\right] + 16k$ $= \dfrac{2^4\left[(2^5)^{16} - 1\right]}{2^5 - 1}(a-k) + 16k$ $= \dfrac{16}{31}\left[2^{80} - 1\right](a-k) + 16k$

Commented [OMFJ4]: amended parts

13(a)	$\int_0^1 2ex-2x^2e^{2-x}dx=2e\left[\frac{x^2}{2}\right]_0^1-2\int_0^1 x^2e^{2-x}dx$ $=e-\left\{2\left[x^2\frac{e^{2-x}}{-1}\right]_0^1-2\int_0^1\frac{e^{2-x}}{-1}(2x)dx\right\}$ $=e+2\left[x^2e^{2-x}\right]_0^1-4\int_0^1 e^{2-x}(x)dx$ $=e+2\left[x^2e^{2-x}\right]_0^1-4\left(\left[x\frac{e^{2-x}}{-1}\right]_0^1-\int_0^1\frac{e^{2-x}}{-1}(1)dx\right)$ $=e+2\left[x^2e^{2-x}\right]_0^1+4\left[xe^{2-x}\right]_0^1-4\left[\frac{e^{2-x}}{-1}\right]_0^1$ $=11e-4e^2$
(b)(i)	$x^2+(y-6)^2=36$ $x^2=36-(y-6)^2$ When $x=2\sqrt{5}$, $\left(2\sqrt{5}\right)^2+(y-6)^2=36$ Then $y=2$ or $10\,(\text{reject})$ <u>Method 1</u> Volume of inert gas needed $=\pi\int_2^{12}36-(y-6)^2\,dy$ $=\pi\left[36y-\frac{(y-6)^3}{3}\right]_2^{12}$ $=\frac{800}{3}\pi$ cm^3 <u>Method 2</u> Total volume of sphere $\frac{4}{3}\pi(6)^3=288\pi$ Volume of the sphere that is not filled by inert gas $=\pi\int_0^2 36-(y-6)^2\,dy$ $=\pi\left[36y-\frac{(y-6)^3}{3}\right]_0^2$ $=\frac{64}{3}\pi$ Volume of inert gas in the light bulb $=288\pi-\frac{64}{3}\pi=\frac{800}{3}\pi$ cm^3
(ii)	$x=\pm\sqrt{36-(y-6)^2}$

Commented [OMFJ5]: formula for sphere not given, unlikely they will apply this method

Commented [TKL6R5]: Can provide as alternative.

	Surface area of spherical part of light bulb $= \int_{2}^{12} 2\pi\left(\sqrt{36-(y-6)^2}\right)\sqrt{1+\left(\frac{-2(y-6)}{2\sqrt{36-(y-6)^2}}\right)^2}\,\mathrm{d}y$ $= 376.991118$
(iii)	Since 1 unit = 100 m, 1 unit^2 = 10000 m^2. Region R = $\left(11\mathrm{e}-4\mathrm{e}^2\right)\left(10^4\right)=3448.757\text{ m}^2$ Number of light bulbs the art student would acquire $=\frac{3448.757}{0.5}=6897.514\approx 6898$ The irregular shape of region R would result the light bulbs lighting up overlapping zones.