H2 Topic 03 – Dynamics

In the 17th century, Sir Isaac Newton formulated his 3 laws of motion. The process required Newton to deploy the concept of an "external agent" that can transport action at a distance, provide instantaneous motion and not be subject to resistive forces. Newton's personal reading habits at that time resulted in him imagining these "external agents" as angels. He gradually trimmed their wings and transformed this new agent into a purely objective "force". Today, Newtonian mechanics is useful for many engineering efforts in our everyday scale, like how an artillery shell travels in air, and it describes many phenomena observed. Occasionally, we can take time off to marvel at the inspiration behind "force" – that the wings of angels are ever beating invisibly and constantly providing instant messaging between objects - so that forces may exist in our modern world.

Content

- Newton's Law of motion
- Linear momentum and its conservation

Learning Outcomes

Candidates should be able to:
(a) State each of Newton's laws of motion
(b) Show an understanding that mass is the property of a body which resists change in motion (inertia)
(c) Describe and use the concept of weight as the effect of a gravitational field on a mass
(d) Define and use linear momentum as the product of mass and velocity
(e) Define and use impulse as the product of force and time of impact
(f) Relate resultant force to the rate of change of momentum
(g) Recall and solve problems using the relationship $F = ma$, appreciating that resultant force and acceleration are always in the same direction
(h) State the principle of conservation of momentum
(i) Apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension (knowledge of the concept of coefficient of restitution is not required)
(j) Show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation
(k) Show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

3.0 Introduction

In dynamics, we study the *forces* that act on a body to cause motion. The vector sum of these forces gives a resultant force that causes the body to *accelerate*. This resultant force causes change in motion.

Dynamics explain the reasons behind kinematics where we describe how a body moves under *constant acceleration*.

The Tesla Model 3 did very well in crash-testing *because its engineers understood Dynamics very well in ensuring the safety of the passengers.*

3.1 Newton's Third Law of Motion

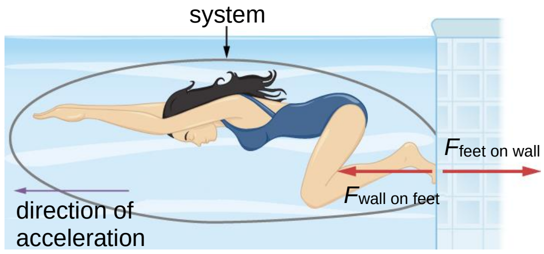
Newton's Third Law of Motion states that

when body A exerts a force on body B,

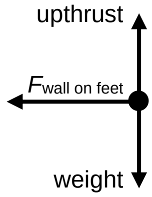
body B exerts on body A a force
of the same type,

equal in magnitude and
opposite in direction.

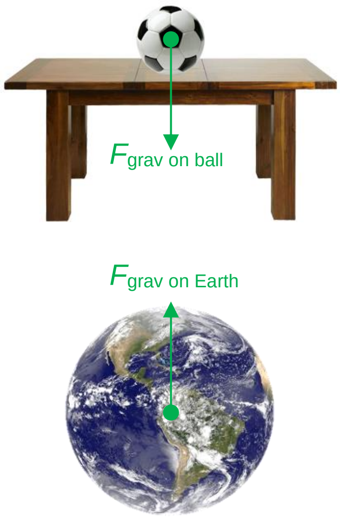
What the 3rd law refers to as "a body" can also refer to a collection of bodies. We can regard these bodies as "a system".



Free Body Diagram of a swimmer in pure horizontal motion. *The net force on swimmer is provided by the force of the wall on the feet. Newton's 3rd Law applies at the wall: the same type of force (contact force) acts different bodies (the swimmer and the wall) with equal magnitude and opposite directions.*



Example 1

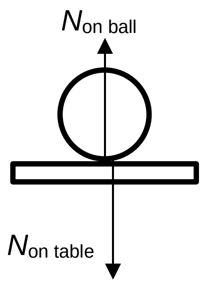


True or False: the weight of the ball and the normal contact force on ball by table are Newton's 3rd law pair of action-reaction forces.

Solution

False. The weight of the ball is the gravitational force that the Earth acts on the ball. By Newton's 3rd Law, the reaction force should be the gravitational force that the ball acts on the Earth.

For the normal contact force on the ball, the Newton's 3rd Law pair of action-reaction is the normal contact force on table by the ball.



3.2 Newton's First Law of Motion

Newton's First Law of Motion states that

an object stays at rest or continues to move at constant velocity

unless a resultant force acts on it.

Newton's 1st Law gives rise to the idea *inertia:* that a body is reluctant to change its "status quo" of motion.

The **mass** of a body is

the property of a body which resists change in motion.

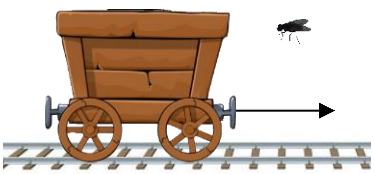
The **weight** of a body is

the force acting on the body due to a gravitational field

The weight of a body at a point (location) in space is given by $W = mg$, where m is the mass of the body and g is the gravitational field strength at that point in space.

The mass of a body remains constant anywhere in the universe while the weight changes with gravitational field strength that the body is situated in.

Example 2
A fly hovers stationary in front of an open-top rail cart that is at rest. The cart starts to move forward. Explain why the fly will hit the cart.



Solution
Fly is stationary so is in translational equilibrium with no resultant force. By Newton's 1st Law, it continues to be at rest.

Cart moves from rest so there is change in velocity hence acceleration. Frictional force by track acts on the wheels and cart accelerates towards the fly.

3.3 Linear Momentum

The **linear momentum** of a body is

the product of its mass and its velocity.

$$p = mv$$

p : linear momentum (kg m s^{-1}) or (N s).)
m : mass (kg)
v : velocity (m s^{-1})

Linear momentum is a vector quantity and it takes the same direction as the velocity of the body.

It takes work done to accelerate a body so that it gains momentum. Conversely, the more momentum a body has, the "harder" it is to reduce the momentum to zero to stop it (see Newton's 2^{nd} Law).

3.3.1 Newton's Second Law of Motion

Newton's Second Law of Motion states that the rate of change of momentum of a body is

[magnitude] directly proportional to the resultant force acting on it and

[direction] takes place in the direction of the resultant force.

Resultant force is related to

the rate of change of momentum.

$$F_{\text{net}} = \frac{\mathrm{d}p}{\mathrm{d}t}$$

The formal definition of Newton's 2nd Law reads mathematically as

$$F_{\text{net}} = k\frac{\mathrm{d}p}{\mathrm{d}t}$$

with k denoting the proportionality constant.

Considering SI units and regarding 1 N as the force which results in an acceleration of 1 m s^{-2} when it is applied to a mass of 1 kg:

$$F_{\text{net}} = (1)\frac{\mathrm{d}p}{\mathrm{d}t}$$

If the mass is constant, then by product rule

$$\begin{aligned} F_{\text{net}} &= \frac{\mathrm{d}p}{\mathrm{d}t} = \frac{\mathrm{d}}{\mathrm{d}t}(mv) \\ &= m\frac{\mathrm{d}v}{\mathrm{d}t} + v\cancelto{0}{\frac{\mathrm{d}m}{\mathrm{d}t}} \\ &= ma \end{aligned}$$

it reduces to a more familiar form.

Example 3

A cricket player catches a fast moving ball with his bare hands. Explain why it is preferable that his palms draw back while catching the ball.

Solution

To catch a ball is to reduce the momentum from just before touching the hands, to zero.

$$F_{\text{net}} = \frac{\mathrm{d}p}{\mathrm{d}t} \approx \frac{\Delta(mv)}{\Delta t} = \frac{p_{\text{final}} - p_{\text{initial}}}{\Delta t} = \frac{0 - mu}{\Delta t}$$

[N2L] For this <u>same change</u> in <u>*ball's*</u> momentum $\Delta(mv) = mu - 0$, the *time interval* Δt *when the force is applied by hand on <u>ball</u> to slow it down* is lengthened.

[force, magnitude] By Newton's 2nd Law, the average force on <u>ball</u> by hand is reduced.

[N3L] By Newton's 3rd Law, the average force on hand *by ball* is reduced. so hand experiences less pain when catching the ball.

Note: (i) F_{net} on ball is in a negative direction i.e. opposing the initial velocity of the ball – in order to slow the ball down. (ii) the body that is having the change in momentum is the body that is experiencing a net force exerted (in this case, the ball). Therefore, it is very common for explanations to demand the action-reaction pair of the force instead (in this case, the hand). Consequently, many explanation-type questions involve both N2L and N3L in a similar fashion.

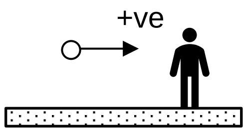
Example 4

In a game of dodge ball, a bouncy ball and a non-bouncy ball both of mass 0.095 kg are thrown directly towards you at the same speed of 20 m s^{-1}. After impact, the bouncy ball rebounds off in the opposite direction with the same speed and the non-bouncy ball comes to a stop. They both come into contact with your arm for 0.10 s.

Calculate the magnitudes of the forces acting on you by each ball.

Solution

Take direction towards "you" as positive:



$$\text{initial momentum } p_{\text{initial}} = mu = (0.095)(20) = 1.9 \text{ Ns}$$

$$\begin{aligned}\text{force on bouncy ball} &= F_{\text{on bouncy}} \\ &\approx \frac{\Delta p}{\Delta t} = \frac{p_{\text{final}} - p_{\text{initial}}}{\Delta t} = \frac{(-mv)-(mu)}{\Delta t} \\ &= \frac{(-1.9)-(1.9)}{0.10} = -38 \text{ N}\end{aligned}$$

$$\begin{aligned}\text{force on non-bouncy ball} &= F_{\text{on non-bouncy}} \\ &\approx \frac{\Delta p}{\Delta t} = \frac{p_{\text{final}} - p_{\text{initial}}}{\Delta t} = \frac{(-mv)-(mu)}{\Delta t} \\ &= \frac{(0)-(1.9)}{0.10} = -19 \text{ N}\end{aligned}$$

By Newton's 3rd Law, there is a contact force on hand by ball that is equal in magnitude and opposite in direction. Therefore same magnitudes of force: 19 N by non-bouncy ball and 38 N by bouncy ball.

Steps to solve dynamics problems:

1. Sketch a labelled force diagram. Identify and label all the forces acting <u>on</u> body (and that body <u>only</u>).
2. Determine the net force acting on the body, which will have same direction as acceleration.
3. Equate $\sum F_{\text{contributing}} = F_{\text{net}} = ma$. This mass is the mass of the body considered.

Example 5

A 1000 kg block hangs on a rope. Find the tension in the rope if

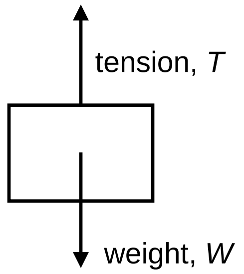
(a) the block is stationary;

(b) the block is moving upward at a constant speed of 5.0 m s^{-1};

(c) the block is accelerating upward at 5.0 m s^{-2}.

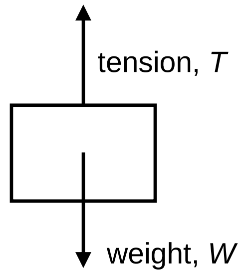
Solution: take upwards as positive

(a) $a = 0$



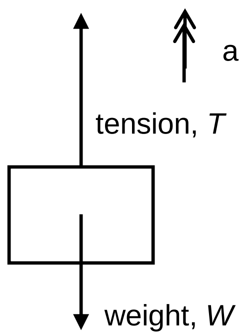
$$F_{net} = ma$$
$$T - W = 0$$
$$T - (1000)(9.81) = 0$$
$$T = 9810 \text{ N}$$

(b) constant speed so $a = 0$



$$F_{net} = ma$$
$$T - W = 0$$
$$T - (1000)(9.81) = 0$$
$$T = 9810 \text{ N}$$

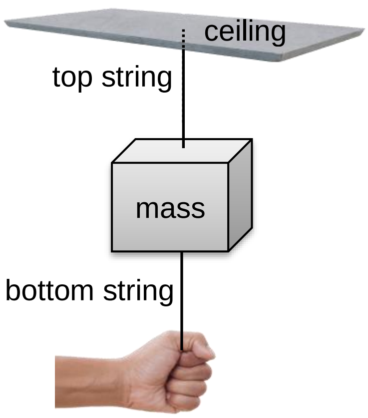
(c) $a = 5.0$ m s^{-2}



$$F_{net} = ma$$
$$T - W = ma$$
$$T - mg = 0$$
$$T - (1000)(9.81) = (1000)(5.0)$$
$$T = 14800 \text{ N}$$

Example 6

Two identical, light, <u>extensible</u> strings suspend a mass from a ceiling as shown. A person suddenly pulls down on the bottom string. Explain which string is likely to break first.



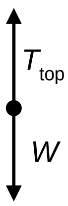
Solution

no tension in bottom string initially.

mass suspended so no resultant force in any direction because of equilibrium.

Consider vertical forces: $T_{top} = mg$

initial:

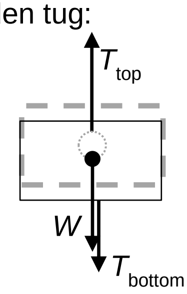


Take downwards positive.

On tug, string extends so mass accelerates down

$$F_{net} = ma$$
$$= mg + T_{bottom} - T_{top}$$
$$m(a-g) = T_{bottom} - T_{top}$$

sudden tug:



<u>Possible outcomes:</u>

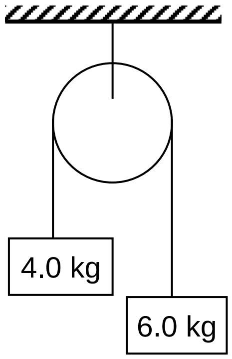
if $a > g$, then $T_{bottom} > T_{top}$
bottom string likely break first

if $a < g$, then $T_{bottom} < T_{top}$
top string likely break first

if $a = g$, then $T_{bottom} = T_{top}$
both strings equally likely to break

Example 7

A light, inextensible string connects a 4.0 kg mass and a 6.0 kg mass over a smooth, fixed pulley. Find the (i) tension in the string and (ii) the magnitude of accelerations acting on both masses.

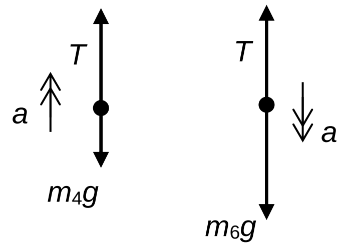


Solution

tension is same throughout string

$$F_{\text{net, }4} = T - m_4 g = m_4 a$$
$$4a = T - 4g \quad \_\_\_\_\_(1)$$

$$F_{\text{net, }6} = m_6 g - T = m_6 a$$
$$6a = 6g - T \quad \_\_\_\_(2)$$



Solving (1) and (2)

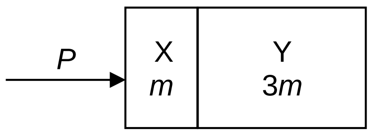
a = 1.96 m s^{-2}

T = 47.1 N

Note: magnitude of *a* is the same for both masses as string cannot be stretched and remains taut.

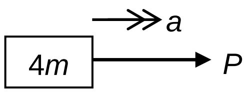
Example 8

A constant force *P* is applied to block X which is adjacent to block Y. The blocks are of masses *m* and 3*m* respectively and sit on a smooth horizontal surface. Determine the force exerted by X on Y.



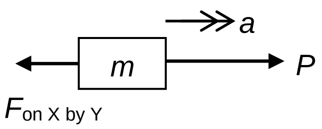
Solution

Consider X and Y as 1 system:



$$F_{\text{net}} = m_{\text{total}} a$$
$$P = 4ma \quad \_\_\_\_(1)$$

Option 1: Consider *only* forces on X:

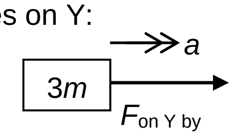


$$F_{\text{net}} = ma$$
$$ma = P - F_{\text{on X by Y}} \quad \_\_\_(2a)$$

Solving (1) and (2a)

$$F_{\text{on X by Y}} = P - ma = 4ma - ma$$
$$= 3ma$$
$$= \frac{3}{4}P$$

Option 2: Consider *only* forces on Y:



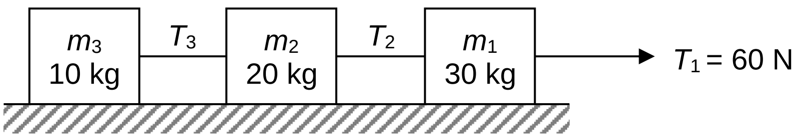
$$F_{\text{net}} = 3ma$$
$$3ma = F_{\text{on Y by X}} \quad \_\_\_(2b)$$

Newton's 3rd Law action-reaction pair

Note: (i) The action-reaction pairs $F_{\text{on X by Y}}$ and $F_{\text{on Y by X}}$ are (both) normal contact forces, between the vertical surfaces. (ii) Even though normal contact forces by floor and weight are not involved in the calculations, you need to draw and label these forces **if** asked to draw labelled force diagrams.

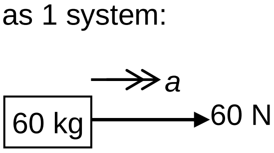
Example 9

3 blocks are connected by light, inextensible strings on a horizontal smooth surface. A force of T_1 = 60 N acts to the right on m_1. If m_1 = 30 kg, m_2 = 20 kg and m_3 = 10 kg, find the tensions T_2 and T_3.



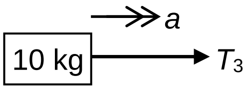
Solution

Consider all masses as 1 system:



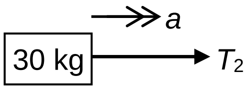
$$F_{net} = m_{total}a$$
$$60 = 60a$$
$$a = 1.0 \text{ m s}^{-2}$$

Consider *only* m_3:



$$\begin{aligned} F_{net} &= ma \\ T_3 &= 10a \\ &= 10(1.0) \\ &= 10 \text{ N} \end{aligned}$$

Consider m_3 and m_2 as a system:



$$\begin{aligned} F_{net} &= ma \\ T_2 &= (30)(1.0) \\ &= 30 \text{ N} \end{aligned}$$

Note: An alternative intermediate step can be to regard m_2 and m_1 as 1 system but the mathematics will be slightly more complicated because T_3 will act left while T_1 acts right. Through practice, we can learn to decide which combinations of masses to consider as a system to do less math.

Example 10

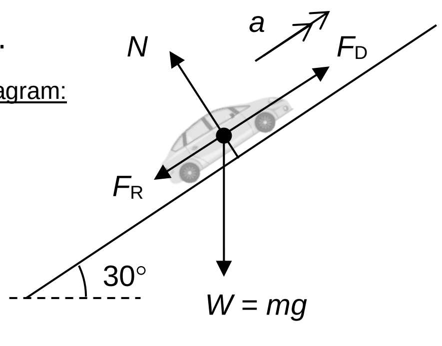
A car of mass 800 kg is moving up a hill inclined at 30° to the horizontal. The total resistive force F_R on the car is 1000 N. Calculate the driving force F_D when the car is

(a) accelerating up the incline at 2.0 m s^{-2}.

(b) moving with a steady velocity of 15 m s^{-1} up the incline.

Solution

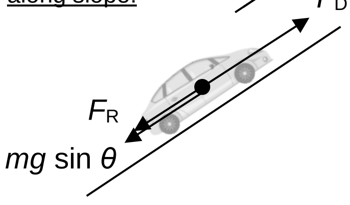
Free body diagram:



(a)
$$\begin{aligned} F_{net} &= ma \\ F_D - F_R - mg\sin\theta &= ma \\ F_D &= ma + F_R + mg\sin\theta \\ &= 800(2) + 1000 + 800(9.81)(\sin 30°) \\ &= 6520 \text{ N} \end{aligned}$$

(b)
$$\begin{aligned} F_{net} &= ma \\ 0 &= F_D - F_R - mg\sin\theta \\ F_D &= F_R + mg\sin\theta \\ &= 1000 + 800(9.81)(\sin 30°) \\ &= 4920 \text{ N} \end{aligned}$$

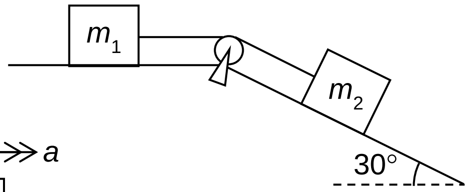
Consider motion along slope:



Note: if asked to draw a labelled force diagram, you are expected to show N (perpendicular to the incline) and $W = mg$ (vertically down).

Example 11

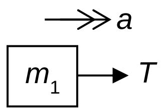
Two blocks m_1 and m_2 both of mass 5.0 kg are connected by a light inextensible string passing over a smooth pulley as shown below. The contact surfaces are smooth. Find the tension in the string.



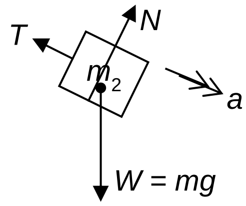
Solution

$$F_{\text{net}} = m_1 a$$
$$T = m_1 a \qquad \_\_\_(1)$$

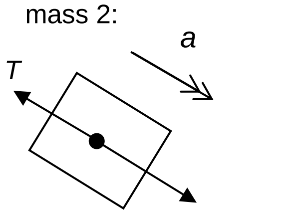
mass 1:



$$F_{\text{net}} = m_2 a$$
$$m_2 g(\sin 30°) - T = m_2 a \qquad \_\_\_(2)$$



mass 2:

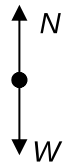


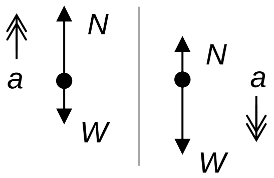
Solve (1) and (2):

$$T = 12.3 \text{ N}$$

3.3.2 Weight and 'Weightlessness'

We do not measure our weight (gravitational force on us by the Earth) directly. A weighing scale measures the *normal contact force* exerted on us when we are on it. When we are stationary, the normal contact force on us is constant, at the same magnitude as our weight.



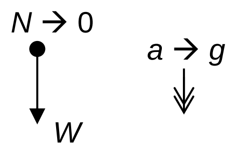


We feel "compressed" when lifts *start* to move up, or "a sense of dropping" when lifts *start* to move down. This funny sensation occurs when the lift accelerates, resulting in a change in the magnitude of normal contact force that the floor exerts on us. In other words, we are used to feeling N, the normal contact force that a floor exerts on us, as the sensation of 'weight'.

If a lift has its cables cut and all brakes missing, the lift falls freely at acceleration of g = 9.81 m s^{-2}. A person in the lift will feel the sensation of 'weightlessness'. This does not mean that the person has no weight.

'Weightlessness' refers to the state where a body experiences no contact force(s). In this case, the lift floor exerts zero contact force on the person, as both lift and person fall at the same rate.

If that person in the falling lift is originally standing on a weighing scale, the reading on the scale will read zero – recall that weighing scales do not measure our true weight. They measure the normal contact forces between our bodies and the weighing scale.

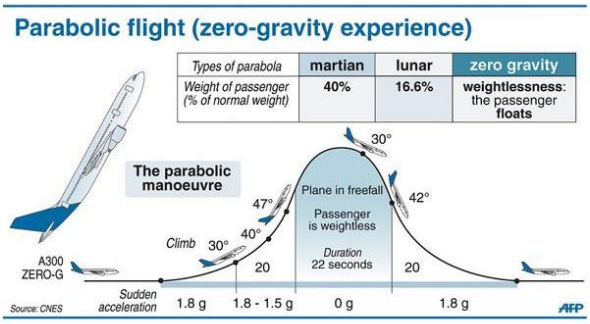


True weight does not change as long as the gravitational field is constant and uniform. *Apparent weight* is given by the reading on a weighing scale, and changes with the normal contact force between our bodies and the weighing scale.

Examples of "weightlessness"

- **only true weight acts on person so person accelerates at *g*:**

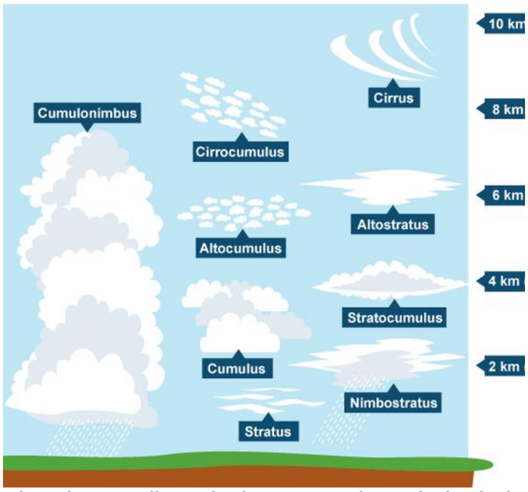
- A free-falling parachutist before parachute is deployed (ignoring air resistance)
- A free-falling bungee jumper before the cord experiences tension (ignoring air resistance)
- An astronaut inside the International Space Station (both astronaut and space station falls towards centre of Earth at the same rate)



***Zero G flights simulate weightlessness** not by "removing" gravity, but by allowing both the aircraft and its passengers to free fall at the same rate. Therefore, the "zero gravity experience" can only last a short while.*

> A scuba diver floating underwater does **not** feel weightless: diver can feel upthrust of water.

> A parachutist descending with deployed parachute does **not** feel weightless: the harness of the parachute is pulling (exerting a force upwards) on the person.



Air resistance allow rain drops to reach terminal velocity. *Otherwise, they can reach from 40% to nearly the speed of sound dropping to Earth, the latter of which can fracture skulls.*

In practice near Earth's surface, states of 'weightlessness' cannot last long due to air resistance. Falling objects tend to reach a terminal velocity.

How is terminal velocity achieved?

[initial conditions] object starts with zero speed so it experiences acceleration of *g*

[recall, forces] As object accelerates, relative speed between object and air increases so magnitude of air resistance increases

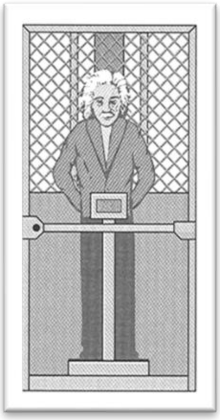
[force, magnitude] resultant force is vector sum of downwards weight and upwards air resistance which reduces to zero when air resistance is equal to weight

no net force, no acceleration so object reaches constant velocity.

See H202 Kinematics for more treatment of motion of falling objects.

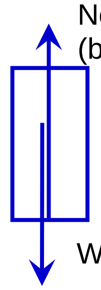
Example 12

A man of mass m stands on a platform scale in a lift.



(a) Draw a free-body diagram to show the forces acting on the man.

Normal contact force, N
(by scale on man), N



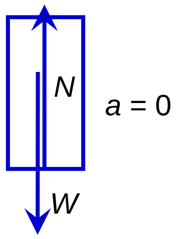
Weight, W

(b) Determine the reading on the scale when the:

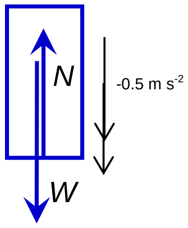
- (i) lift is stationary
- (ii) lift is going down to a lower level:
  1. accelerating at 0.5 m s^{-2}
  2. moving at constant speed v
  3. decelerating at 0.5 m s^{-2}

Taking upwards as positive:
$F_{net} = ma$

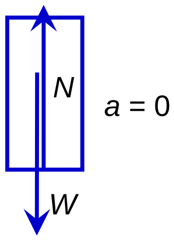
Lift is stationary
(i) $N - W = 0$
$N = W$



Lift is accelerating downwards
(ii) 1. $N - W = m(-0.5)$
$N = W - 0.5m$

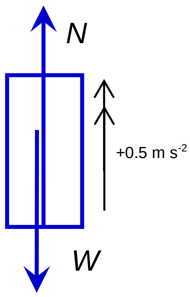


Lift is moving at constant speed
(ii) 2. $N - W = 0$
$N = W$



Lift is moving downwards and decelerating

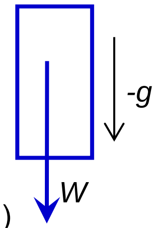
(ii) 3. $N - W = 0.5m$
$N = W + 0.5m$



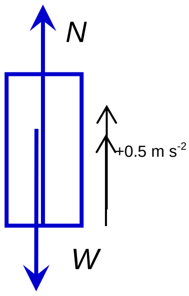
- (iii) lift is faulty and free-falling
- (iv) lift is going up to a higher level:
  1. accelerating at 0.5 m s^{-2}
  2. moving at constant speed v
  3. decelerating at 0.5 m s^{-2}

Taking upwards as positive:
$F_{net} = ma$

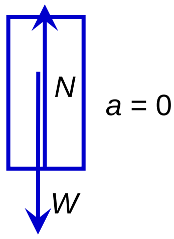
Lift is free-falling
(iii) $N - W = ma$
free-falling so acceleration is $-g$,
$N - W = m(-g)$
$N - mg = -mg$
$N = 0$ ("weightlessness")



Lift is accelerating upwards
(iv) 1. $N - W = 0.5m$
$N = W + 0.5m$

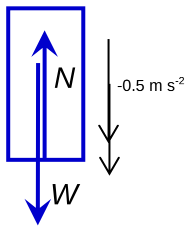


Lift is moving at constant speed
(iv) 2. $N - W = 0$
$N = W$



Lift is moving upwards and decelerating

(iv) 3. $N - W = m(-0.5)$
$N = W - 0.5m$



3.3.3 Newton's Second Law involving Flowing Mass

Certain real life situations involve a "flowing mass". Typical scenarios include a stream of water, a column of moving air, a jet of combusted gases or a continuous flow of powder or sand. In such cases, it useful to consider the change in momentum of a small mass in 1 second.

Example 13

A conveyor belt system transfers luggage at an airport. At a particular section, a horizontal belt moves at a constant speed of 1.5 m s^{-1}. The rate at which luggage is placed vertically on to the belt is 20 kg s^{-1}. Find the magnitude of average driving force *F* generated by the horizontal belt.

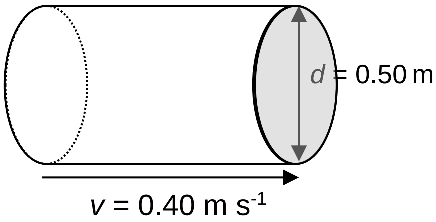
Solution

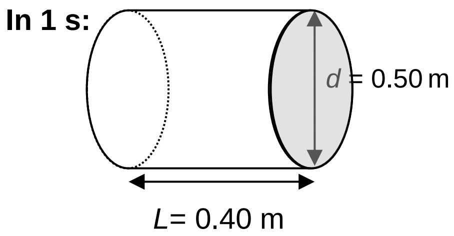
$$\langle F_{net} \rangle = \frac{p_{final} - p_{initial}}{\Delta t} = \frac{(20)(1.5) - 0}{1} = 30\text{ N}$$

Note: we can interpret this answer as the average force needed to accelerate the luggage from an initial horizontal speed of zero to that of the constant speed of 1.5 m s^{-1}.

Example 14

Water leaves a hose of diameter d = 0.050 m at a speed v = 0.40 m s^{-1}. The water hits a wall perpendicularly without rebound. The density of water is ρ = 1000 kg m^{-3}. Calculate the force exerted on the wall by the water.





Solution

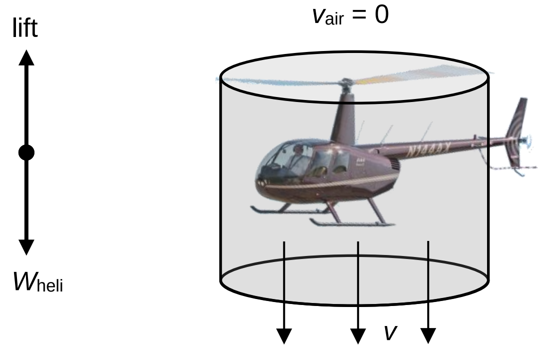
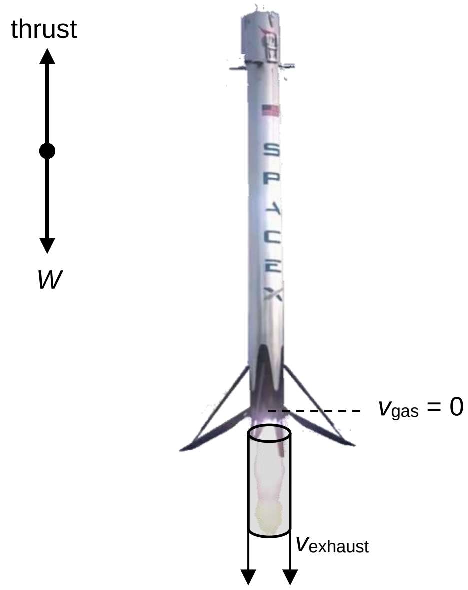
In 1 s, a cylindrical-shaped mass of water (with diameter 0.050 m and length 0.40 m) hits the wall. Final horizontal momentum $p_{\text{final}} = 0$ since the water flows downwards without rebound.

$$p_{initial} = mv = \rho(\text{volume})v$$
$$= \rho\left(\pi\left(\frac{d}{2}\right)^2 L\right)v = (1000)\pi\left(\frac{0.050}{2}\right)^2(0.40)(0.40) = 0.314\text{ kg m s}^{-1}$$

$$\langle F_{\text{on water}} \rangle = \frac{p_{final} - p_{initial}}{\Delta t} = \frac{0 - 0.314}{1} = -0.314\text{ kg m s}^{-1}$$

By Newton's 3rd Law, magnitude of force on water by wall is the same as magnitude of force on wall by water. So average force on wall is 0.314 N in the direction of the velocity of water.

Related Examples

A helicopter generates lift by pushing a column of air downwards via its rotors.  By Newton's 2nd Law, average force <u>on air</u> $\langle F_{net} \rangle = \dfrac{p_{final} - p_{initial}}{\Delta t}$ By Newton's 3rd Law, average lift on helicopter has same magnitude as force on air.	A rocket burns fuel and emits gases to produce thrust. As fuel is burnt, the mass (and weight) of the rock also decreases 

3.4 Impulse

Impulse is defined as

the product of force and time duration of impact.

$$\boxed{\Delta p = F_{\text{net}} \Delta t}$$

Δp : impulse (N s) or (kg m s^{-1})
F_{net} : net force (N)
Δt : time of impact (s)

We can think of *impulse* as the area under a (net) force-time graph which *gives the change in momentum* Δp in this time interval. It is useful for working with force(s) that vary with time.

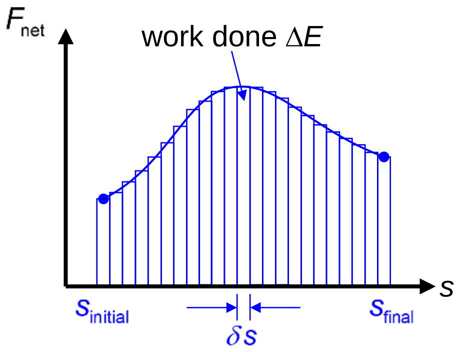
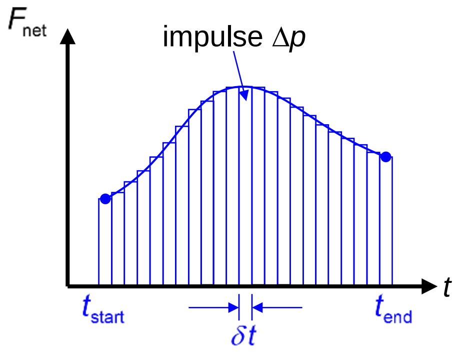
From Newton's 2nd Law,

$$F_{\text{net}} = \frac{\mathrm{d}p}{\mathrm{d}t}$$

$$\int F_{\text{net}}\ \mathrm{d}t = \Delta p$$

Hence $\quad \Delta p = \langle F_{\text{net}} \rangle (\Delta t)$ for constant or average F

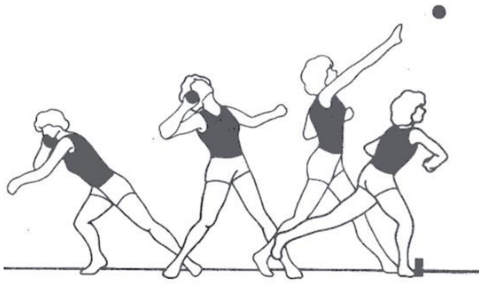
This should look familiar to you because we have worked with a force which varies across displacement (rather than time) in H205 Work Energy Power. Here are the parallels:

Work-Energy Theorem	Impulse-Momentum Theorem
Area under force-displacement graph gives work done by force (process w/ energy transfer).	Area under force-time graph gives impulse which is change in momentum.
	

Example 15
A shotput athlete draws her hand back before pushing the shot forward. By considering impulse, explain how this increases the launch velocity of the shot.

Solution
$\Delta p = \langle F_{net} \rangle \Delta t$

Increases time over which force is applied,
so increases the change in momentum of shot.
For a shot of constant mass,
momentum is directly proportional to velocity
so the launch velocity is increased.

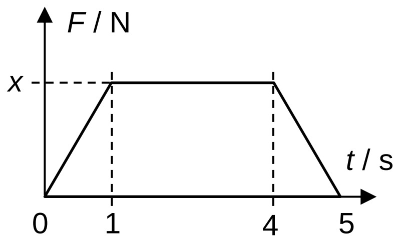


Example 16
A varying force F is applied to a mass of 10 kg. The mass gains 40 kg m s^{-1} of momentum. Find x.

Solution
area under force-time graph gives impulse

$$\frac{1}{2}(1)(x)+(4-1)(x)+\frac{1}{2}(1)(x)=40$$

$$x = 10 \text{ N}$$



3.5 Principle of Conservation of Linear Momentum

By Newton's 2nd Law of motion, when there is *no net force* acting on an isolated system (comprising more than one body), the system must experience *no change of linear momentum*.

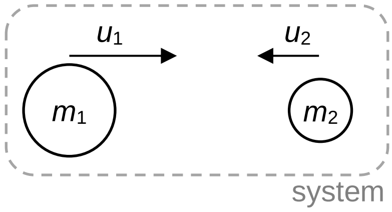
This is consistent with Newton's 1st Law, as the *system remains at rest or remains at constant velocity* when no net external force acts on the system.

The **Principle of Conservation of Linear Momentum** states that

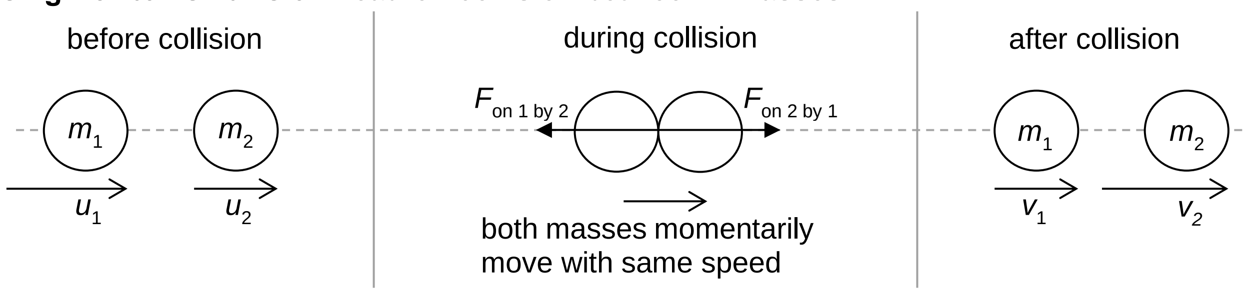
the total linear momentum of an isolated system
of interacting bodies
before and after collision
remains constant if no net external force
acts on the system.

The above idea leads to the Principle of Conservation of Linear Momentum. For a system of 2 masses in which they collide head-on:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$



Using Newton's Laws on head-on collision between 2 masses



Take rightwards as positive, and Δt as the small-time interval during which $F_{\text{on 2 by 1}}$ & $F_{\text{on 1 by 2}}$ act,

from Newton's 3rd Law: $F_{\text{on 2 by 1}} = -F_{\text{on 1 by 2}}$

from Newton's 2nd Law: $\dfrac{\Delta p_2}{\Delta t} = -\dfrac{\Delta p_1}{\Delta t} \Rightarrow \Delta p_2 = -\Delta p_1$

$$p_{\text{final, 2}} - p_{\text{initial, 2}} = -\left(p_{\text{final, 1}} - p_{\text{initial, 1}}\right)$$

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

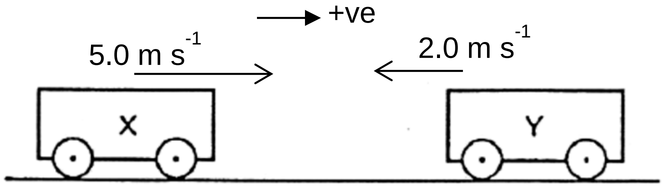
Hence, total linear momentum of both masses before and after collision remains constant.

Note:

1. Although the total momentum of the system (consisting of bodies 1 and 2) remains unchanged, the momenta of body 1 and body 2 *individually* has changed due to the net force that they exert on one another during the collision.
2. Head-on means that the velocities of the centres-of-mass of the bodies are all along 1 line of action (mathematically we say that the velocities are *collinear*)

Example 17

Two trolleys X and Y of masses 3.0 and 8.0 kg respectively collide. The final momentum of X is 2.0 N s in a direction opposite from the initial. Find the final velocity of Y.



Solution

By Principle of Conservation of Linear Momentum:

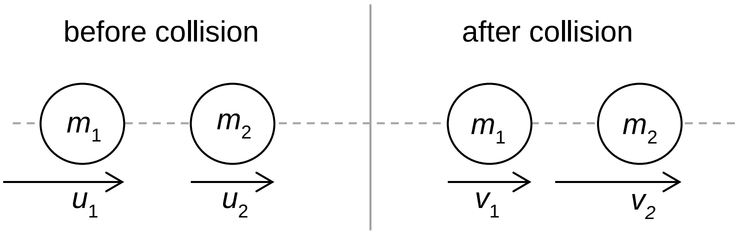
$$m_X u_X + m_Y u_Y = m_X v_X + m_Y v_Y$$
$$(3)(5) + (8)(-2) = (-2) + (8)v_Y$$
$$v_Y = +0.13 \text{ m s}^{-1} \text{ to the right}$$

3.5.1 Energy Considerations during Collisions

While the *total linear momentum of an isolated system is always conserved* in interactions between bodies, some change in kinetic energy usually takes place. In reality, collisions can result in a loss of energy through thermal energy, sound, work done through deformation of the bodies or even emission of electromagnetic radiation. Based on energy, we categorize:

<table>
<tr><th>type</th><th>elastic collisions</th><th>inelastic collisions</th><th>perfectly inelastic collisions</th></tr>
<tr><th>definition</th><td>total kinetic energy of system of bodies before and after collision remains the same</td><td colspan="2">total kinetic energy of system of bodies after collision is less than before</td></tr>
<tr><th>special result</th><td>relative speed of approach = relative speed of separation</td><td>-</td><td>masses stick together and move off with same velocity after collision</td></tr>
</table>

3.5.1 Relative Speeds in an Elastic Collision



By Principle of Conservation of Linear Momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad \_\_\_ (1)$$

Elastic collision so total kinetic energy is conserved:

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

$$m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2)$$

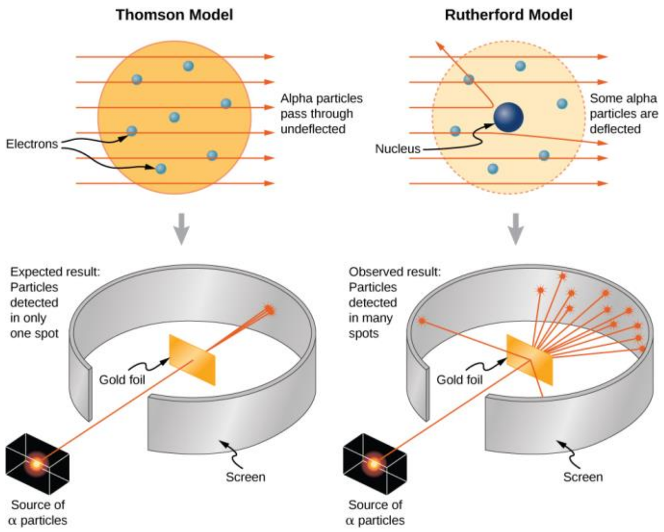
$$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2)$$

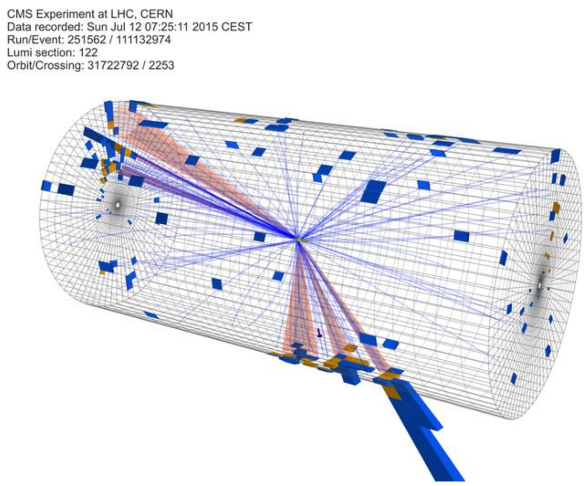
sub (1) $\quad \cancel{m_1(u_1 - v_1)}(u_1 + v_1) = \cancel{m_2(v_2 - u_2)}(v_2 + u_2)$

$$u_1 - u_2 = v_2 - v_1$$

"Relative speed of approach = relative speed of separation" is a special result that applies to 2 masses (can be different mass!) undergoing head-on elastic collision. You can use this result without the need to prove (unless otherwise instructed). If ever asked to prove, remember that the mathematical move of $a^2 - b^2 = (a+b)(a-b)$ is used in simplifying the KE-conservation.

Note: The relationship is derived base on the given directions. For different scenario, you must consider the directions of the moving bodies and adjust the equation accordingly.

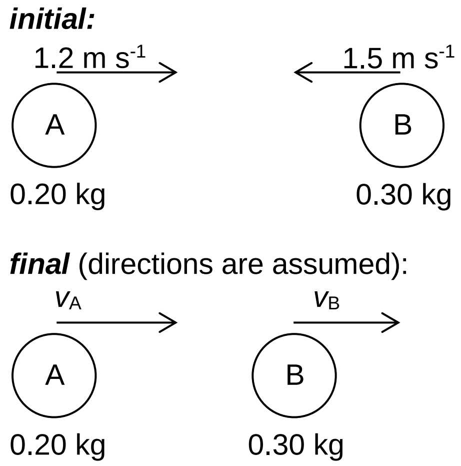




Not conserving kinetic energy may sound "bad" *but they provide very useful experimental data. These "scattering experiments" combine information on loss of energy as well as geometries of rebound particles to identify and characterize the resulting observations post-collisions.*

Example 18

Two balls A and B undergo an elastic head-on collision. Find their final velocities.



Solution

Take right as positive direction,
by Principle of Conservation of Linear Momentum:

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$(0.2)(1.2) + (0.3)(-1.5) = (0.2)v_A + (0.3)v_B$$

$$-0.21 = 0.2v_A + 0.3v_B \quad \_\_\_ (1)$$

<u>Method 1</u>

elastic collision so sum of kinetic energy before and after collision is conserved

$$\frac{1}{2}m_A u_A^2 + \frac{1}{2}m_B u_B^2 = \frac{1}{2}m_A v_A^2 + \frac{1}{2}m_B v_B^2$$

$$\frac{1}{2}(0.2)(1.2)^2 + \frac{1}{2}(0.3)(-1.5)^2 = \frac{1}{2}(0.2)v_A^2 + \frac{1}{2}(0.3)v_B^2$$

$$0.4815 = 0.1v_A^2 + 0.15v_B^2 \quad \_\_\_\_ (2a)$$

solve (1) and (2a) simultaneously

$$v_A = -2.04 \text{ m s}^{-1}$$
$$= 2.04 \text{ m s}^{-1} \text{ to the left}$$
$$v_B = 0.660 \text{ m s}^{-1}$$

<u>Method 2</u>

elastic collision so
relative speed of approach
= relative speed of separation

$$u_A - u_B = v_B - v_A$$

$$1.2 - (-1.5) = v_B - v_A = 2.7 \quad \_\_\_ (2b)$$

solve (1) and (2b) simultaneously

$$v_A = -2.04 \text{ m s}^{-1}$$
$$= 2.04 \text{ m s}^{-1} \text{ to the left}$$
$$v_B = 0.660 \text{ m s}^{-1}$$

Note: final directions are assumed; a negative velocity calculated simply means to the left since we have defined positive as the right.

Example 19

Two balls of the same mass m undergo an elastic head-on collision. The first mass has an initial speed of u and the second mass is at rest. Find the final velocities of each of the masses in terms of u.

Solution

Take right as positive direction,
by Principle of Conservation of Linear Momentum:

$$mu + 0 = mv_1 + mv_2$$

$$u = v_1 + v_2 \quad \_\_\_ (1)$$

elastic collision so
relative speed of approach
= relative speed of separation

$$u - 0 = v_2 - v_1$$

$$u = v_2 - v_1 \quad \_\_\_ (2)$$

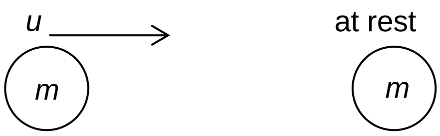
sub (1) in (2)

$$v_2 - v_1 = v_1 + v_2$$

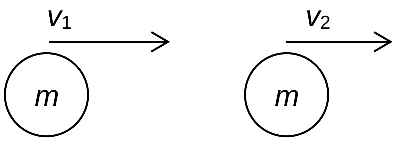
$$2v_1 = 0$$

$$\left.\begin{aligned} v_1 &= 0 \\ v_2 &= u \end{aligned}\right\}$$

initial:



final (directions are assumed):



Note: for elastic collisions involving equal masses, if the moving mass hits a stationary mass, there is complete transfer of momentum: the initially-moving mass stops and the initially-stationary mass moves off with the same velocity. Apply this to the steel balls on a Newton's cradle.

Example 20

Two balls of the same mass *m* moves towards each other at the same speed *u* and collides head-on elastically. Find the final velocities of each of the masses in terms of *u*.

Solution

Take right as positive direction,
by Principle of Conservation of Linear Momentum:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
$$mu + m(-u) = mv_1 + mv_2$$
$$0 = v_1 + v_2$$
$$v_2 = -v_1 \quad \_\_\_(1)$$

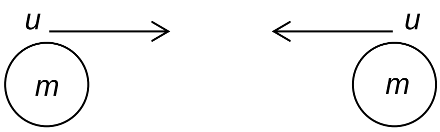
elastic collision so
relative speed of approach = relative speed of separation

$$u - (-u) = v_2 - v_1$$
$$2u = v_2 - v_1 \quad \_\_\_(2)$$

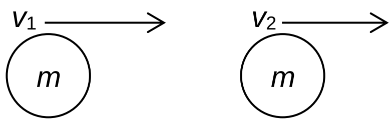
sub (1) in (2)

$$\left.\begin{aligned} v_1 &= -u \\ v_2 &= u \end{aligned}\right\}$$

initial:



final (directions are assumed):



Note: here masses move off with same speeds but in opposite directions

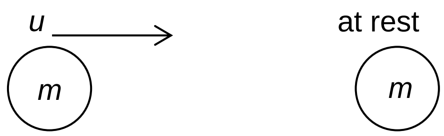
Challenge: Show that in general for elastic head-on collisions of same masses, the bodies exchange velocities before and after the collision.

Example 21

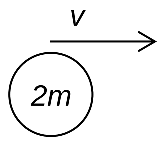
A particle of mass *m* moving with speed *u* makes a head-on collision with an identical particle which is initially at rest. The particles coalesce (stick together) and move off with a common velocity.

(a) Find the common speed of the particles after the collision.
(b) Find the ratio of the kinetic energy of the system after the collision to that before.
(c) Explain what happens to the kinetic energy that is 'lost'.

initial:



final (direction is assumed):



Solution

Take right as positive direction,

(a) by Principle of Conservation of Linear Momentum:

$$mu + 0 = (2m)v$$

$$2v = u \quad \_\_\_(1)$$

(b)

$$E_{\text{K, initial}} = \frac{1}{2}mu^2 + 0$$

$$E_{\text{K, final}} = \frac{1}{2}(2m)v^2 + 0$$

$$\text{take ratio:} \frac{E_{\text{K, final}}}{E_{\text{K, initial}}} = \frac{mv^2}{\frac{1}{2}mu^2} = \frac{2v^2}{u^2} \quad \_\_\_(2)$$

sub (1) in (2):

$$\text{ratio } = 2\left(\frac{v}{2v}\right)^2 = 0.50$$

(c) kinetic energy of system converted to thermal energy

Note:

1. For exams, you need to evaluate ratios and fractions into decimal numbers – i.e. you cannot leave answers as ½.
2. For part (c), there is also sound energy – you can whimsically think of collisions in real life as "*oh got 'tok' sound so not elastic already!*". However, for exam purposes please quote thermal energy (heat) as it is the main source of energy dissipation. Also, please see structure of answers for such energy questions: need to address the conversion *from (which type) to (other type).*

3.5.2 Separation of objects

A single body can separate into smaller parts, usually through an explosion or a spontaneous disintegration (see topic H220 Nuclear Physics). The principle of conservation of momentum allows us to work out the directions and the speeds of the constituent pieces.

Example 22

(a) Find recoil velocity of a rifle of mass 5.0 kg firing a bullet of mass 20 g at a speed of 620 m s^{-1}.
(b) Explain which between the rifle or the bullet has a higher kinetic energy.

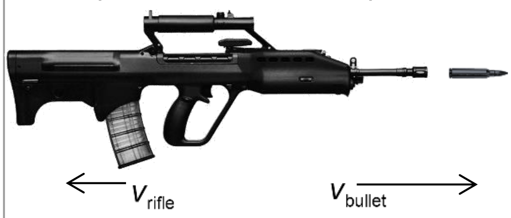
Solution

Take right as positive direction,
by Principle of Conservation of Linear Momentum:

(a)
$$m_{\text{rifle}}u_{\text{rifle}} + m_{\text{bullet}}u_{\text{bullet}} = m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}}$$
$$(m_{\text{rifle}} + m_{\text{bullet}})(0) = m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}}$$
$$m_{\text{rifle}}v_{\text{rifle}} = -m_{\text{bullet}}v_{\text{bullet}} \quad \_\_\_ (*)$$
$$(5)v_{\text{rifle}} = -620(0.02)$$
$$v_{\text{rifle}} = -2.5 \text{ m s}^{-1}$$
$$= 2.5 \text{ m s}^{-1} \text{ backwards}$$

initial:

final (direction is assumed):



(b)
$$\frac{1}{2}mv^2 = \frac{1}{2}\frac{(mv)^2}{m} = \frac{p^2}{2m}$$

from $(*)$, magnitude of momentum is same for rifle and bullet

take ratio: $\dfrac{E_{\text{K, bullet}}}{E_{\text{K, rifle}}} = \dfrac{\dfrac{p^2{}_{\text{bullet}}}{2m_{\text{bullet}}}}{\dfrac{p^2{}_{\text{rifle}}}{2m_{\text{rile}}}} = \dfrac{m_{\text{rifle}}}{m_{\text{bullet}}} = \dfrac{5}{0.02} = 250$

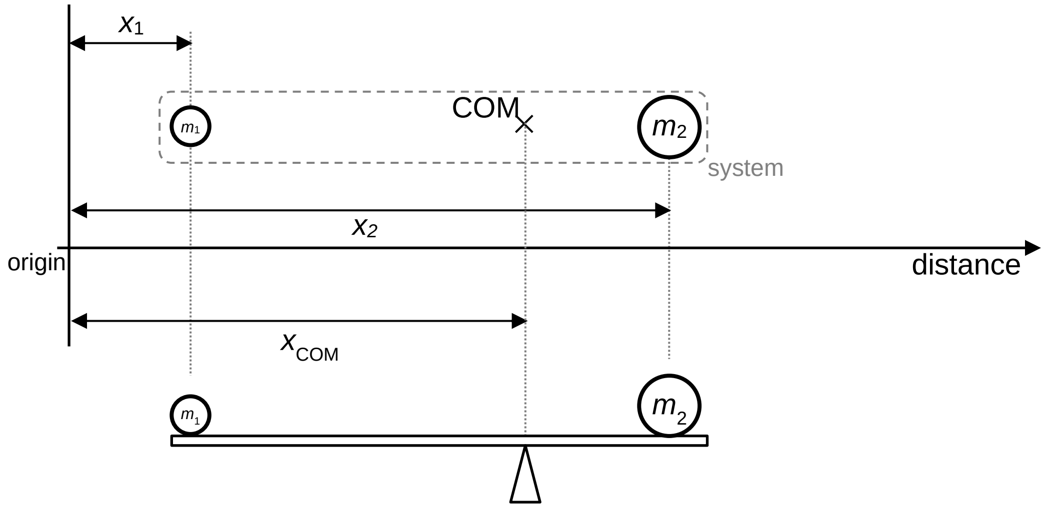
Mass of rifle is 250 times the mass of the bullet, so the kinetic energy of the exiting bullet is 250 times that of the recoiling rifle

Note: the alternative form of $E_K = \dfrac{p^2}{2m}$ is very useful for comparing ratios of momentum, masses and will appear quite often in H219, the topic of Quantum Physics.

3.5.3 Motion of a System (of Two Bodies) – Centre-of-Mass

Since the total linear momentum of an isolated system remains constant before and after a collision, we can regard the system as a single body: we represent the system by its centre-of-mass ("COM") having a constant velocity:

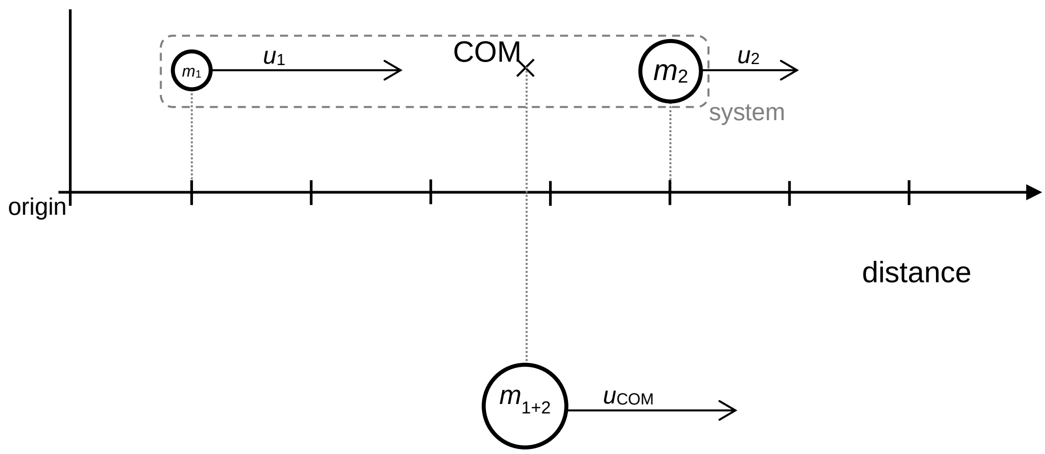
$$p_{\text{system}} = m_{\text{total}} v_{\text{system}}$$



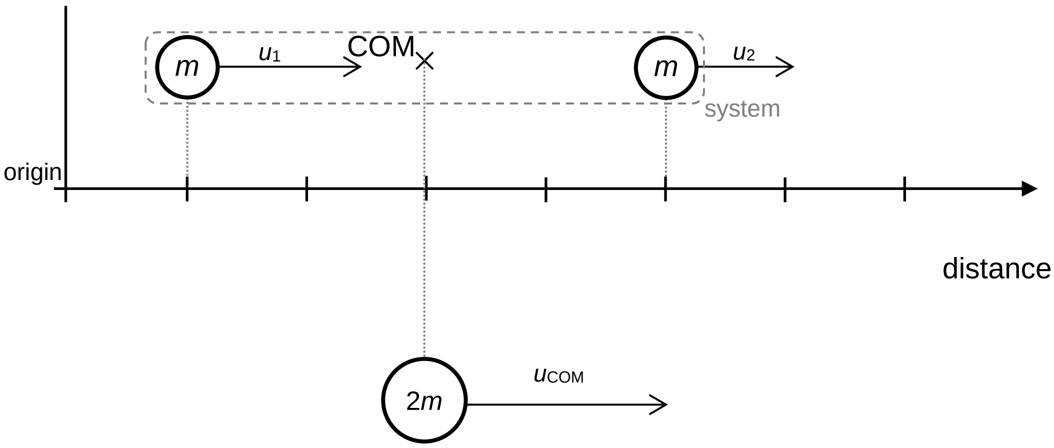
We can imagine the COM as a pivot that allows us to balance the two masses on see-saw. It is akin to centre-of-gravity, the single point where the weight of a body may be considered to act. We "take moments about origin O":

$$\text{considering moments about origin:}$$
$$\text{sum of moments due to each individual mass} = \text{moment due to centre of mass of system}$$
$$m_1 x_1 + m_2 x_2 = m_{\text{total}} x_{\text{COM}}$$
$$x_{\text{COM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

We can then represent the system of 2 bodies via 1 single point:



For instance, if the 2 bodies are of the same mass, it should not be a surprise that the COM is in the middle of the 2 masses:



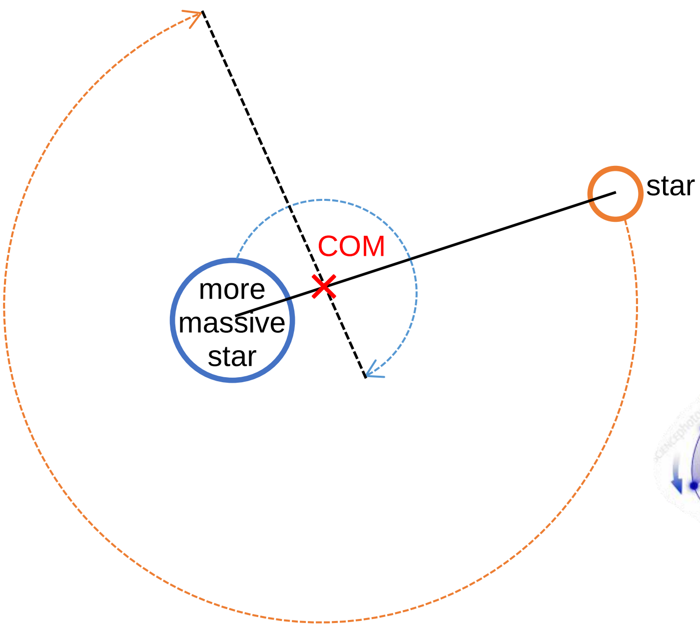
$$x_{COM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{x_1 + x_2}{2}$$

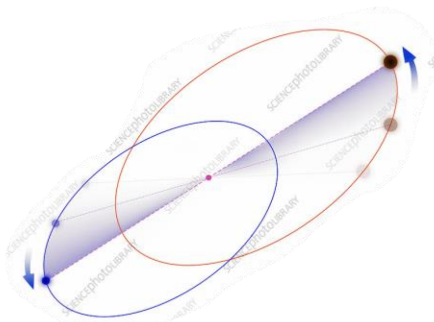
By Principle of Conservation of Linear Momentum:

$$mu_1 + mu_2 = (2m)u_{COM}$$

$$u_{COM} = \frac{u_1 + u_2}{2}$$

On the other hand, if one of the bodies is more massive, the COM will lie closer to the more-massive body. We can look at binary (pair of) stars. A pair of stars exert gravitational force on each other which are internal forces – as a system these forces cancel out. It is an isolated system where no external forces act. Therefore, despite the rotation, the COM obeys Newton's 1st Law.



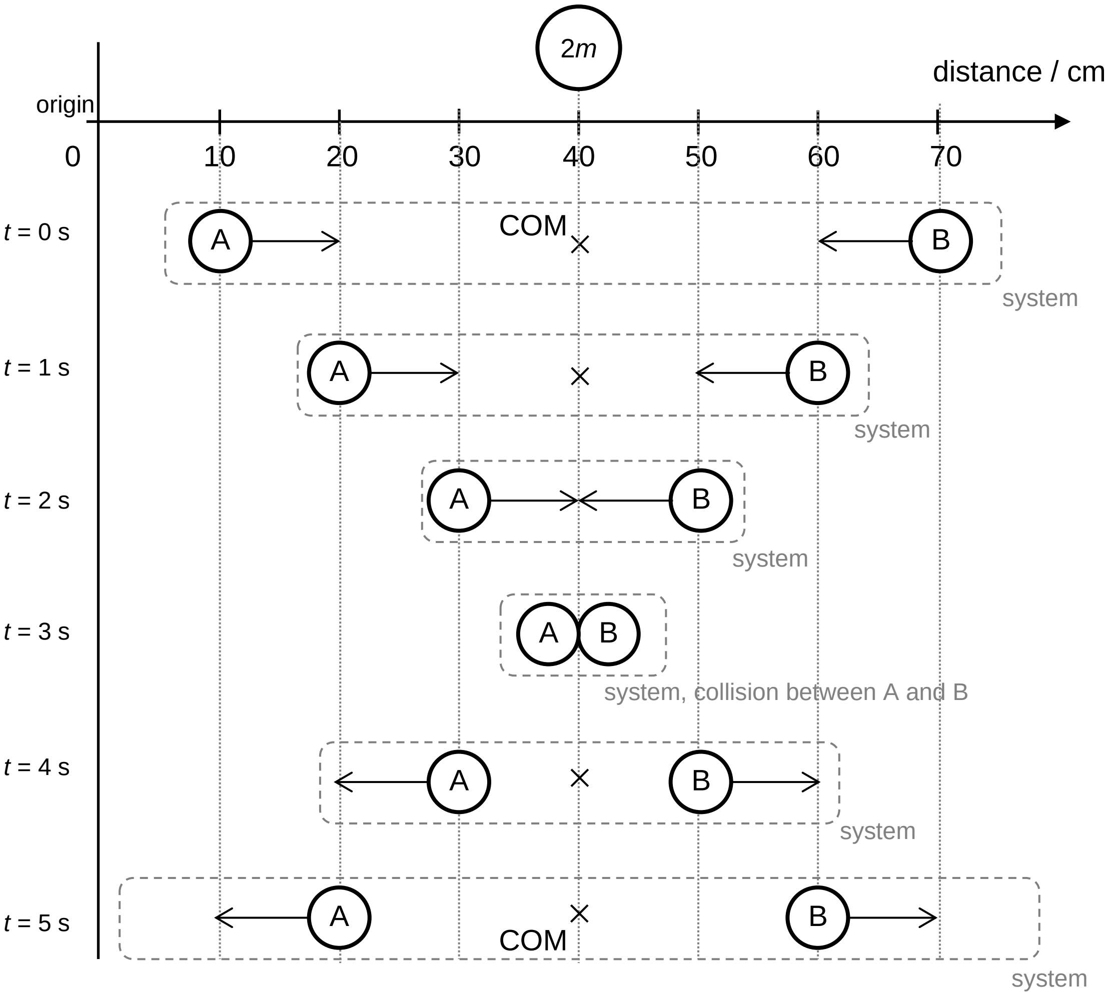


In reality *most planetary orbits are elliptical rather than perfectly circular*

Example 23

Two particles A and B of the same mass and same speed of 10 cm s^{-1} move towards each other and collide elastically head-on. Describe the changes, if any, to the centre of mass of the two particles before and after collision.

Solution: The total linear momentum of the isolated system is zero and there is no net external force acting on the system. The principle of conservation of linear momentum applies and by Newton's first law, the centre of mass remains in its state of rest.



Note: During collision, by Newton's 3rd Law both particles exert forces on each other and each particle experiences a force. But when considering the whole system, these forces are *internal* forces within the system which cancel out.

Example 24

Particles A and B have the same mass. Particle A moves right at a speed of 20 cm s^{-1} towards a stationary B and they collide elastically head-on. Describe the changes, if any, to the centre of mass of the 2 particles before and after collision.

Solution:

By Principle of Conservation of Linear Momentum,

$$m_A u_A + 0 = m_A v_A + m_B v_B$$
$$20 = v_A + v_B \quad \_\_\_ (1)$$

Elastic so relative speed of approach = relative speed of separation

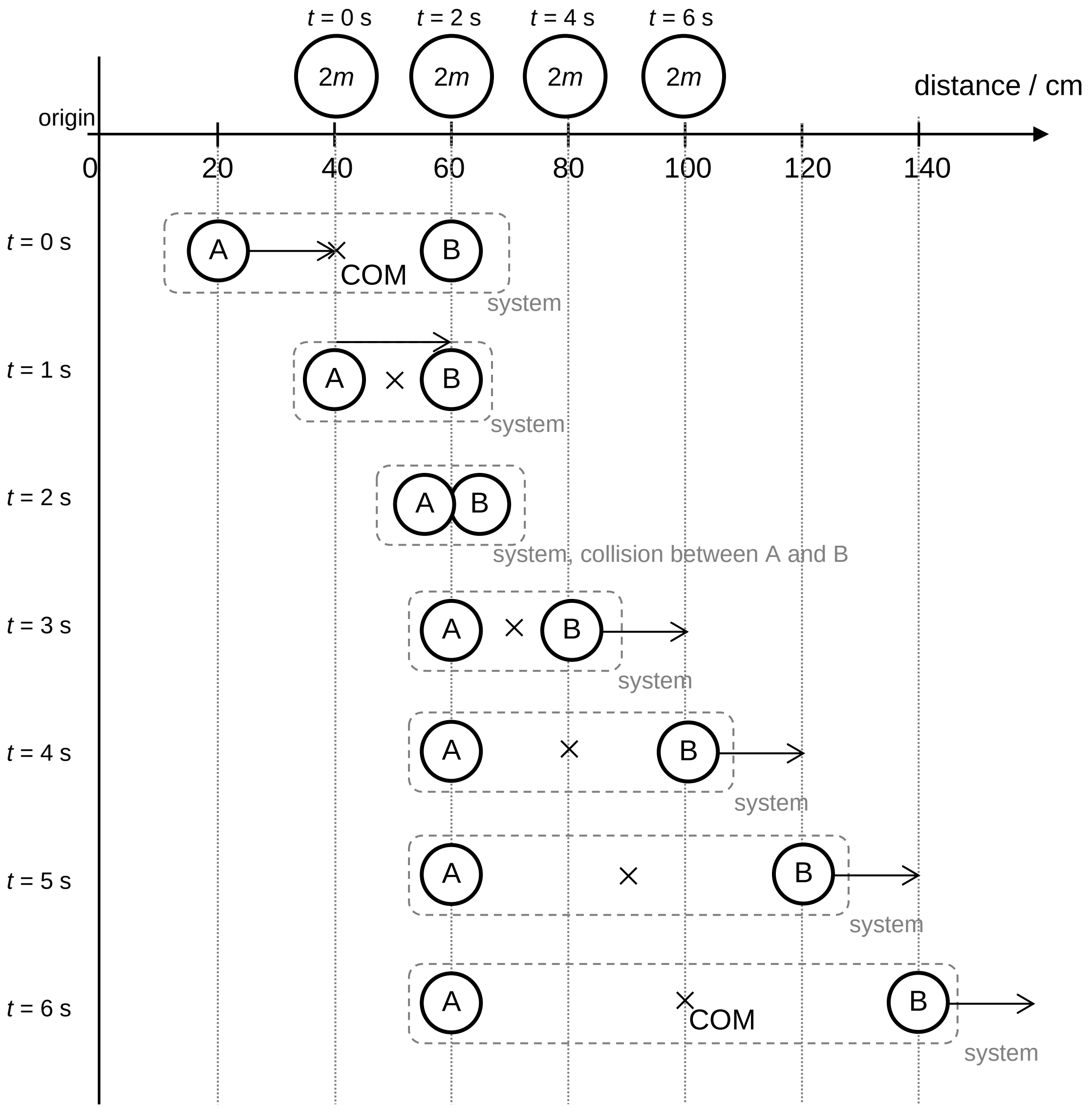
$$u_A = v_B - v_A$$
$$20 = v_B - v_A \quad \_\_\_ (2)$$

sub (1) in (2):

$$\left.\begin{aligned} v_A &= 0 \\ v_B &= 20 \text{ cm s}^{-1} \end{aligned}\right\}$$

Considering momentum of system:

$$20m = (2m)u_{COM}$$
$$u_{COM} = 10 \text{ cm s}^{-1}$$



Note: centre of mass moves at a constant velocity at 10 cm s^{-1} to the right as there are no external forces on the system throughout the entire motion, even though a collision took place.

3.6 Summary

Newton's 1st Law of motion
an object continues at rest or with constant velocity unless acted upon by a resultant force

Newton's 2nd Law of motion
rate of change of momentum of a body is directly proportional to the resultant force acting on it and in the direction of the resultant force

$$F_{\text{resultant}} \propto \frac{dp}{dt}$$

proportionality constant = 1

force
rate of change of momentum

$$F = \frac{dp}{dt}$$

linear momentum
product of mass and velocity

$$p = mv$$

no net force, constant p

constant mass

$$F_{\text{net}} = ma$$

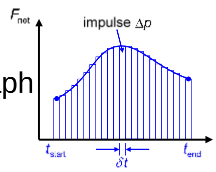
$$F_{\text{net}} \approx \frac{\Delta p}{\Delta t}$$

impulse
product of force and time duration of impact

$$\Delta p = \langle F_{\text{net}} \rangle \Delta t$$

varying force

area under force-time graph gives impulse



Newton's 3rd Law of motion
when body A exerts a force on body B, body B exerts on body A, a force of same type, equal in magnitude and opposite in direction

2 bodies mutually exert force for same duration

Principle of Conservation of Linear Momentum
total linear momentum of an isolated system of interacting bodies before and after collision remains constant if no net external force acts on the system

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

consider as 1 system

centre-of-mass has constant velocity

$$m_1u_1 + m_2u_2 = m_{\text{total}}u_{\text{COM}}$$

elastic collision
total kinetic energy of system of bodies *before and after* collision remains the same

$$\frac{1}{2}m_Au_A^2 + \frac{1}{2}m_Bu_B^2 = \frac{1}{2}m_Av_A^2 + \frac{1}{2}m_Bv_B^2$$

special result, use $a^2 - b^2 = (a-b)(a+b)$

relative speed of approach = relative speed of separation

$$u_1 - u_2 = v_2 - v_1$$

inelastic collision
total kinetic energy of system not conserved

perfectly inelastic collision
masses stick together after collision

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

3.7 Ending Notes

H203 Dynamics closes off an important milestone in Physics thus far because it requires putting together knowledge from the past few topics (Forces, Work Energy & Power, and Kinematics). Together, the topics form a foundation of Mechanics, which will be completed with the next topic H206 Motion in a Circle. Do digest the material carefully.

Why science teachers should not be given playground duty.

The space below is for your own summary mind-map.