Chapter 4: Normal Distribution

1.	ACJC JC2 Prelim 8865/2019/Q8

The masses, in kg, of lemons and tangerines sold by a wholesaler have independent normal distributions with means and standard deviations as shown in the following table.

	Mean	Standard deviation
Lemons	0.18	0.02
Tangerines	0.22	0.03

(i) Find the probability that the mass of a randomly chosen lemon is less than 3% below the mean mass. [2]

(ii) Find the mass *m* such that only 2% of the tangerines have mass more than *m* kg. [2]

(iii) By first stating clearly the mean and variance of the distribution that you use, find the probability that 2 randomly chosen lemons have a total mass not exceeding twice the mass of a randomly chosen tangerine. [3]

(iv) A fruit stall owner prepares fruit juice by extracting the pulp of the two fruits. The mass of each lemon pulp is 90% of the mass of each lemon while the mass of each tangerine pulp is 80% of the mass of each tangerine.
By first stating clearly the mean and variance of the distribution that you use, find the probability that the total mass of the pulp from 3 randomly chosen lemons and 4 randomly chosen tangerines is between 1.1 kg and 1.2 kg. [3]

Answer: (i) 0.394 (ii) 0.282 (iii) 0.886 (iv) 0.511

1.	ACJC JC2 Prelim 8865/2019/Q8 (Solutions)
(i)	Let L be the random variable 'mass of a lemon'. $L \sim \mathrm{N}\left(0.18, 0.02^2\right)$ $\mathrm{P}\left(L < 0.97 \times 0.18\right) = \mathrm{P}\left(L < 0.1746\right) = 0.394$
(ii)	Let T be the random variable 'mass of a tangerine'. $T \sim \mathrm{N}\left(0.22, 0.03^2\right)$ $\mathrm{P}\left(T > m\right) = 0.02 \;\Rightarrow\; m = 0.282$
(iii)	$L_1 + L_2 - 2T \sim \mathrm{N}\left(2\times 0.18 - 2\times 0.22, 2\times 0.02^2 + 2^2 \times 0.03^2\right)$ $L_1 + L_2 - 2T \sim \mathrm{N}\left(-0.08, 0.0044\right)$ $\mathrm{P}\left(L_1 + L_2 \le 2T\right) = \mathrm{P}\left(L_1 + L_2 - 2T \le 0\right) = 0.886$
(iv)	Let $X = 0.9L$ and $Y = 0.8T$ Let J be the total mass of the pulp from 3 lemons & 4 tangerines. $J = X_1 + X_2 + X_3 + Y_1 + Y_2 + Y_3 + Y_4$ $J \sim \mathrm{N}\left(3\times 0.9\times 0.18 + 4\times 0.8\times 0.22, 3\times 0.9^2\times 0.02^2 + 4\times 0.8^2 \times 0.03^2\right)$ $J \sim \mathrm{N}\left(1.19, 0.003276\right)$ $\mathrm{P}\left(1.1 < J < 1.2\right) = 0.511$

2.	ASRJC JC2 Prelim 8865/2019/Q11

A factory manufactures paperweights consisting of glass mounted on a wooden base. The volume of glass, in cm^3 and the volume of the wooden base, in cm^3 for a randomly selected paperweight follows independent normal distributions with means and standard deviations as shown in the table below:

	Mean	Standard deviation
Glass	58.6	2.1
Wooden base	38.5	3.3

(i) Find the probability that the total volume of a randomly selected paperweight is more than 98 cm^3 [2]

(ii) Find the probability that two paperweights selected at random have volumes that differ by less than 2 cm^3 [2]

Glass weighs 4.4 grams per cm^3 and wood weighs 0.85 grams per cm^3.

(iii) Find the probability that the total weight of ten randomly selected paperweights is between 2800 and 2900 grams. [4]

An events company has a long-standing relationship with the factory. The delivery time (in mins) of a randomly selected trip to the events company follows a normal distribution with mean μ and standard deviation σ. 10% of the trips take less than 24.3 minutes, while 18% of the trips take longer than 33.3 minutes.

(iv) Find the values of μ and σ. [3]

The events company is given a discount if the delivery time is more than *k* mins.

(v) Find the minimum value of *k* if the events company gets a discount on 5 % of the trips. [1]

Answer: (i) 0.409 (ii) 0.282 (iii) 0.426 (iv) 36.3

2.	ASRJC JC2 Prelim 8865/2019/Q11 (Solutions)
(i)	Let X be random variable denoting the volume of glass, and Y random variable denoting the volume of wood, of a randomly selected paperweight. $X \sim N(58.6, 2.1^2)$ and $Y \sim N(38.5, 3.3^2)$ Let $W = X + Y \sim N(58.6+38.5, 2.1^2+3.3^2)$ i.e $N(97.1, 15.3)$ $\text{Prob} = P(W > 98) = 0.409$
(ii)	$W_1 - W_2 \sim N(0, 15.3+15.3)$ i.e $N(0, 30.6)$ $P(-2 < W_1 - W_2 < 2) = 0.282$
(iii)	Mass of one paperweight, $M = 4.4X + 0.85Y$ Total mass of one paperweights, $T = M_1 + M_2 + ...M_{10}$ $T \sim N(10(4.4\times58.6+0.85\times38.5),\ 10(4.4^2\times2.1^2+0.85^2\times3.3^2))$ $T \sim N(2905.65,\ 932.45625)$ $P(2800 < T < 2900) = 0.426$
(iv)	Let V be the random variable denoting time taken for a randomly selected trip

$V \sim N(\mu, \sigma^2) \qquad Z = \dfrac{V-\mu}{\sigma} \sim N(0,1)$

$P(V < 24.3) = 0.10$

$\Rightarrow P(Z < \dfrac{24.3-\mu}{\sigma}) = 0.10$

$\Rightarrow \dfrac{24.3-\mu}{\sigma} = -1.2815516 ------(1)$

$\Rightarrow \mu - 1.2815516\sigma = 24.3$

$P(V > 33.3) = 0.18$

$\Rightarrow P(Z > \dfrac{33.3-\mu}{\sigma}) = 0.18$

$\Rightarrow \dfrac{33.3-\mu}{\sigma} = 0.915365 ------(2)$

$\Rightarrow \mu + 0.915365\sigma = 33.3$

μ =29.550 $\qquad \sigma$ = 4.09665

(v) $P(V > k) = 0.05 \Rightarrow k = 36.288$

Minimum k is 36.3

3. DHS JC2 Prelim 8865/2019/Q11

The speeds of an e-scooter (X km/h) and a pedestrian (Y km/h) measured on a particular stretch of footpath are normally distributed with mean and variance as follows:

	mean	variance
X	12.3	9.9
Y	μ	σ^2

It is known that $\mathrm{P}(Y < 5.2) = \mathrm{P}(Y \geq 7.0) = 0.379$.

(i) State the value of μ and find the value of σ. [3]

(ii) Given that the speeds of half of the e-scooters measured are found to be within a km/h of the mean, find a. [2]

(iii) A LTA officer stationed himself at the footpath and measured the speeds of 50 e-scooters at random. Find the probability that the 50th e-scooter is the 35th to exceed LTA's legal speed limit of 10 km/h. [3]

Answer: (i) 6.1, 2.92 (ii) 2.12 (iii) 0.0468

3. DHS JC2 Prelim 8865/2019/Q11 (Solutions)

(i) By symmetry, $\mu = \dfrac{5.2+7.0}{2} = 6.1$

	$P(Y < 5.2) = P(Y \geq 7.0) = 0.379$ $P\left(Z < \frac{5.2-6.1}{\sigma}\right) = 0.379$ $\frac{-0.9}{\sigma} = -0.308108$ $\sigma = 2.92105 = 2.92$
(ii)	$X \sim N(12.3, 9.9)$ $P(\lvert X - 12.3 \rvert < a) = 0.5$ $P(12.3 - a < X < 12.3 + a) = 0.5$ From GC, $12.3 - a = 10.1777$ $a = 2.1223 = 2.12$ Alternative $P(\lvert X - 12.3 \rvert < a) = 0.5$ $P(\lvert Z \rvert < \frac{a}{\sqrt{9.9}}) = 0.5$ $P(Z < -\frac{a}{\sqrt{9.9}}) = 0.25$ $-\frac{a}{\sqrt{9.9}} = -0.674489$ $a = 2.1222 = 2.12$
(iii)	Let W be number of e-scooters that exceed speed limit, out of 49 $W \sim B(49, P(X > 10))$ i.e. $W \sim B(49, 0.76761)$ Probability required $= P(W = 34) \times 0.76761$ $= 0.0468$

4. CJC JC2 Prelim 8865/2019/Q12

A vegetable stall in the wet market sells carrots at 25 cents per 100 grams and potatoes at 30 cents per 100 grams. The seller gives a 10% discount for customers who spend above $5 before discount.

The masses of carrots and potatoes have independent normal distributions with means and standard deviations as shown in the following table.

	Mean (grams)	Standard Deviation (grams)
Carrot	65	10
Potato	170	30

(i)	Find the probability that a randomly chosen carrot costs more than 20 cents and a randomly chosen potato costs more than 60 cents.	[3]
(ii)	Find the probability that the total cost of a randomly chosen carrot and a randomly chosen potato exceeds $0.80.	[4]
(iii)	Explain why your answers in parts (i) and (ii) differ.	[1]
(iv)	A customer decides to purchase carrots in bulk in order to benefit from the discount. Find the minimum number of carrots he needs to buy such that the probability of qualifying for the discount exceeds 0.5.	[4]
(v)	On a particular day, fourteen customers buy some potatoes at the stall. Find the probability that at least five customers purchase at most 140 grams of potatoes.	[3]
	Answer: (i) 0.0106 (ii) 0.0861 (iv) 31 (v) 0.0576	

4. CJC JC2 Prelim 8865/2019/Q12 (Solutions)

(i) Let C and D be the random variables denoting the mass, in grams, of a randomly chosen carrot and potato respectively.

$C \sim \mathrm{N}\left(65, 10^2\right)$

$D \sim \mathrm{N}\left(170, 30^2\right)$

Method 1:

$$\mathrm{P}\left(\frac{25}{100}C > 20\right)\mathrm{P}\left(\frac{30}{100}D > 60\right)$$
$$= \mathrm{P}(C > 80)\,\mathrm{P}(D > 200)$$
$$= (0.0668072287)(0.1586552596)$$
$$= 0.0105993182$$
$$= 0.0106 \text{ (3 s.f.)}$$

Method 2:

$$\frac{25}{100}C \sim \mathrm{N}\left(\frac{25}{100}(65), \left(\frac{25}{100}\right)^2 10^2\right)$$
$$\sim \mathrm{N}(16.25, 6.25)$$
$$\frac{30}{100}D \sim \mathrm{N}\left(\frac{30}{100}(170), \left(\frac{30}{100}\right)^2 30^2\right)$$
$$\sim \mathrm{N}(51, 81)$$
$$\mathrm{P}\left(\frac{25}{100}C > 20\right)\mathrm{P}\left(\frac{30}{100}D > 60\right)$$
$$= (0.0668072287)(0.1586552596)$$
$$= 0.0105993182$$
$$= 0.0106 \text{ (3 s.f.)}$$

(ii)	$\frac{25}{100}C+\frac{30}{100}D \sim \mathrm{N}\left(\frac{25}{100}(65)+\frac{30}{100}(170),\left(\frac{25}{100}\right)^2 10^2+\left(\frac{30}{100}\right)^2 30^2\right)$ $\sim \mathrm{N}(67.25, 87.25)$ $\mathrm{P}\left(\frac{25}{100}C+\frac{30}{100}D>80\right)=0.0861291438$ $=0.0861$ (3 s.f.)
(iii)	<u>There are more cases in part (ii) than part (i)</u>. For example, the case when a carrot costs 19 cents while a potato costs 62 cents is considered in part (ii) but not in part (i).
(iv)	Let the number of carrots bought be n. $C_1+C_2+...+C_n \sim \mathrm{N}(65n, 10^2 n)$ $\mathrm{P}\left(\frac{25}{100}(C_1+C_2+...+C_n)>500\right)>0.5$ $\mathrm{P}(C_1+C_2+...+C_n>2000)>0.5$ <table><tr><th>n</th><th colspan="2">$\mathrm{P}(C_1+C_2+...+C_n>2000)$</th></tr><tr><td>30</td><td>0.1807</td><td>< 0.5</td></tr><tr><td>31</td><td>0.6062</td><td>> 0.5</td></tr><tr><td>32</td><td>0.9214</td><td>> 0.5</td></tr></table> Minimum number of carrots to buy is 31.
(v)	Let W be the random variable denoting the number of customers who purchase at most 140 g of potatoes. $W \sim \mathrm{B}(14, \mathrm{P}(D \le 140))$ $W \sim \mathrm{B}(14, 0.1586552596)$ $\mathrm{P}(W \ge 5)$ $=1-\mathrm{P}(W<5)$ $=1-\mathrm{P}(W \le 4)$ $=0.0576137002$ $=0.0576$ (3 s.f.)

5. MI PU2 Prelim 8865/2019/Q12

Maxi Durian shop sells two types of durian, *MSW* and *BG*. The masses, in kilogram, of each type of durian have independent normal distributions. The means and standard deviations of these distributions, are shown in the following table.

	Mean (kg)	Standard deviation (kg)
MSW durian	2.5	0.05
BG durian	5	0.15

(i) Find the probability that the mass of a randomly chosen *MSW* durian is within 0.1kg of its mean mass. [2]

(ii) The probability that a randomly chosen *BG* durian has a mass between 4.9 kg and m kg is 0.7. Find the value of m. [2]

(iii) Find the probability that the total mass of the three randomly chosen *MSW* durians is less than 7.3 kg. [3]

(iv) Find the probability that 50% of the mass of one randomly chosen *BG* durian exceeds the mass of one randomly chosen *MSW* durian by at least 0.125 kg. [4]

(v) The selling price for a *MSW* durian is \$20 per kg and the selling price for a *BG* durian is \$15 per kg.

Stating clearly the mean and variance of the distribution that you use, find the probability that the total selling price of four randomly chosen *MSW* durians and two randomly chosen *BG* durians is more than \$355. [4]

Answer: (i) 0.954 (ii) 5.25 (iii) 0.0105 (iv) 0.0828 (v) 0.0917

5. MI PU2 Prelim 8865/2019/Q12 (Solutions)

(i) Let S and L be the random variables denoting the mass, in kilograms, of a randomly chosen a *MSW* durian and a *BG* durian respectively. Then $S \sim \mathrm{N}\left(2.5, 0.05^2\right)$ and $L \sim \mathrm{N}\left(5, 0.15^2\right)$.

$\mathrm{P}\left(2.4 < S < 2.6\right)$

$= 0.9544999$

$= 0.954$ (3 s.f.)

(ii) <u>Method 1</u>

$\mathrm{P}\left(4.9 < L < m\right) = 0.7$

$\mathrm{P}\left(L < m\right) - \mathrm{P}\left(L \le 4.9\right) = 0.7$

$\mathrm{P}\left(L < m\right) - 0.25249 = 0.7$

$\mathrm{P}\left(L < m\right) = 0.95249$

By GC InvNorm,

$m = 5.2504 = 5.25$ (3 s.f.)

<u>Method 2</u>

	$\mathrm{P}(4.9 < L < m) = 0.7$ $1 - \mathrm{P}(L \le 4.9) - \mathrm{P}(L > m) = 0.7$ $0.74750 - \mathrm{P}(L > m) = 0.7$ $\mathrm{P}(L > m) = 0.047508$ By GC InvNorm, $m = 5.2504 = 5.25$ (3 s.f.)
(iii)	$\mathrm{E}(S_1 + S_2 + S_3) = 3(2.5) = 7.5$ $\mathrm{Var}(S_1 + S_2 + S_3) = 3(0.05^2) = 0.0075$ $S_1 + S_2 + S_3 \sim \mathrm{N}(7.5,\ 0.0075)$ $\mathrm{P}(S_1 + S_2 + S_3 < 7.3) = 0.010461 = 0.0105$ (3 s.f.)
(iv)	<u>Method 1</u> $\mathrm{E}\left(\frac{1}{2}L - S\right) = \frac{1}{2}(5) - 2.5 = 0$ $\mathrm{Var}\left(\frac{1}{2}L - S\right) = \left(\frac{1}{2}\right)^2 (0.15)^2 + 0.05^2 = 0.008125$ $\frac{1}{2}L - S \sim \mathrm{N}(0,\ 0.008125)$ $\mathrm{P}\left(\frac{1}{2}L - S \ge 0.125\right) = 0.082759 = 0.0828$ (3 s.f.) <u>Method 2</u> $\mathrm{P}\left(\frac{1}{2}L - S \ge 0.125\right) = \mathrm{P}(L - 2S \ge 0.25)$ $\mathrm{E}(L - 2S) = 5 - (2)(2.5) = 0$ $\mathrm{Var}(L - 2S) = (0.15)^2 + (2^2)0.05^2 = 0.0325$ $L - 2S \sim \mathrm{N}(0,\ 0.0325)$ $\mathrm{P}(L - 2S \ge 0.25) = 0.082759 = 0.0828$ (3 s.f.)
(v)	$\mathrm{E}\left(20(S_1 + S_2 + S_3 + S_4) + 15(L_1 + L_2)\right)$ $= 20(4)\mathrm{E}(S) + 15(2)\mathrm{E}(L)$ $= 20(4)(2.5) + 15(2)(5)$ $= 350$

	$\text{Var}\left(20\left(S_1+S_2+S_3+S_4\right)+15\left(L_1+L_2\right)\right)$ $=20^2(4)\text{Var}(S)+15^2(2)\text{Var}(L)$ $=20^2(4)\left(0.05^2\right)+15^2(2)\left(0.15^2\right)$ $=14.125$ $20\left(S_1+S_2+S_3+S_4\right)+15\left(L_1+L_2\right)\sim\text{N}\left(350,\ 14.125\right)$ $\text{P}\left(20\left(S_1+S_2+S_3+S_4\right)+15\left(L_1+L_2\right)>355\right)$ $= 0.091697 = 0.0917$ (3 s.f.)

6. NJC JC2 Prelim 8865/2019/Q12

In this question you should state clearly the values of the parameters of any normal distribution you use.

The masses, in kilograms, of Red Snappers and Golden Snappers, are normally distributed. The means and standard deviations of these distributions, and the selling prices, in \$ per kilogram, are shown in the following table.

	Mean mass (kg)	Standard deviation (kg)	Selling price (\$ per kg)
Red Snappers	0.875	0.04	19.50
Golden Snappers	0.925	0.05	24.40

(i) Three Red Snappers are randomly selected. Find the probability that exactly one of the snappers has mass less than 0.85 kg and each of the other two snappers has mass more than 0.9 kg. [2]

(ii) Find the probability that the mass of a randomly chosen Red Snapper is greater than the mass of a randomly chosen Golden Snapper. [2]

(iii) Show that the probability that the total mass of six randomly chosen Red Snappers is within 1.3 kg of four times the mass of a randomly chosen Golden Snapper is 0.1308, correct to 4 decimal places. [4]

(iv) Phillip buys four Red Snappers. Donald buys two Golden Snappers. Find the probability that Phillip pays \$25 or more than Donald. [4]

State an assumption needed for your calculation in parts (ii), (iii) and (iv) to be valid. [1]

Answer: (i) 0.0565 (ii) 0.217 (iii) 0.1308 (iv) 0.208

6.	NJC JC2 Prelim 8865/2019/Q12 (Solutions)
(i)	Let R be the random variable denoting the mass of a Red Snapper. Then $R \sim \text{N}(0.875, 0.04^2)$ $\text{P}(R < 0.85) = 0.2659854678$

	$P(R > 0.9) = 0.2659854678$ The required probability $= P(R < 0.85)P(R > 0.9)P(R > 0.9)$ $+ P(R > 0.9) P(R < 0.85)P(R > 0.9)$ $+ P(R > 0.9)P(R > 0.9) P(R < 0.85)$ $= P(R < 0.85)P(R > 0.9)^2 \times 3$ $= 0.056454034 = 0.0565$
(ii)	Let G be the random variable denoting the mass of a Golden Snapper. Then $G \sim N(0.925, 0.05^2)$ $E(R - G) = E(R) - E(G) = 0.875 - 0.925 = -0.05$ $Var(R - G) = Var(R) + Var(G) = 0.04^2 + 0.05^2 = 0.0041$ Thus $R - G \sim N(-0.05, 0.0041)$ $P(G < R) = P(0 < R - G) = 0.2174397605 = 0.217$
(iii)	To find $P(-0.02 < (R_1 + R_2 + \ldots + R_6) - 4G < 0.02)$. Let $T = (R_1 + R_2 + \ldots + R_6) - 4G$ $E(T) = E((R_1 + R_2 + \ldots + R_6) - 4G) = 6E(R) - 4E(G)$ $= 6 \times 0.875 - 4 \times 0.925 = 1.55$ $Var(T) = Var((R_1 + R_2 + \ldots + R_6) - 4G) = 6Var(R) + 4^2Var(G)$ $= 6\times0.04^2 + 4^2\times0.05^2 = 0.0496$ Then T ~ N(1.55,0.0496) $P(-1.3 < T < 1.3)$ = 0.130817913056483 = 0.1308(4dp)[AG]
(iv)	Let \$$V$ and \$$W$ be the amount that Phillip and Donald pay respectively. Then $V = 19.50(R_1 + R_2 + \ldots + R_4)$ and $W = 24.40(G_1 + G_2)$ $E(V) = 19.50E(R_1 + R_2 + \ldots + R_4)$ $= 19.50\times4\times0.875 = 68.25$ $Var(V) = 19.50^2 Var(R_1 + R_2 + \ldots + R_4)$ $= 19.50^2\times4\times0.04^2 = 2.4336$ $E(W) = 24.40E(G_1 + G_2)$ $= 24.40\times2\times0.925 = 45.14$ $Var(W) = 24.40^2 Var(G_1 + G_2)$ $= 24.40^2\times2\times0.05^2 = 2.9768$ Need to compute $P(V - W > 25)$. $E(V - W) = E(V) - E(W) = 23.11$ $Var(V - W) = Var(V) + Var(W) = 5.4104$ Thus $V - W \sim N(23.11, 5.4104)$ $P(V - W > 25) = 0.208239651906010 = 0.208$
	An assumption needed for your calculation in parts (ii), (iii) and (iv) to be valid is the normal distributions are independent

7.	TJC JC2 Prelim 8865/2019/Q12
	A medicinal shop sells Type A herbs in packets. The masses of packets of Type A herbs are normally distributed with mean 30 grams and standard deviation 1.2 grams.

(i) Find the probability that the mass of a packet of Type *A* herbs is within 2 grams of the mean mass of a packet of Type *A* herbs. [1]

(ii) Two packets of Type *A* herbs are randomly chosen. Find the probability that the mass of one packet is greater than 31 grams and the mass of the other packet is less than 31 grams. [2]

(iii) Ten packets of Type *A* herbs are randomly chosen. Find the probability that at most four out of the ten packets have masses that exceeds 31 grams. [2]

The medicinal shop also sells Type *B* herbs in packets. The masses of packets of Type *B* herbs are normally distributed with mean 32 grams and standard deviation 1.5 grams. The masses of packets of herbs are independent of one another. The cost of brewing a packet of Type *A* herbs is \$0.30 per gram and the cost of brewing a packet of Type *B* herbs is \$0.25 per gram.

(iv) Find the probability that the total mass of two randomly chosen packets of Type *A* herbs exceeds twice the mass of a randomly chosen packet of Type *B* herbs. [3]

(v) Joshua orders a special concoction of 4 packets of Type *A* herbs and 3 packets of Type *B* herbs to boost his immunity. Find the probability that Joshua needs to pay at most \$61 to brew the special concoction. [4]

Answer: (i) 0.904 (ii) 0.323 (iii) 0.966 (iv) 0.123 (v) 0.849

7. TJC JC2 Prelim 8865/2019/Q12 (Solutions)

(i) Let X be the random variable denoting the mass of a packet of Type *A* herbs in grams

$X \sim \mathrm{N}(30, 1.2^2)$

$\mathrm{P}(28 < X < 32) = 0.90441 = 0.904$ (3 s.f.)

(ii) Required probability $= \mathrm{P}(X > 31) \times \mathrm{P}(X < 31) \times 2 = 0.32278 = 0.323$ (3 s.f.)

(iii) Let *S* be the random variable denoting the number of packets of Type *A* herbs (out of 10) that has mass exceeding 31 grams.

$S \sim \mathrm{B}(10, 0.20232)$

$\mathrm{P}(S \leq 4) = 0.96564 = 0.966$ (3 s.f.)

(iv) Let *Y* be the mass of a packet of Type *B* herbs in grams

$Y \sim \mathrm{N}(32, 1.5^2)$

$X_1 + X_2 - 2Y \sim \mathrm{N}(30 \times 2 - 32 \times 2, 1.2^2 \times 2 + 1.5^2 \times 2^2)$

i.e. $X_1 + X_2 - 2Y \sim \mathrm{N}(-4, 11.88)$

$\mathrm{P}(X_1 + X_2 > 2Y) = \mathrm{P}(X_1 + X_2 - 2Y > 0) = 0.12291 = 0.123$ (3 s.f.)

(v)	$0.30(X_1 + X_2 + X_3 + X_4) \sim \mathrm{N}(0.30 \times 30 \times 4, 0.30^2 \times 1.2^2 \times 4)$ $0.25(Y_1 + Y_2 + Y_3) \sim \mathrm{N}(0.25 \times 32 \times 3, 0.25^2 \times 1.5^2 \times 3)$ Let the total cost $C = 0.30(X_1 + X_2 + X_3 + X_4) + 0.25(Y_1 + Y_2 + Y_3)$ $C \sim \mathrm{N}(60, 0.940275)$ $\mathrm{P}(C \le 61) = 0.84879 = 0.849$ (3 s.f.)

8. YIJC JC2 Prelim 8865/2019/Q11

(a) A fruit grocer sells red and yellow dragon fruits. The masses, in kg, of these two types of dragon fruits, have independent normal distributions with means and standard deviations as shown in the following table.

Dragon Fruit	Mean	Standard deviation
Red	0.5	σ
Yellow	0.35	0.06

It is found that 40% of the red dragon fruits have a mass less than 0.49 kg.

(i) Show that σ is 0.0395 kg, correct to 3 significant figures. [2]

(ii) Find the probability that the total mass of 2 randomly chosen red dragon fruits and 3 randomly chosen yellow dragon fruits exceeds 2.2 kg. [3]

(iii) Two red dragon fruits are chosen at random. Find the probability that one of the red dragon fruits has mass more than 0.49 kg and the other has mass less than 0.49 kg. [2]

Red dragon fruits cost \$4 per kg and yellow dragon fruits cost \$5 per kg.

(iv) Find the probability that the total cost of 3 randomly chosen yellow dragon fruits is within \$0.90 of twice the cost of a randomly chosen red dragon fruit. [5]

(v) The fruit grocer packs 24 red dragon fruits in a carton. Find the probability that there are at least 9 red dragon fruits, each with mass less than 0.49 kg, in the carton. [2]

Answer: (i) 0.0395 (ii) 0.102 (iii) 0.48 (iv) 0.282 (v) 0.672

8.	YIJC JC2 Prelim 8865/2019/Q11 (Solutions)
(i)	Let R be the random variable denoting the mass of a red dragon fruit, in kg.

	$R \sim \text{N}(0.5, \sigma^2)$ $\text{P}(R < 0.49) = 0.4$ $\text{P}\left(Z < \frac{0.49-0.5}{\sigma}\right) = 0.4$ $\text{P}\left(Z < \frac{-0.01}{\sigma}\right) = 0.4$ $\frac{-0.01}{\sigma} = -0.25335$ $\sigma = 0.039471 = 0.0395$ (shown)
(ii)	Let Y be the random variable denoting the mass of a yellow dragon fruit, in kg. $Y \sim \text{N}\left(0.35, 0.06^2\right)$ Let $T = R_1 + R_2 + Y_1 + Y_2 + Y_3$ $T \sim \text{N}\left(2(0.5)+3(0.35),\ 2\left(0.0395^2\right)+3\left(0.06^2\right)\right)$ $T \sim \text{N}\left(2.05,\ 0.013920\right)$ $P\left(T > 2.2\right) = 0.102$
(iii)	Required probability $= \text{P}(R > 0.49) \times \text{P}(R < 0.49) \times 2$ $= 0.6 \times 0.4 \times 2$ $= 0.48$
(iv)	Let $W = 5\left(Y_1 + Y_2 + Y_3\right) - 2\left(4R\right)$ $\text{E}\left(W\right) = 5(3)(0.35) - 2(4)(0.5) = 1.25$ $\text{Var}\left(W\right) = 5^2\left(3\text{Var}\left(Y\right)\right) + 2^2\left(4^2\right)\text{Var}\left(R\right)$ $= 5^2(3)\left(0.06^2\right) + 2^2\left(4^2\right)\left(0.0395^2\right)$ $= 0.36986$ $W \sim \text{N}\left(1.25,\ 0.36986\right)$ $\text{P}\left(\lvert W \rvert \le 0.9\right) = \text{P}\left(-0.9 \le W \le 0.9\right)$ $= 0.282$
(v)	Let Y be random variable denoting the number of red dragon fruits, each with mass less than 0.49 kg, out of 24.

	$Y \sim B(24, 0.4)$ $\mathrm{P}(Y \geq 9) = 1 - \mathrm{P}(Y \leq 8)$ $= 0.6720777427 \approx 0.672$

9.	JPJC JC2 Prelim 8865/2019/Q12

Bee's journey to work every day consists of a bus journey, a train journey and a walk. The journey times, in minutes, by bus, by train and by walking have independent normal distributions. The means and standard deviations of these distributions are shown in the following table.

	Mean (min)	Standard Deviation (min)
Bus	25	4
Train	17	3
Walk	15	2

(i) Find the probability that in a randomly chosen day, Bee's total journey time to work is more than 1 hour. [2]

(ii) Bee starts work at 8 am. By what time does she have to leave home so that she is 95% certain that she will be on time for work. [3]

Bee works 5 days a week, find the probability that in a randomly chosen week,

(iii) her journey to work is more than 1 hour on each of the 5 days. [2]

(iv) she takes more than 300 minutes travelling to work in total. [3]

Dee's daily journey times to work are normally distributed with mean 31 minutes and variance 16 minutes^2. Dee also works 5 days a week.

(v) Find the probability that in any particular 5-day work week, Bee's total journey times exceed twice of Dee's total journey times. You should state the mean and variance of any distribution that you use. [4]

Answer: (i) 0.289 (ii) 6.54am (iii) 0.00201 (iv) 0.106 (v) 0.123

9.	JPJC JC2 Prelim 8865/2019/Q12 (Solutions)
	Let B, T and W be the random variables denoting the journey times by bus, train and walking. $B \sim \mathrm{N}(25, 4^2)$ $T \sim \mathrm{N}(17, 3^2)$ $W \sim \mathrm{N}(15, 2^2)$
(i)	Let $X = B + T + W$ $\mathrm{E}(X) = 25+17+15 = 57$ $\mathrm{Var}(X) = 4^2 + 3^2 + 2^2 = 29$

	$X \sim \mathrm{N}(57, 29)$ $\mathrm{P}(X > 60) = 0.28873 \approx 0.289$ (3 sf)
(ii)	$\mathrm{P}(X < m) = 0.95$ $m = 65.858$ Bee should leave home by 6.54 am
(iii)	Required Probability = $\left[\mathrm{P}(X > 60)\right]^5 = 0.0020066 \approx 0.00201$ (3 sf)
	Or $Y \sim$ no. of days she took more than 1 hour for the journey out of 5 $Y \sim \mathrm{B}(5, 0.28873)$ $\mathrm{P}(Y = 5)$ $= 0.0020066 \approx 0.00201$ (3 sf)
(iv)	Let U represent Bee's total journey time to work in 5 days $U = X_1 + X_2 + X_3 + X_4 + X_5$ E(U) = 5 × 57 = 285 Var (U) = 5 × 29 = 145 $U \sim \mathrm{N}(285\ ,\ 145)$
	$\mathrm{P}(U > 300) = 0.10644 \approx 0.106$ (3 sf)
(v)	Let D be the random variable denoting Dee's daily journey times to work $D \sim \mathrm{N}\ (31,16)$ Let V represent Dee's total journey time to work in 5 days $V = D_1 + D_2 + D_3 + D_4 + D_5$ E(V) = 5 × 31 = 155 Var (V) = 5 × 16 = 80 $V \sim \mathrm{N}(155\ ,\ 80)$ E($U - 2V$) = 285 – (2×155) = – 25 Var ($U - 2V$) = 145 + (2^2× 80) = 465 $U - 2V \sim \mathrm{N}(-25\ ,\ 465)$ $P(U > 2V) = P(U - 2V > 0)$ = 0.12316 ≈ 0.123