Name : _____ Class : _____

Section A : Pure Mathematics [40 marks]

1 **(i)** Given that

$$\mathrm{h}(\theta) = \frac{1-\sqrt{2(1-\cos 3\theta)}}{2+\sin\theta},$$

and that θ is a sufficiently small angle, show that

$$\mathrm{h}(\theta) \approx \frac{1-3\theta}{2+\theta}.$$ [2]

(ii) Given that n is a positive integer, by expanding binomially, show that

$$\left(\mathrm{h}(\theta)\right)^n \approx a + b\theta,$$

where a and b are constants to be found in terms of n. [2]

(iii) State the range of values of θ for which the expansion is valid. [1]

(iv) Verify your answers in **(ii)** by finding a suitable Maclaurin's series. [2]

2 The complex number z satisfies the relations

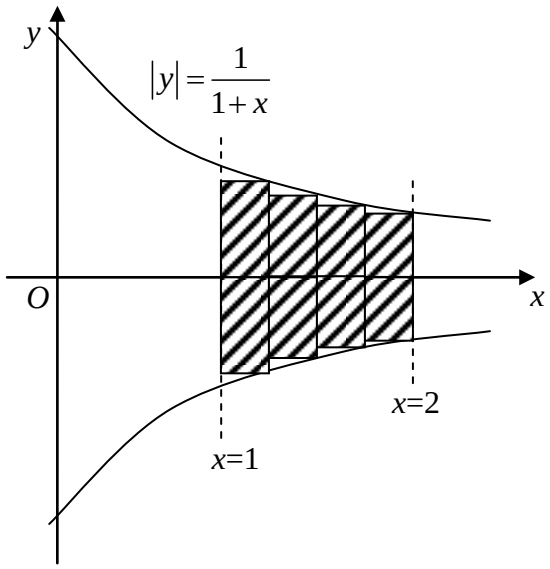
$$\frac{|z-4\mathrm{i}|}{|2+z|} \le 1, \quad |z-2\mathrm{i}| \ge 2, \quad \arg(z-2)^{\frac{1}{3}} = \frac{\pi}{4}.$$

(i) Illustrate these loci on a single Argand diagram, showing clearly the locus of z. [5]

(ii) Find the complex number z such that $|z|$ is the smallest, giving your answer in exact form. [3]

3 The diagram shows the parts of the graph of $|y| = \frac{1}{1+x}$. The four rectangles each of equal width, as shown in diagram below, are rotated through π radians about the *x*-axis. Prove that the volume generated may be expressed as

$$\sum_{r=1}^{4} \frac{4\pi}{(8+r)^2}. \qquad [2]$$



n rectangles, each of equal width under the curve between $x = 1$ and $x = 2$, are used to estimate the volume of solid obtained when the region bounded by the graph, the lines $x = 1$ and $x = 2$, is rotated through π radians about the *x*-axis. Find the estimated volume in the form $\sum_{r=1}^{n} \mathrm{f}(r)$, where f(*r*) is to be determined in terms of *r* and *n*. [2]

(i) Deduce that $\sum_{r=1}^{n} \frac{1}{(2n+r)^2} < \frac{1}{6n}$. [3]

(ii) State the value of $\sum_{r=1}^{n} \frac{n}{(2n+r)^2}$ as $n \to \infty$. [1]

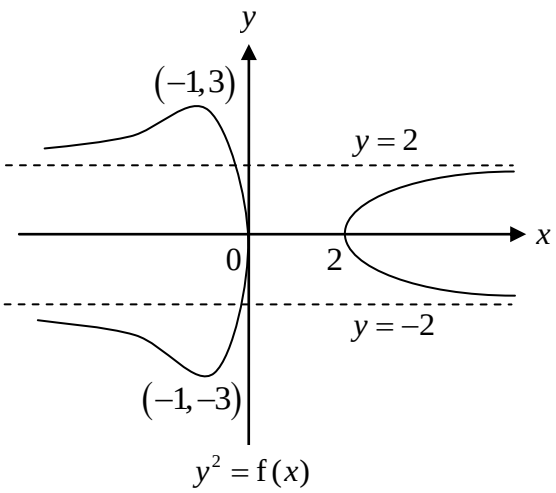
4 **(a)** *A*, *B* and *C* are constants such that

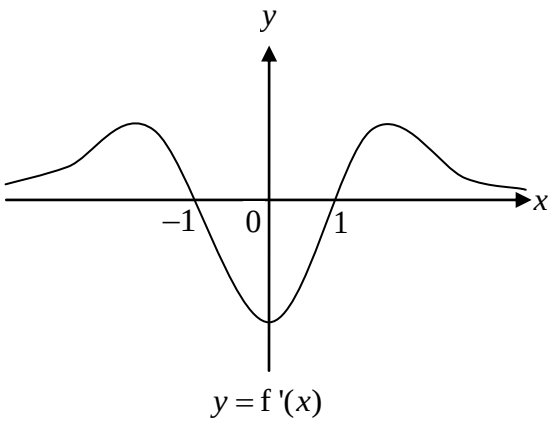
$$\frac{x^2}{x^2+4} = A + \frac{B}{(Cx)^2+1}$$

for all values of *x*. Find the values of *A*, *B* and *C*. [1]

State precisely a sequence of transformations by which the graph of $y = \frac{x^2}{x^2+4}$ may be obtained from the graph of $y = \frac{1}{x^2+1}$. [3]

(b)





The sketches above show the graphs of $y^2 = \mathrm{f}(x)$ and $y = \mathrm{f}'(x)$ for a certain function f.

(i) Sketch the graph of $y = \mathrm{f}(x)$. [2]

(ii) Sketch the graph of $y = \mathrm{f}(|x|+1)$. [2]

5 The functions f and g are defined as follows

$$\mathrm{f}: x \to \frac{x-\alpha}{x-\beta}, \qquad x \in \mathbb{R},\ x \neq \beta, \text{ and } \alpha < \beta,$$

$$\mathrm{g}: x \to 3x^2 + 6x + 2, \quad x \in \mathbb{R}.$$

(i) Show that f^{-1} exists. [2]

(ii) Show that $\mathrm{f}^{-1}(x) = \mathrm{f}(x)$ if $\beta = 1$. Deduce the value of $\mathrm{f}^{(2013)}(\alpha)$. [3]

(iii) Given that the composite function fg does not exist, show that $\beta \geq -1$. [2]

(iv) Given that the composite function fg exists and that $\mathrm{fg}(0) = 2$, find the range of values of α. [2]

Section B : Statistics [60 marks]

6 A feedback manager of a mobile internet service provider, which has a large pool of subscribers, aims to determine the quality of his customers' experience with its service.

(i) Suggest the most informative sampling method that can be used to help the manager achieve his aim. [1]

(ii) Describe precisely how the manager should carry out your suggestion. [3]

7 Find the number of 6-digit codes that can be formed using the digits 1 to 9 if

(i) there are no restrictions, [1]

(ii) there are three distinct pairs of identical digits, [2]

(iii) there is more than one odd digit and all odd digits are separated. [3]

8 An event *A* occurs at random times, at an average rate of λ times per day.

(a) Fifty days are chosen at random, and the number of times *A* occurs in each day is recorded. Given that $\lambda = 8$, find the probability that the average number of times *A* occurs is not more than 7. [3]

(b) Given that $\lambda = m + 0.5$ where m is a large positive integer, estimate the probability of *A* occurring at most *m* times in a randomly chosen day. [3]

9 In a university, a 3 year degree programme is offered to students. The proportion of students in the first, second and third year of study are in arithmetic progression. The probabilities that a candidate did not complete the first year and third year of study are 0.1 and 0.12 respectively.

(i) Show that the probability that a randomly chosen student from the degree programme will be in his second year of study is $\frac{1}{3}$. [1]

(ii) Given that the probability that a randomly chosen student is a second year student and has completed his second year of study is $\frac{14}{45}$, find the probability that a student will not complete his second year of study. [2]

(iii) It was found that if a student did not complete his year of study, the probability that he is in his third year of study is 0.6. Find the proportion of third year students in the degree programme. [4]

(iv) Are the 2 events "a randomly chosen student did not complete his year of study" and "a randomly chosen student is in his third year" independent? Justify your answer. [1]

10 A certain number of mobile phones purchased at a handphone shop is monitored and the number of those phones that are camera phones is denoted by C.

(i) State, in context, one assumption needed for C to be well modelled by a Binomial distribution. [1]

(ii) Given that 1 in 20 phones purchased are non-camera phones and the probability that there are more than 5 non-camera phones purchased is less than 0.1, find the largest number of mobile phones purchased that can be monitored. [3]

(iii) The records kept by the hand phone shop suggested that 98% of the mobiles phones purchased are camera phones. Using a suitable approximation, find the probability that less than 171 mobile phones purchased are camera phones out of 180 mobile phones purchased. [4]

11 Ten randomly selected people were asked to indicate the number of hours they spent listening to radio, *s*, and watching TV, *t*, during a 2-week period, as shown in the table below.

s	3.8	2.4	3.0	7.2	9.4	3.6	8.2	2.6	2.0	4.8
t	47	19	30	78	84	38	76	23	10	64

(i) Calculate the product moment correlation coefficient for the data. [1]

(ii) Give a sketch of the scatter diagram for the data using the model $t = a + bs$ and comment on the value of the product moment correlation coefficient found in **(i)**. [2]

(iii) State, with reason, which of the following models is more appropriate.

A : $t = a + \dfrac{b}{s}$ **B** : $t = a + bs^2$ **C** : $t = a + b\ln s$ [1]

(iv) Explain how the product moment correlation coefficient can be used to justify that your choice in **(iii)** is a better model than in **(ii)**. [1]

(v) Using the appropriate regression line, calculate an estimate for the time spent on listening to radio, given that the time spent on watching TV is 5 hours. Comment on your answer. [3]

12 A promoter of a certain type of battery claimed that his battery can last for at least 2000 hours when used continuously in a graphic calculator. Tom heard this and is keen to recommend the battery to his classmates only if the battery's life-span is as claimed. He bought less than 20 of this battery to test out on his own graphic calculator. In the process, however, he misplaced some readings and were left only with the following records of battery's life-span in hours:

1995 1998 1997 2002 2012 1999 2003 1998 1992 1980

(i) Using the above data, find unbiased estimates of the mean and variance of the battery's life span. [1]

(ii) Carry out an appropriate test at the 0.5% significance level for Tom, stating any assumption to be made. Use your test's conclusion to help Tom come to a decision. [4]

(iii) Explain, in the context of the question, the meaning of 'at the 0.5% significance level'. [1]

(iv) Tom later realised that the standard deviation of the battery's lifespan is indicated on the battery, and is verified by a government authority to be 5 hours. He decided to redo the hypothesis test using the same estimate for the mean of the battery's lifespan in **(i)**. Obtain an inequality involving the sample size of batteries he should use so that he will end up with a different decision from **(ii)**. [3]

13 A manufacturing process produces pencils with diameter normally distributed with mean 0.9 cm and pencil sharpeners with holes (to fit the pencils) with diameters normally distributed with mean 0.91cm. This process allows the manufacturer to adjust the standard deviations of the diameter for both the pencils and the sharpener holes.

(i) If the standard deviation of both pencils and sharpener holes are adjusted to 0.005 cm, find the probability that a randomly chosen pencil will not be able to fit into a randomly chosen sharpener. [3]

(ii) State an assumption needed for your calculations in **(i)**. [1]

(iii) Pencils with diameters that differ from its mean by more than 0.02 cm are considered as substandard. Find the standard deviation that should be set for the pencil's diameter so that the manufacturer can keep the percentage of substandard pencils to be 2.2%. [3]

The manufacturer runs quality check by sampling n pencils and n sharpeners randomly and measuring their circumferences. If he finds that the total circumference length of the pencils and sharpener holes exceeds 30 cm, he will stop production so that he can service his machines.

(iv) Given that the standard deviation of the diameter of the pencils and sharpener holes have been set to 0.003 cm and 0.001 cm respectively and that he is willing to stop production for not more than 5% of the time, find the maximum n. [4]

BLANK PAGE