Topic 17
Electromagnetic Induction

Content
- Magnetic Flux
- Laws of Electromagnetic Induction

Learning Outcomes
Candidates should be able to:

(a) define magnetic flux and the weber

(b) recall and solve problems using $\Phi = BA$

(c) define magnetic flux linkage

(d) infer from appropriate experiments on electromagnetic induction:
  i. that a changing magnetic flux can induce an e.m.f.
  ii. that the direction of the induced e.m.f. opposes the change producing it
  iii. the factors affecting the magnitude of the induced e.m.f.

(e) recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law

(f) explain simple applications of electromagnetic induction

17.0 Introduction

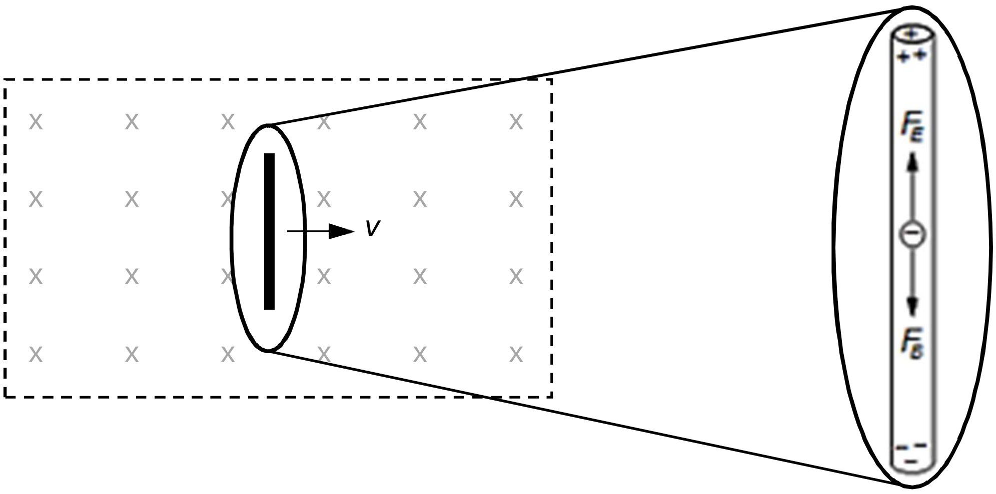
In the earlier topic, two ways in which electricity and magnetism are related were highlighted:

i. an electric current produces a magnetic field, and
ii. a magnetic field exerts a force on an electric current or moving electric charge.

These discoveries were made in 1820. As nature is often symmetric, the discovery that electric currents produce magnetic fields led scientists to suspect that magnetic fields could produce electric currents.

17.1 Motion Induced Electromotive Force

Consider the situation in which a thin straight conductor of length l moving at a constant speed v perpendicular to a uniform magnetic field that is directed into the plane of the page as shown.



By Fleming's Left-Hand Rule, the electrons in the conductor will experience a downward magnetic force F_B. This causes the electrons to migrate towards the lower end of the conductor.

Thus charge separation occurs within the conductor with the accumulation of negative charge on one end and positive charge on the opposite end.

This charge separation will in turn produce an electric field along the rod, which causes the electrons to experience an upward electric force F_E.

Steady state is reached when there is no further charge separation. This occurs when the magnetic force and electric force acting on the electrons are balanced.

i.e. $F_B = F_E$

$$Bqv = q\frac{\varepsilon_{induced}}{l}$$

Hence, the induced e.m.f. across the ends of the conductor is given by

$$\varepsilon_{induced} = Blv$$

17.2 Magnetic Flux and Magnetic Flux Linkage

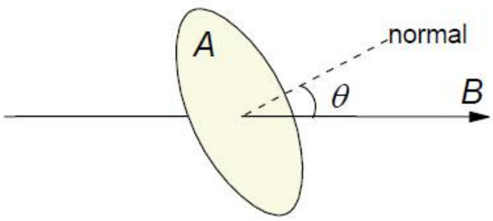
Ten years after the discovery that an electric current could produce a magnetic field, experiments conducted independently by Joseph Henry in United States and Michael Faraday in England showed that a changing magnetic field could induce an electric current in a circuit. The results of these experiments led to a basic and important law known as Faraday's law. But before delving into the Laws of Electromagnetic Induction, the concept of magnetic flux, and consequently magnetic flux linkage, must first be understood.

17.2.1 Magnetic Flux

In the earlier topic, the term *magnetic flux* was introduced and is defined as the product of the magnetic flux density normal to the surface and the area of the surface.

The general equation for magnetic flux Φ is

$$\Phi = \int \vec{B} \cdot d\vec{A}$$



If the magnetic field is **uniform**, the integral reduces to

$$\Phi = B_{\perp} A$$
$$= \left(B\cos\theta\right)A$$

, where θ is the angle between the normal to A and $\vec{B}$.

Magnetic flux is a scalar quantity and its SI unit is the *weber* (Wb), which is defined as the amount of magnetic flux that when reduced to zero in one second induces an e.m.f. of one volt.

17.2.2 Magnetic Flux Linkage

The *magnetic flux linkage* through a loop is the product of the magnetic flux through the loop and the number of turns of wire in the loop.

Thus, if the area A is bounded by a coil and the coil has N turns, the magnetic flux linkage through the coil, is

$$N\Phi = NB_{\perp}A$$
$$= N\left(B\cos\theta\right)A$$

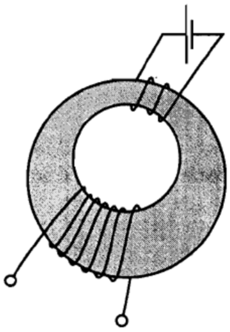
From the equation, magnetic flux linkage through a coil depends on

- ✓ the number of turns in the coil N,
- ✓ the magnitude of magnetic flux density B,
- ✓ the orientation of the coil with respect to the direction of B, and
- ✓ the area bounded by the coil A.

A change in any of these variables results in a change of magnetic flux linkage.

Example 1

A soft iron ring of variable cross-section has two coils wound around it. One coil has 7 turns. The other, of 3 turns, is connected to a d.c. supply.



Which quantity is the same inside each coil?

A magnetic field

B magnetic flux

C magnetic flux density

D magnetic flux linkage

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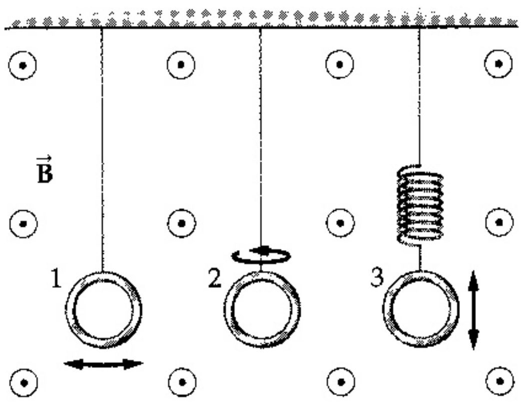
Example 2

The three loops of wire shown are all in a region of uniform magnetic field.



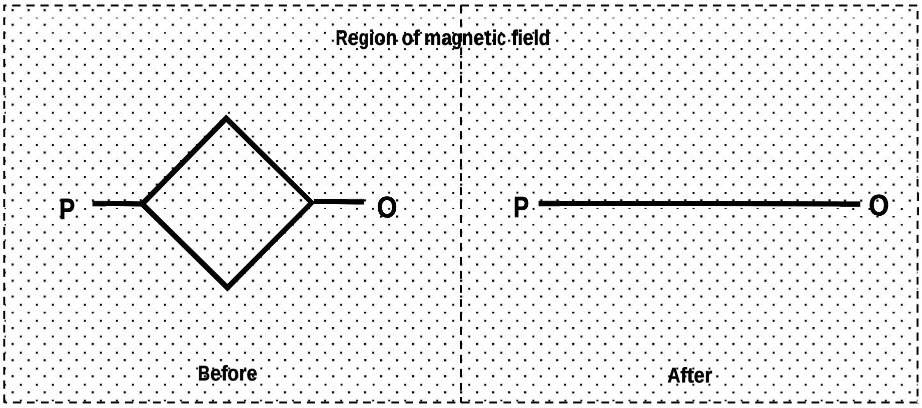
Loop 1 swings back and forth as the bob on a pendulum.
Loop 2 rotates about a vertical axis.
Loop 3 oscillates vertically on the end of a spring.

Which loop(s) have a magnetic flux that changes with time?

Only loop 2 has a changing magnetic flux as its orientation relative to the field changes as it rotates.

Example 3

A coil of 10 turns and an enclosed area of 0.35 m^2 is placed perpendicular to a magnetic field of flux density 2.0 T as shown in the diagram on the left. Points P and Q are pulled apart until the coil becomes a straight line as shown in the diagram on the right.



What is the change in magnetic flux linkage through the coil?

$$N\Phi_i = N(B\cos\theta)A$$

$$= 10(2.0\cos 0^o)(0.35) = 7.0 \text{ Wb}$$

$$N\Phi_f = 0 \text{ Wb}$$

$$\Delta N\Phi = N\Phi_f - N\Phi_i = -7.0 \text{ Wb}$$

17.3 Laws of Electromagnetic Induction

17.3.1 Faraday's Law

Faraday's Law of electromagnetic induction states that <u>the induced e.m.f. is proportional to the rate of change of the magnetic flux linkage.</u>

Mathematically: $$\varepsilon_{induced} \propto \frac{dN\Phi}{dt}$$

The polarity of the induced e.m.f. is governed by another law known as Lenz's Law.

17.3.2 Lenz's Law

Lenz's Law of electromagnetic induction states that <u>a current produced by the induced e.m.f. will flow in a direction such that the magnetic field it produces opposes the original change in magnetic flux linkage</u>.

Lenz's Law is a consequence of the principle of conservation of energy.

Consider moving a magnet towards the face of a solenoid. According to Lenz's Law, if the magnet's pole facing the solenoid was North, the induced current in the solenoid would flow such that the end facing the magnet would be a North pole as well.

Suppose the induced currents' directions were opposite to those prescribed by Lenz's Law; the north pole of an approaching magnet would induce a south pole in the near face of the solenoid. The attractive force between these poles would accelerate the magnet's approach and make the magnetic field increase more quickly. This in turn would increase the current induced in the solenoid, strengthening the magnetic field, which would in turn increase the forces of attraction and hence the acceleration of the magnet. Both the kinetic energy of the magnet and the rate of energy dissipation in the solenoid (due to heat dissipation) would increase. A small energy input would produce a large energy output, thereby violating the law of conservation of energy

Hence, the two laws of electromagnetic induction can be summarized succinctly as

$$\boxed{\varepsilon_{induced} = -\frac{dN\Phi}{dt}}$$

with the negative sign indicating that the induced e.m.f. opposes the change in magnetic flux linkage. This is also the relationship that gives rise to the definition of the weber given in the earlier section.

Example 4

A square coil of side 5 cm lies perpendicular to a magnetic field of flux density 4.0 T. The coil consists of 200 turns of wire.

(a) What is the magnetic flux linkage through the coil?

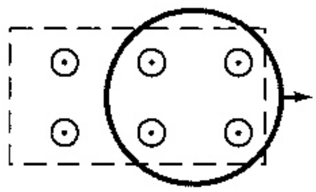
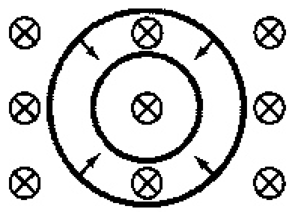
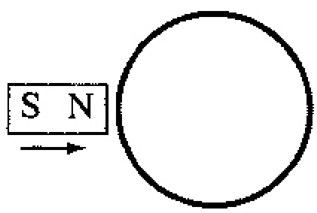
$$N\Phi = N(B\cos\theta)A$$
$$= 200(4.0\cos 0^o)(0.05^2) = 2.0 \text{ Wb}$$

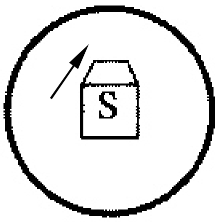
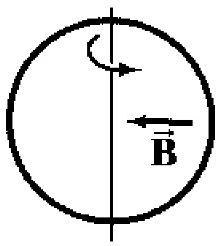
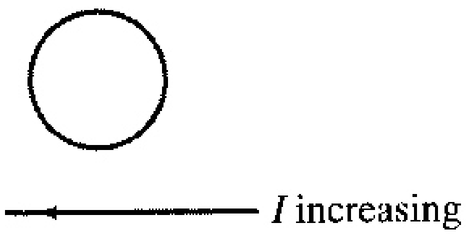
(b) The coil is rotated through an angle of 180^o in 0.4 seconds. Calculate the magnitude of the average e.m.f. induced in the coil while it is being rotated.

$$\varepsilon_{induced} = -\frac{dN\Phi}{dt}$$
$$= -\frac{dN(B\cos\theta)A}{dt}$$
$$\approx -NBA\frac{\Delta\cos\theta}{t}$$
$$= -(200)(4.0)(0.05^2)\frac{\cos 180^o - \cos 0^o}{0.4} = 10 \text{ V}$$

Example 5

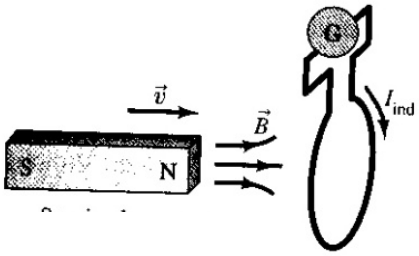
Using Lenz's Law, predict the direction of the current due to the induced e.m.f. in the circular coil for each of the following cases.

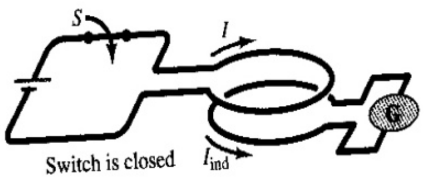
Action	Direction of Current due to Induced e.m.f.
(a) Pulling the coil to the right out of a magnetic field pointing out of page 	As the coil is pulled out of the field, magnetic flux linkage through the coil decreases. The current due to the induced e.m.f. will flow counter-clockwise to produce its own magnetic field pointing out of the page so as to increase the magnetic flux linkage through the coil.
(b) Shrinking a coil in a magnetic field pointing into page 	Since the coil area gets smaller, the magnetic flux linkage through the coil decreases. The current due to the induced e.m.f. will flow clockwise to produce its own magnetic field pointing into the page so as to increase the magnetic flux linkage through the coil.
(c) Magnetic north pole moving in the plane of the page towards the coil 	As the magnetic field is in the plane of the coil, the magnetic flux linkage through the coil remains at zero throughout. Since there is no change in the magnetic flux linkage through the coil, no e.m.f. is induced and hence no current flows in the coil.

Action	Direction of Current due to Induced e.m.f.
(d) Magnetic north pole moving into the page towards coil without passing through it 	As the magnet moves towards the coil, the magnetic field into the page experienced by the coil gets stronger. Hence the <u>magnetic flux linkage through the coil increases</u>. The current due to the induced e.m.f. will flow <u>counter-clockwise</u> to produce its own magnetic field pointing out of the page so as to decrease the magnetic flux linkage through the coil.
(e) Rotating the coil within a magnetic field pointing to the left. 	At this instant when the loop is rotated, the <u>magnetic flux linkage through the coil increases</u>. The current due to the induced e.m.f. will flow <u>counter-clockwise</u> to produce its own magnetic field with a component to the right of the page so as to decrease the magnetic flux linkage through the coil.
(f) Increasing the current in a wire that is placed below a coil in the plane of the page 	As the current in the straight wire increases, the <u>magnetic flux linkage through the coil increases</u>. The current due to the induced e.m.f. will flow <u>counter-clockwise</u> to produce its own magnetic field pointing out of the page so as to decrease the magnetic flux linkage through the coil. Note: The magnetic field produced by the straight wire within the coil points into the page.

Example 6

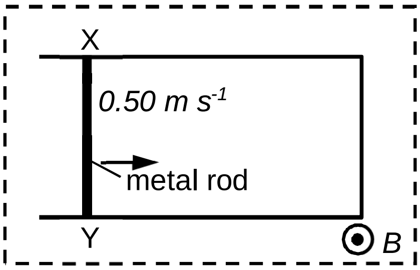
Using the Laws of Electromagnetic Induction, explain the following observations:

Experimental Observation	Explanation
 When the bar magnet is moved towards the wire loop, there is a deflection in the galvanometer. A faster movement of the magnet results in a larger deflection in the galvanometer. If the magnet is moved away from the wire loop, the deflection in the galvanometer is in the opposite direction.	When the magnet moves towards or away from the wire loop, the magnetic flux linkage through the loop changes. Hence, according to Faraday's law, an e.m.f. is induced in the loop. Since the loop is a closed circuit, current flows and hence a deflection in the galvanometer is observed. A faster movement of the magnet gives rise to a higher rate of change of magnetic flux linkage through the loop. This, by Faraday's Law, leads to a larger induced e.m.f. and thus a larger current which causes a larger deflection in the galvanometer. When the magnet moves towards the loop, the magnetic flux linkage through the loop increases. By Lenz's Law, the current due to the induced e.m.f. flows in a direction to decrease the magnetic flux linkage through the loop. However, when the motion is reversed, the magnetic flux linkage through the loop decreases and the current due to the induced e.m.f. flows in the opposite direction so as to increase the flux linkage through the loop. This caused the galvanometer to be deflected in the opposite direction.

Experimental Observation	Explanation
 When switch is closed in the upper loop, a momentary deflection in the galvanometer connected to the lower loop is observed. When the switch is subsequently opened, the deflection of the galvanometer is in the opposite direction.	When the switch is closed, the current flow in the upper loop produces a magnetic field. As the current in the upper loop builds up from zero to its steady value, it magnetic field that is produced changes, resulting in a changing magnetic flux linkage through the lower loop. According to Faraday's law, this changing magnetic flux linkage through the lower loop induces an e.m.f. in the lower loop. Since the lower loop is a closed circuit, current flows and hence a momentary deflection in the galvanometer is observed. According to Lenz's Law, when the magnetic flux linkage through the lower loop increases as the switch is closed, the current due to the induced e.m.f. flows in a direction to decrease the magnetic flux linkage through the lower loop. However, when the switch is opened, the magnetic flux linkage through the lower loop decreases and the current due to the induced e.m.f. flows in the opposite direction so as to increase the flux linkage through the lower loop. This caused the galvanometer to be deflected in the opposite direction.

Example 7

A metal rod XY of length 24 cm and resistance 20 Ω, is moved at a constant velocity of 0.50 m s^{-1} along the limbs of a U-shaped metal wire in a region of uniform magnetic field B. as shown.



The magnetic field has a flux density of 0.35 T and is directed out of the page. The limbs have negligible resistance.

(a) Determine the current in the rod.

$$\varepsilon_{induced} = -\frac{dN\Phi}{dt}$$
$$= -N(B\cos\theta)\frac{dA}{dt}$$
$$= -N(B\cos\theta)lv$$
$$= -(1)(0.35\cos 0^o)(0.24)(0.50) = 0.042 \text{ V}$$

$$I = \frac{\varepsilon_{induced}}{R}$$
$$= \frac{0.042}{20} = 2.1\times10^{-3} \text{ A}$$

(b) State and explain the direction of the current within the rod.

As the rod moves along the metal wire, the magnetic flux linkage through the loop decreases. By Lenz's law, the current (due to the induced e.m.f). will flow counter-clockwise to produce its own magnetic field pointing out of the page so as to increase the magnetic flux linkage through the loop. Hence the current flows from X to Y within the rod.

(c) Determine the power dissipated in the rod.

$$P = I^2R$$
$$= (2.1\times10^{-3})^2(20) = 8.82\times10^{-5} \text{ W}$$

(d) Calculate the magnitude of the external force required to keep the rod moving at constant velocity.

$$F_{\text{ext}} = F_{\text{B on rod}}$$
$$= BIL$$
$$= (0.35)(2.1\times10^{-3})(0.24) = 1.76\times10^{-4} \text{ N}$$

(e) Hence determine the rate of work done by the external force.

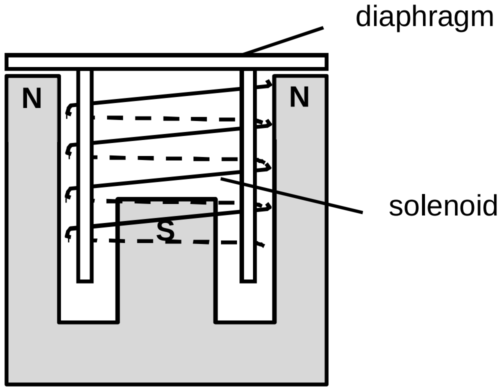
$$P_{\text{ext}} = F_{\text{ext}}v$$
$$= (1.76\times10^{-4})(0.50) = 8.82\times10^{-5} \text{ W}$$

(f) Comment on the answers to **(c)** and **(e)**.

The values are the same because by the law of conservation of energy, the work done by the external force on the rod is converted to the electrical energy that is dissipated in the rod.

17.4 Applications of Electromagnetic Induction

17.4.1 Microphones



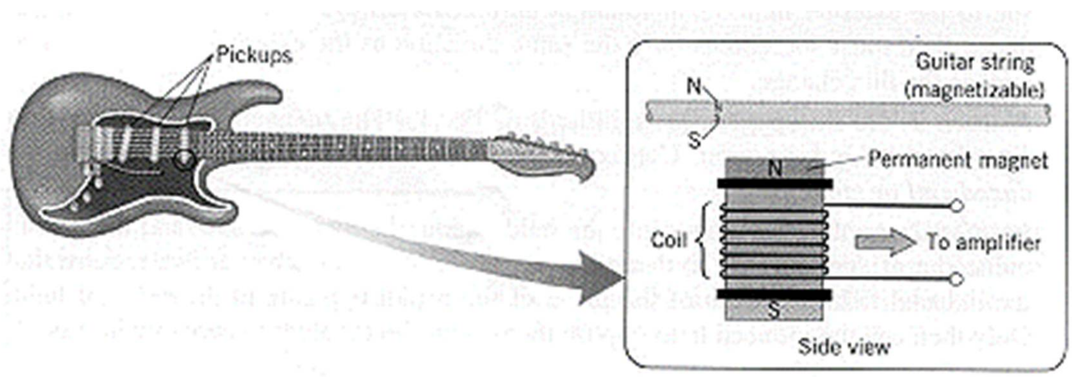
In microphones, there is usually a stationary permanent magnet and a wire coil attached to a movable diaphragm as shown.

When a sound wave strikes the microphone, the pressure differential within the wave causes the diaphragm to oscillate, moving the coil to and fro from the magnet.

This movement changes the magnetic flux linkage through the coil, and by Faraday's Law, produces an induced e.m.f. This induced e.m.f. signal can be transmitted and amplified to recreate the sound.

17.4.2 Electric Guitars

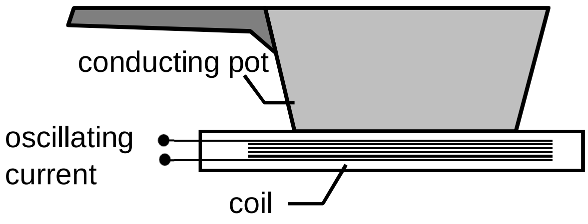
An acoustic guitar depends on the acoustic resonance produced in the hollow body of the instrument by the oscillations of strings. An electric guitar is, on the other hand, a solid instrument, so there is no body resonance. Instead, the oscillations of the metal strings are sensed by electric "pickups" that send signals to an amplifier and a set of speakers.



For each pickup, wire is coiled around a small magnet and this wire is connected to an amplifier. The magnetic field of the magnet produces a north and south pole in the section of the metal strong above the magnet. This section of string then has its own magnetic field.

When the string is plucked and made to oscillate, its motion relative to the coil causes a change of magnetic flux linkage through the coil, thus by Faraday's Law, induces an e.m.f. As the string oscillates towards and away from the coil, the induced e.m.f. varies at the same frequency as the string's oscillations and this signal can be transmitted and amplified to recreate this frequency at the speakers.

17.4.3 Induction Cookers



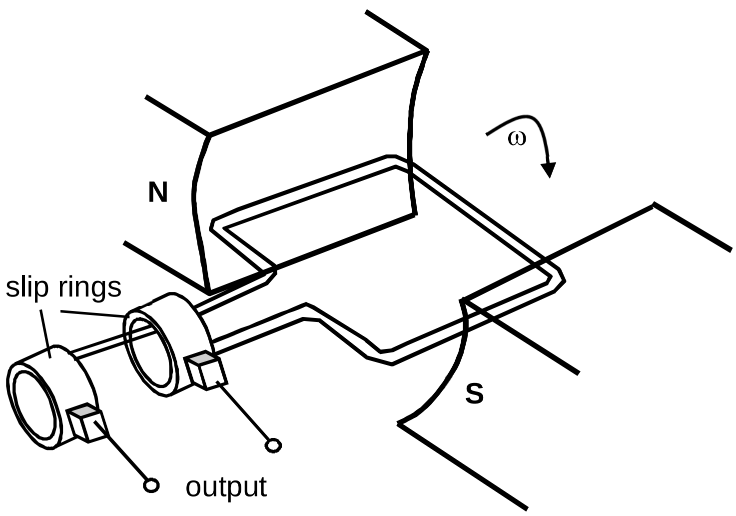
The oscillating current in the coil within the induction cooker produces an alternating magnetic field which results in periodic change in flux linkage in the base of the pot.

Hence by Faraday's law, an e.m.f. is induced which produces (eddy) currents in the conducting pots. The large amount of heat generated in the pot due to currents is used to cook food.

Hence, the pot is usually made of metal as it needs to be a conductor of electricity. The cooking surface is a non-conductor (usually ceramic) and thus eddy currents cannot flow in it. Thus the temperature of the cooking surface only rises to the extent that heat is conducted to it from the pan, i.e. the cooking surface is no hotter than the bottom of the pan.

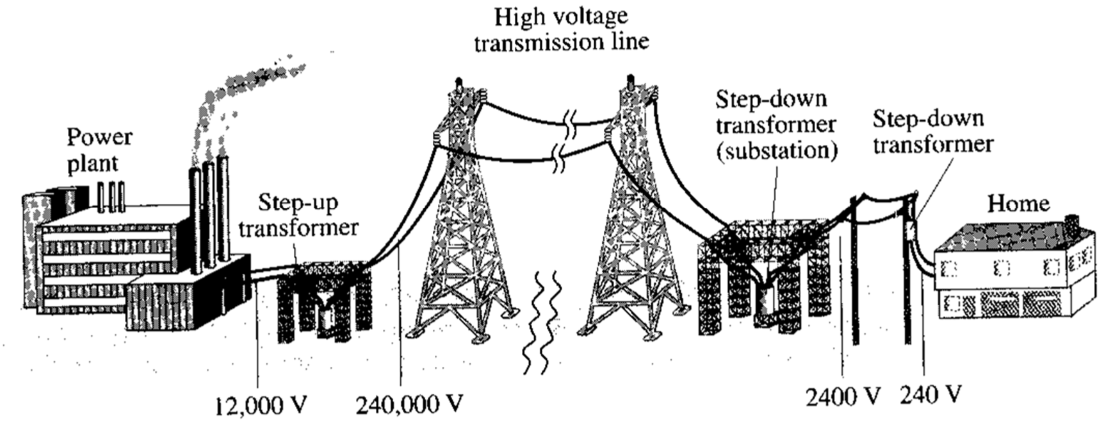
17.4.4 Alternating Current Generator

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

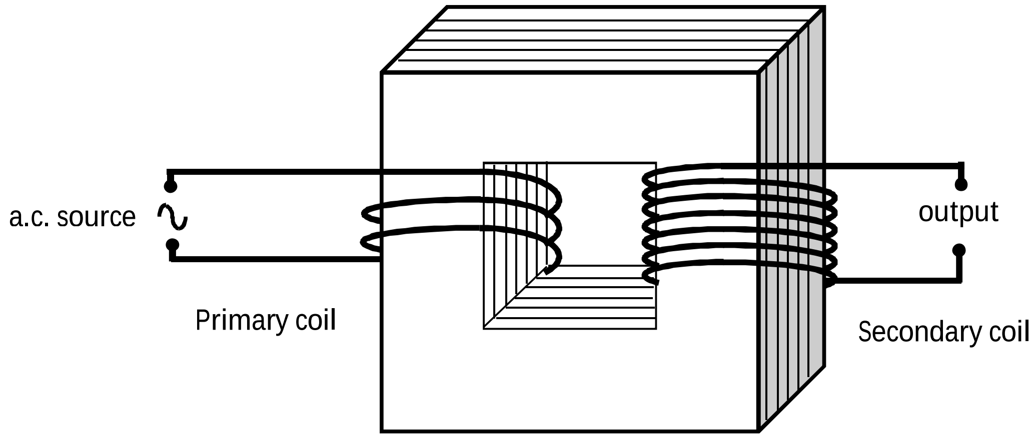


When the coil is rotated in the magnetic field, there will be a change of magnetic flux linkage through the coil due a change in the orientation of coil relative to the magnetic field. Hence by Faraday's law, there is an induced e.m.f.

17.4.5 Transmission of Electricity



Our entire system for distributing electricity is based on transformers, devices that uses the concept of electromagnetic induction. Transformers raise voltages for transmission over long distances across power lines. Transformers then reduce voltages for safe use in businesses and homes.



A simple transformer which consists of two coils wound on a common soft iron core so that nearly all the magnetic field lines produced by the primary coil pass through each turn of the secondary coil. Whenever the magnetic field produced in the primary coil changes due to the alternating current, the magnetic flux linkage in the secondary coil also changes.

Hence by the laws of Electromagnetic Induction, an alternating e.m.f. is thus induced in the secondary coil. By adjusting the ratio of the number of turns in the primary and secondary coil, the magnitude of the induced e.m.f. in the secondary coil may be varied. More details regarding the transformer will be covered in the next topic.