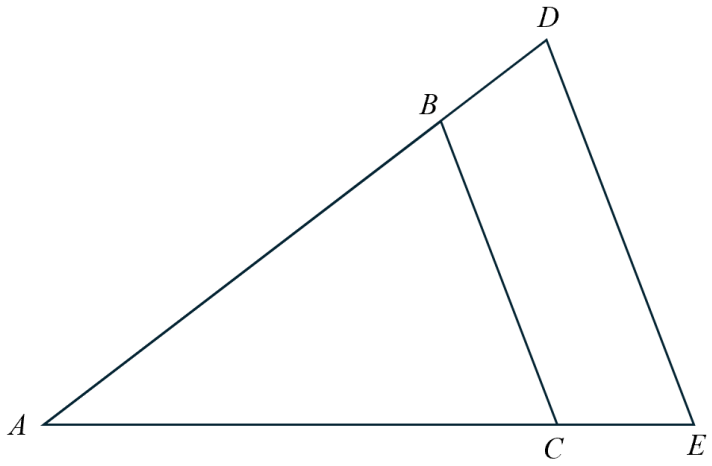
Congruence and similarity

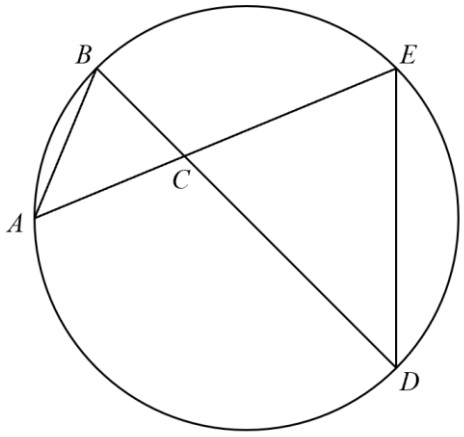
1



Show that triangle *ABC* and triangle *ADE* are similar. [3]

Angle *ABC* = angle *ADE* (corr. ∠, *BC*//*DE*) **[1]**
Angle *BAC* = angle *DAE* (common ∠) **[1]**
∴ triangle *ABC* and triangle *ADE* are similar. (AA Test) (shown) **[1]**

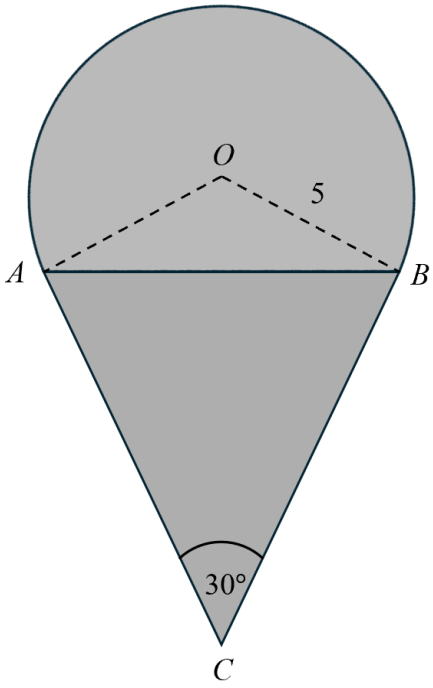
2



Show that triangle *ABC* and triangle *DEC* are similar. [3]

Angle *ABC* = angle *DEC* (angle in same segment) **[1]**
Angle *ACB* = angle *DCE* (vert. opp. ∠) **[1]**
∴ triangle *ABC* and triangle *DEC* are similar. (AA Test) (shown) **[1]**

3

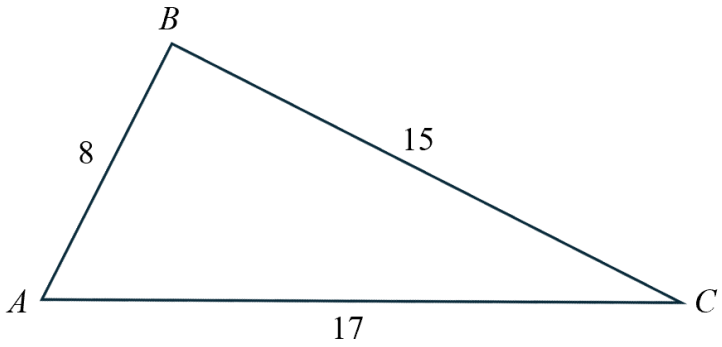


Show that triangle *AOC* and triangle *BOC* are congruent. [4]

Angle OAC = angle OBC (radius ⊥ tangent) (R) **[1]**
OC (common hypotenuse) (H) **[1]**
OA = OB (radius) (S) **[1]**
∴ triangle *AOC* and triangle *BOC* are congruent. (RHS Test) (shown) **[1]**

Pythagoras Theorem/Right-angled triangle

1



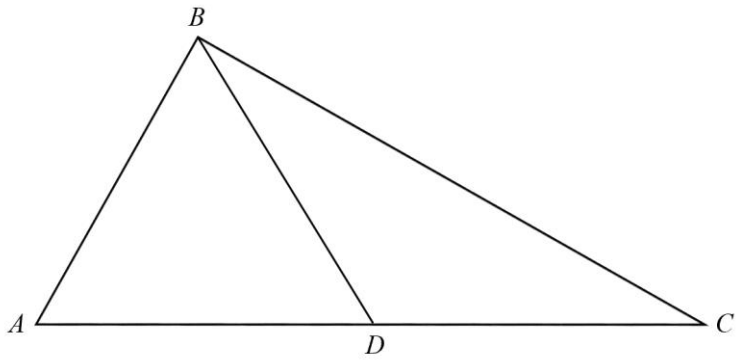
Show that angle $ABC = 90°$. [3]

$AB^2 + BC^2 = 8^2 + 15^2 = 289$ **[1]**

$AC^2 = 17^2 = 289$ **[1]**

Since $AB^2 + BC^2 = AC^2$, by converse of Pythagoras Theorem, hence triangle ABC is a right-angle triangle and angle $ABC = 90°$ (shown) **[1]**

2 (O level 2023 P1 Qn 14)



ABC is a triangle.

D is a point on AC such that it is equidistant from A, B and C.

Explain why angle ABC is a right angle. (Ans from theannexproject)

let $\angle ADB = 1°$, then $\angle ABD = \frac{180-1}{2} = (90-\frac{1}{2})°$ (AD = BD, base $\angle$s of isosceles $\triangle$)

$\angle BDC = 180° - 1°$ (adj. $\angle$s on a straight line)

$\angle DBC = \frac{180°-(180-1)}{2} = \frac{1°}{2}$ (CD = CD, base $\angle$s of isosceles $\triangle$)

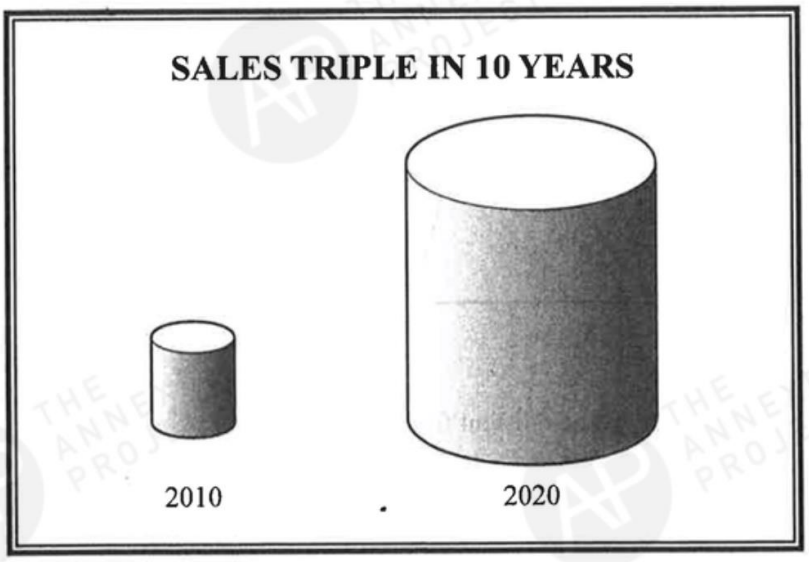
$\angle ABC = \angle ABD + \angle DBC = (90-\frac{1}{2}) + \frac{1}{2} = 90°$

[2]

Data Analysis

1 (O level 2023 P1 Qn 16)

A company sells tinned food.
The company displays the sales figures with this chart.



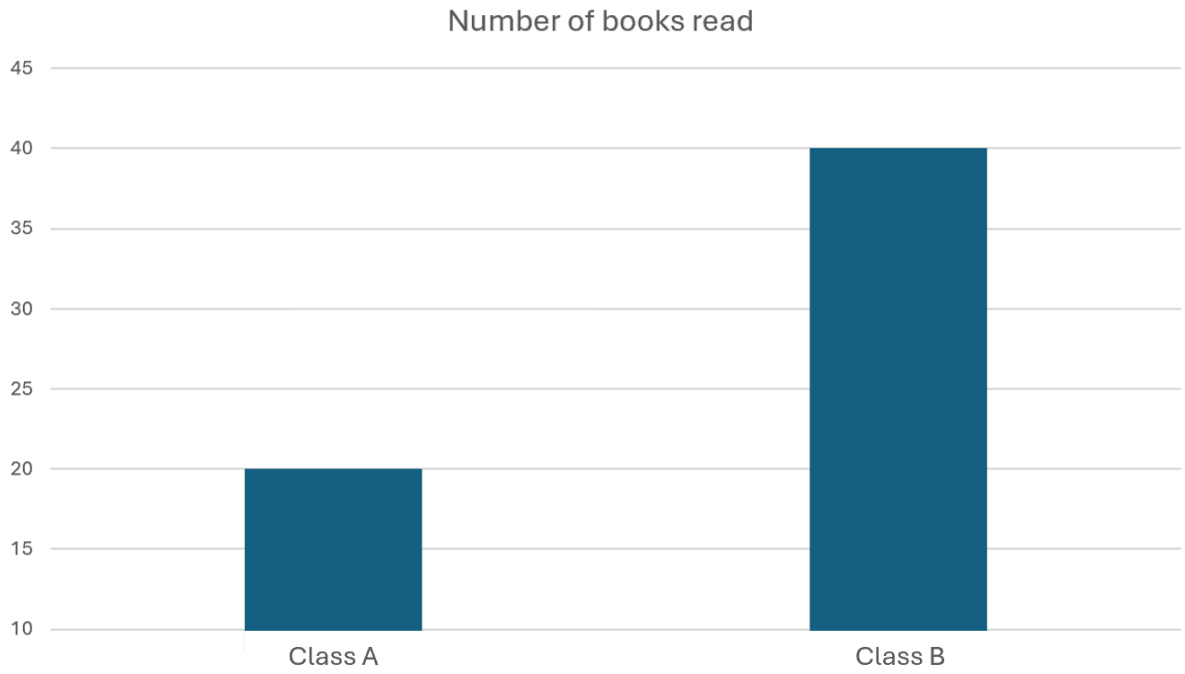
The chart shows drawings of two geometrically similar solids. The dimensions of the tin for 2020 are three times the corresponding dimensions of the tin for 2010.

The chart is misleading.
Explain what it implies about the company's sales of tinned food.

By similarity, $(\frac{1}{3})^3 = \frac{1}{27}$ which suggests sales of tinned food increase by 27 folds, however, in reality sales only tripled. [1]

(Ans from theannexproject)

2



The bar graph shows the number of books read by two class, *A* and *B* this week.

State one aspect of the graph that may be misleading and explain how this may lead to a misinterpretation of the graph. [2]

The *y*-axis does not start from 0. Based on the graph, the number of books read by Class *B* is thrice the number of books read by Class *A*. However, it should be twice.

Numbers/Prime factorisation

1 The prime factorisation of two integers, M and N are shown below.

$$M = p^8 \times q^4 \times r^6$$

$$N = p^{x+1} \times q^4 \times r^{2y-1}$$

Explain why M is a perfect square? [1]

The powers of p, q and r are multiples of 2.

2 Show that $(2n + 3)^2 - 5$ is a multiple of 4 for all integer values of n. [2]

$$\mathbf{(2n + 3)^2 - 5 = 4n^2 + 12n + 9 - 5}$$

$$\mathbf{= 4n^2 + 12n + 4}$$

$$\mathbf{= 4(n^2 + 3n + 1)}$$ **[1]**

Since $(2n + 3)^2 - 5 = 4(n^2 + 3n + 1)$, which can be divisible by 4, hence $(2n + 3)^2 - 5$ is a multiple of 4 for all integer values of n. (shown) **[1]**

3 In the figure, five numbers are shown below to form a pattern.

5, 12, 19, 26, 33

Aaron says that 112 is a term in the pattern.

Is he correct? Explain your answer. [1]

No, he is not correct.

$7n - 2 = 112$, $n = \frac{114}{7}$.

Since the value of n is not an integer, hence 112 is not a term in the pattern.

4 Let x be the number of minutes a pump can generate water out.

Solve the equation $3x^2 - 15x - 100 = 0$.

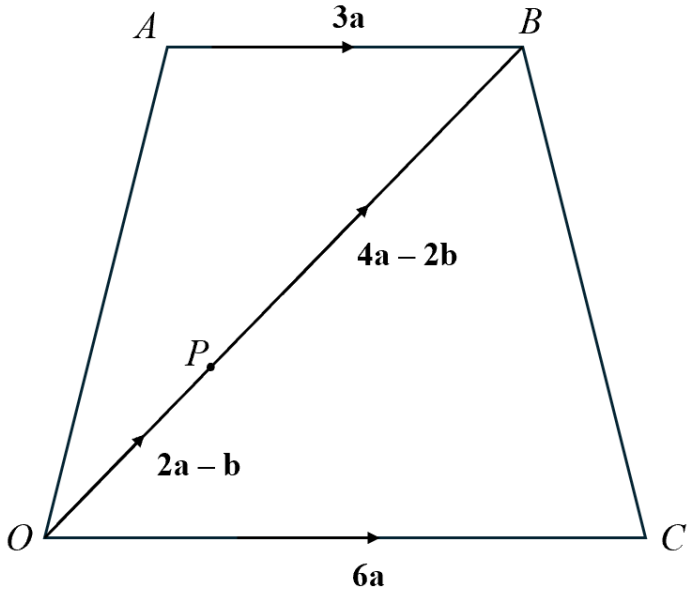
Explain why one of your solutions is not acceptable. [1]

$x = 8.79$ or $x = -3.79$.

Since the number of minutes cannot be negative, hence $x = 8.79$.

Vectors

1



(a) Explain why AB and OC are parallel. [1]

Since $\overrightarrow{OC} = 2\overrightarrow{AB}$, hence AB and OC are parallel.

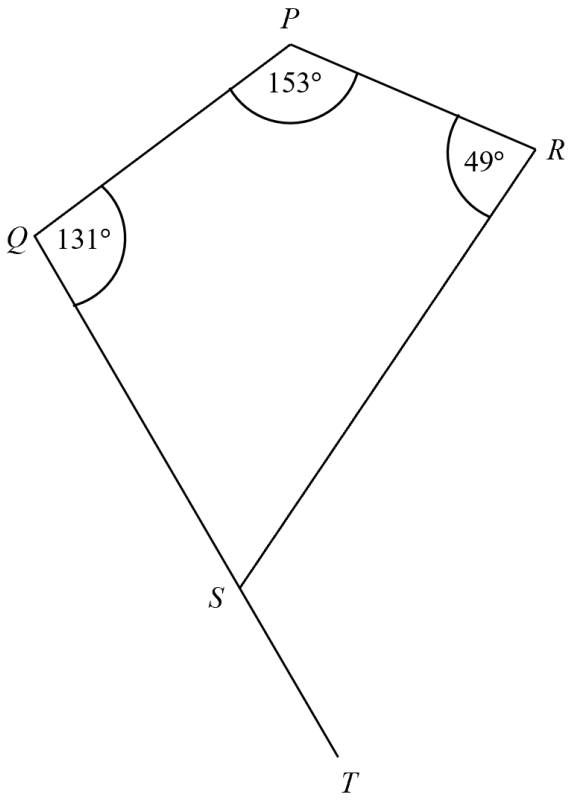
(b) Explain why points O, P and B are collinear. [2]

$\overrightarrow{OP} = 2a - b$ and $\overrightarrow{PB} = 2(2a - b)$ **[1]**

Since $\overrightarrow{PB} = 2\overrightarrow{OP}$, hence points O, P and B are collinear. **[1]**

Circle properties

1

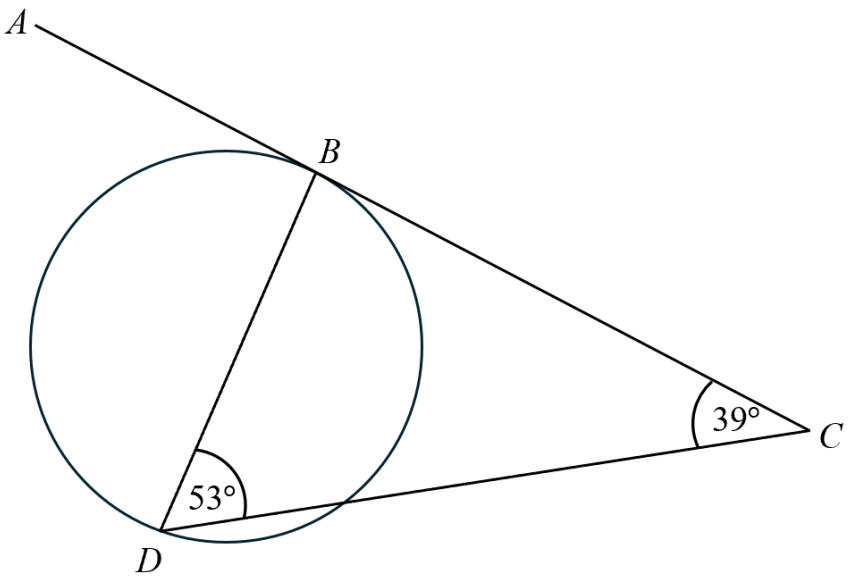


Explain why a circle can be drawn with points *P*, *Q*, *R* and *S*? [1]

Angle PQS + angle PRS = 131° + 49° = 180°

By converse of angle at opposite segment, a circle can be drawn with points *P*, *Q*, *R* and *S*.

2 (O level 2013 P1 Qn 13 (a))



ABC is a tangent to the circle at point *B*.

Explain why *BD* is not a diameter of the circle. [1]

Angle DBC = 180° - 53° - 39° = 88° (∠ sum of Δ)

Since angle DBC ≠ 90°, by converse of radius ⊥ tangent, BD is not a diameter of the circle.