TEMASEK JUNIOR COLLEGE, SINGAPORE

Preliminary Examination 2012
Higher 2

MATHEMATICS 9740/01

Paper 1

13 September 2012

Additional Materials: Answer paper
List of Formulae (MF15)

3 hours

READ THESE INSTRUCTIONS FIRST

Write your Civics group and name on all the work that you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

You are expected to use a graphic calculator.

Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.

Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.

You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
At the end of the examination, fasten all your work securely together.

This document consists of **6** printed pages.

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PASSION PURPOSE DRIVE

1 An ellipse E has equation

$$9x^2 + 16y^2 = 144.$$

(i) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in term of x and y. [1]

(ii) Show that the point P with coordinates ($4\cos\theta,\ 3\sin\theta$) lies on E. [1]

(iii) Find the equation of the normal to the ellipse E at the point P, in the form of $y = mx + c$ where m and c are single trigonometric expressions of θ. [3]

2 Consider the equation

$$2z^3 + (1-2\mathrm{i})z^2 - (a+b\mathrm{i})z + 2 + 2\mathrm{i} = 0,$$

where a and b are real.

Given that -2 is a root of the equation, find the values of a and b. [3]

Given also that $1+\mathrm{i}$ is another root, find the third root of the equation. [3]

3 HIMHEYS' Confectionery recently created three types of chocolate: Organic white, Organic milk and Organic dark, available in 250g bars. Cocoa butter, an essential ingredient in chocolate bars, makes up 25%, 20% and 15% of the mass of an Organic white, Organic milk and Organic dark chocolate bar respectively.

To prepare for the official launch of their chocolate bars at an upcoming Food Expo, HIMHEYS' decides to manufacture a total of 300 bars for the event, with more than 70 bars of each type.

The confectionary intends to use 14kg of cocoa butter in the production of the above batch of chocolate bars. If the number of milk chocolate bars is to be smaller than the number of white chocolate bars, determine how many organic chocolate bars of each type can be produced. [6]

4 Referred to an origin O, the position vectors of two points A and B are $\mathbf{a}$ and $\mathbf{b}$ respectively. The points P on OA and Q on AB are such that $OP = 2PA$ and $5AQ = 4QB$. Show that the equation of the line l passing through P and Q can be written as

$$\mathbf{r} = \frac{2}{3}\mathbf{a} + \lambda(4\mathbf{b} - \mathbf{a}),\ \text{where } \lambda \in \mathbb{R}.$$ [4]

The point X on l is such that AX is perpendicular to l. If $|\mathbf{a}| = 2$, $|\mathbf{b}| = 1$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$, show that the position vector of X is $\frac{1}{15}(11\mathbf{a} - 4\mathbf{b})$. [4]

[Turn over

5 The functions f and g are defined by

$$f : x \mapsto \sqrt{6+x-x^2},\ -2 < x < \frac{1}{2},$$

$$g : x \mapsto \ln\left(9-x^2\right),\ -3 < x < 3.$$

Determine whether each of the following functions exists and give a definition (including the domain) of the function if it exists.

(a) f^{-1},

(b) gf . [9]

6 Given that $y = \ln\left(1+\sin x\right)$, show that

(i) $e^y \frac{dy}{dx} = \cos x$, [2]

(ii) $\frac{d^3y}{dx^3} + 3\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right) + \left(\frac{dy}{dx}\right)^3 + \frac{dy}{dx} = 0$. [3]

Find the Maclaurin's series for y up to and including the term in x^3. [3]

Hence, or otherwise, show that $\frac{\cos x}{1+\sin x} \approx 1 - x + \frac{x^2}{2}$. [2]

7 **(a)** A finite arithmetic progression has n terms and common difference d. The first term is 1 and the sum of the last 5 terms exceeds the sum of the first 4 terms by 193.

(i) Show that $5nd - 21d - 192 = 0$. [3]

(ii) Given also that the 6^{th} term of the progression is 16, find n. [2]

(b) A sequence U is formed in which the n^{th} term is given by e^{t_n} where t_n is the n^{th} term of an arithmetic progression with first term $t_1 = 1$.

(i) Show that U is a geometric progression. [2]

(ii) Given that the sum to infinity of even-numbered terms of U is $\frac{8e}{63}$, find the common ratio of U. [3]

[Turn over

8 The point A has position vector $3\mathbf{j} - 4\mathbf{k}$ with respect to an origin O.

The plane π_1 has Cartesian equation $13x - 9y + z = 15$.

(i) If π_2 is a plane parallel to π_1 and contains the point A, write down the equation of π_2 in scalar product form and find the distance from the origin O to π_2. [3]

(ii) Hence or otherwise, find the distance between π_1 and π_2. State with clear explanations whether O and A are on the same side of π_1. [3]

(iii) Another plane π_3 has Cartesian equation $x + py + 3z = q$. If π_2 and π_3 intersect at a line containing A and π_3 is perpendicular to π_1, find the value of p and q. [4]

9 **(a)** Prove by induction that

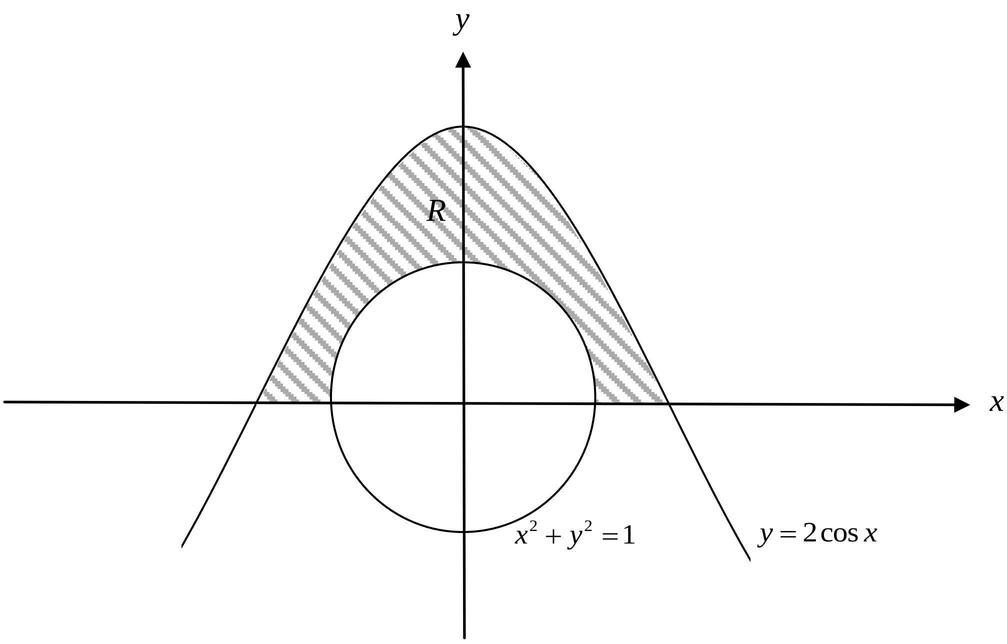
$$\sum_{r=1}^{n}\left(\frac{r}{2^r}+1\right) = (n+2)\left(1-\frac{1}{2^n}\right).$$ [5]

(b) Show that $u^2 + u - 1$ can be written in the form $(u+2)(u+1) - k(u+2) + h$ where h and k are positive constants to be determined. [2]

Hence show that $\displaystyle\sum_{u=1}^{N}\frac{1-u-u^2}{(u+2)!} = \frac{N+1}{(N+2)!} - \frac{1}{2}$. [4]

[Turn over

10



The diagram shows the region R bounded by the x-axis and the two curves $y = 2\cos x$ and $x^2 + y^2 = 1$.

(i) Find the exact area of the region R. [3]

(ii) Using integration by parts, show that

$$\int u^2 \sin u \, du = -u^2 \cos u + 2u \sin u + 2\cos u + C$$

where C is a real constant. [2]

(iii) The region R is rotated π radians about the y-axis to form a solid of revolution S. Show that the volume of S can be expressed as

$$\pi \int_0^m \left(\cos^{-1} \frac{y}{2} \right)^2 dy - k ,$$

where m and k are exact values to be determined. [3]

Hence, by using the substitution $u = \cos^{-1} \frac{y}{2}$ and the result in part (ii), find the exact value of the volume of S. [4]

[Turn over

11 **(a)** Show that

$$\int_0^{\frac{e}{2}} \frac{4x+e^2}{4x^2+e^2}\,\mathrm{d}x = m\ln 2 + ne\,,$$

where m and n are exact values to be determined. [6]

(b) In an experiment to study the spread of a soil disease, an area of 15 m^2 of soil was exposed to infection. In a simple model, it is assumed that the infected area grows at a rate which is proportional to the product of the infected area and the uninfected area. Initially, 5 m^2 was infected and the rate of growth of the infected area was 0.1 m^2 per hour. At time t hours after the start of the experiment, an area x m^2 is infected.

(i) Show that $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{x(15-x)}{500}$. [2]

(ii) Solve the differential equation and express t in terms of x. [4]

(iii) Find the minimum time in hours needed for 95% of the soil area to become infected. [1]

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