Preliminary Examination Paper 2
The *caifan* Exam
(final)

Time: 2 hours 15 minutes

Name: ……………………………

(Paper 2 Question Paper)

Marks: **90**

Topics: everything other than
trigonometry, differentiation, and integration
(do paper 1 first!)

Pages: **30**

READ THE INSTRUCTIONS FIRST
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question. The use of an approved scientific calculator is expected, where appropriate. You are reminded of the need for clear presentation in your answers.

90

Formula List:

1. ALGEBRA

Quadratic Equation

For the equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial expansion

$$(a+b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \frac{1}{2} ab \sin C$$

To all viewers (and holy grail moderators):
setter of this paper is a sec 4 student who has taken a deep and fond interest in my beloved, amath
1) thank you holy grail mods!
2) this paper, if you are using it as practice, is much more difficult from the regular question types. in my opinion, amath questions has gotten more vanilla and less interesting once you start to try more papers, hence this paper showcases how some amath questions can truly be very difficult
this amath paper consists of a mishmash of small miscalleneous chapters, from quadratic functions to plane geometry (which is why it's named the caifan paper)
3) similarly, marks might not be given as fairly in this paper as in normal papers
4) if you really want to do it timed, I suggest giving more time (about 15 minutes) to balance the fairness of this paper
inspiration for some qns:
nchs, tkss, gess, blss, nhhs, mgs

Answer **all** questions.

1 **(a)** Solving the inequality $kx^2 > 2x + n$, the solutions are $x > 3 + 2\sqrt{3}$ and $x < 3 - \sqrt{p}$, where k, n, and p are rational constants.

Find the values of k and n. [2]

(b) The diagonals of a rhombus are of length $(1 + 2\sqrt{3})$ cm and x cm respectively. The area of the rhombus is $(8 + 5\sqrt{3})$ cm^2 .

Find the perimeter of the rhombus, in the form $a\sqrt{b + c\sqrt{3}}$ cm, where a, b, and c are **integers** in simplest possible form. [2]

2 Look at Diagram **2** below closely. It shows a fish jumping out of its tank into another empty tank. After the fish jumps out of the tank, at a certain time in the air, the movement of the fish can be plotted as a parabolic curve below. The height h axis cuts the centre of the fish while the distance d axis lies on the openings of the fish tanks.

The equation of the curve is $h = ad^2 + bd + c$, where a, b and c are constants.

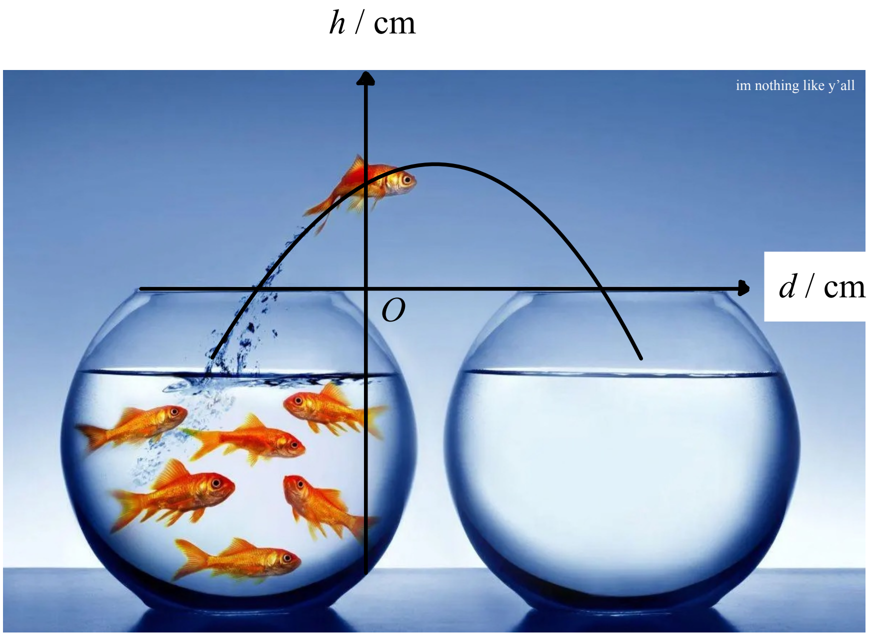

Diagram **2.1**

(i) The maximum value of h is 7.2 cm when the horizontal distance from the h axis was 2 cm. The fish travelled a horizontal distance of 12 cm from the initial opening of the fish tank to the other opening of the fish tank.
Find the equation of the curve, in the form given above. [3]

(ii) The horizontal line where both of the water in the fish tanks lie on the line $h = -4.2$. Find the horizontal distance travelled by the fish from the water in the left tank into the water in the other tank. [1]

(iii) Look at the diagram below. It shows a quadratic graph being drawn below. Its maximum point lies on the x axis.

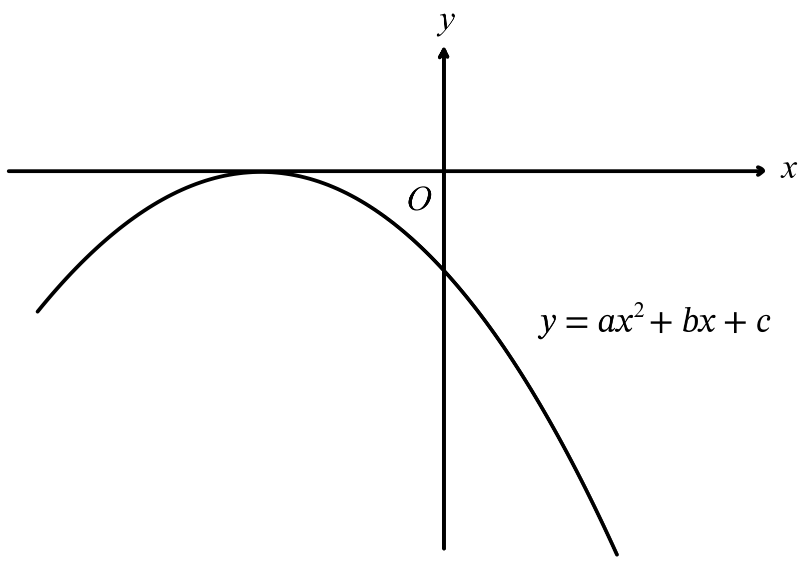

Diagram **2.2**

Determine the conditions of a, b, and c. [1]

3 **(i)** A polynomial P(x) has a function where if P(x) is divided by $x^2 - 9$, its remainder is $-8x + 6$.
When P(x) is divided by $x^2 + 9$, its remainder is $5x - 2$.
Find the remainder of P(x) when it is divided by $x^3 + 3x^2 + 9x + 27$. [3]

(ii) When P(x) Q(x) is divided by $(x - 3)$, where Q(x) is a polynomial, its remainder is k, where $k > 0$.
Find the smallest value V added to Q(x) such that $(x - 3)$ is a factor of (Q(x) + V), in terms of k if needed. [1]

4 **(a)** Express $\dfrac{x^4 + 8x^3 + 12x^2 + 20x + 85}{(x^2 + 5)\,(x + 5)^2}$ in partial fractions. [4]

(b) The polynomial f(x) is such that the coefficient of x^4 is 2.

The roots of the equation f(x) = 0 are $\frac{3}{2}$ and -1.

f(x) has a remainder of -8 when divided by $(x - 1)$ and a remainder of 28 when divided by $(x + 2)$.

(i) Find an expression for f(x), given its highest degree is 4.

Hence determine the number and nature of the roots of $e^{f(x)} = 1$. [3]

(ii) Expand and simplify f(x) in descending powers of x.

Hence solve $2 + y^2 - 5y^6 - 6y^8 = 0$. [2]

5 Solutions to this question by accurate drawing will not be accepted.

The diagram shows a kite $ABCD$ in which A, B and C are points on a circle C_1.
The coordinates of A and B are $(-2, 7)$ and $(5, 8)$ respectively.
The lines AC and BD intersect at point E, the centre of the circle.

BD makes an angle θ with the x axis such that $\cos\theta = \frac{3}{5}$.

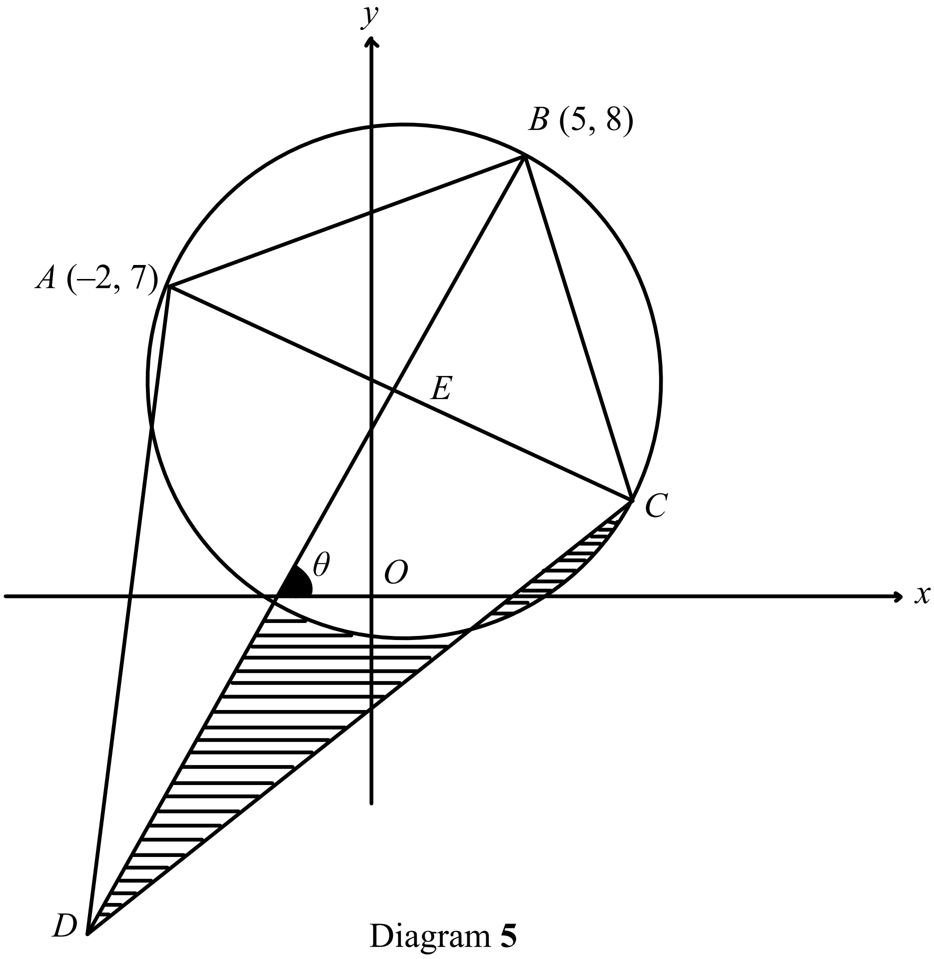

Diagram **5**

(i) Find the coordinates of C. [3]

(ii) It is further given that the ratio of $BE : BD = 2 : 7$.
Find the coordinates of D. [1]

(iii) The centre of another circle lies on BD and touches D.
It intersects the circle C_1 at two distinct points.
Find the range of possible values of the radius r units of the circle. [1]

(iv) Calculate the difference in the shaded area. [1]

6 **(a)** Given that $3.5^k = h$ and $2^k = \frac{h}{2}$,

calculate $\log_{h^2}\sqrt[3]{\frac{49}{h^5}} + \log_{343} h$. [2]

(b) Solve the following equations:

(i) $5^{2a-3} = 8^{2a-1} + 16^{\frac{3}{2}a}$, and [2]

(ii) $\lg b + \log_{100}(b-1) = \lg\sqrt{2b-b^2}$. [2]

(c) Look at Diagram **6** below.

It shows the part of the graph of $y = \log_a(x + b) + c$, where a, b, and c are constants.

The graph passes through the point $(-3\ ,\ 2)$ and intersects the x axis at $x = -\dfrac{20}{9}$.

The graph also approaches the vertical line $x = k$.

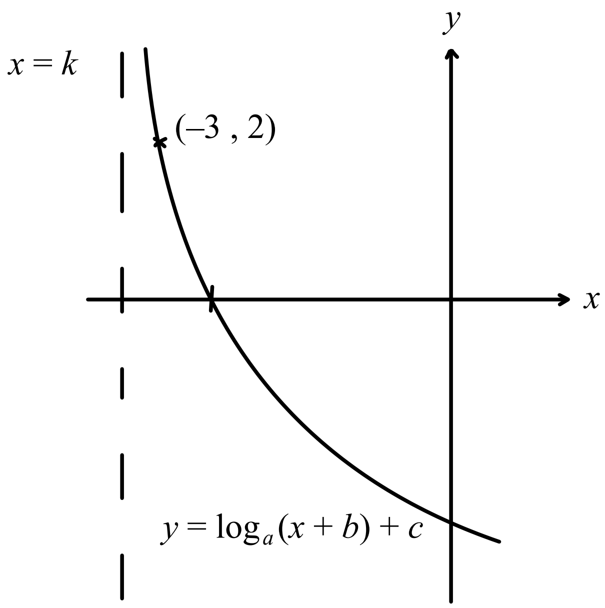

Diagram **6**

(i) Circle the right condition for a below. Explain your answer carefully. [1]

$a < 0$ $\qquad\qquad$ $0 < a < 1$ $\qquad\qquad$ $a > 1$

(ii) Express the relationship between k and either a, b, and/or c. [1]
Express how you got your answer clearly.

(iii) Given that $k = -4$, find the equation of y in the form given. [2]

(d) It is given that $g(t) = \log_t(7 - 2t)$. Explain, with working, whether $g(t)$ is always positive. [1]

7 Determine whether $ax(x + 1) - x(x + 2) > a - \frac{1}{2}$ works for $x \in \mathbb{R}$ for $a \in \mathbb{R}$. [3]

8 The diagram shows the graphs of $y = 2x - 1$ and $y = kx^2$, where k is a positive constant. The graphs intersects at two distinct points A and B.

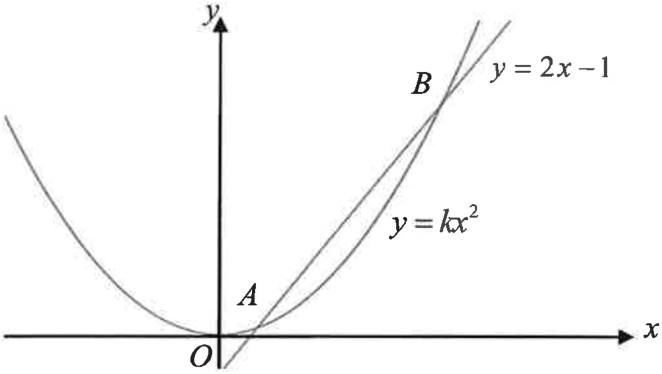

Diagram **8**

(i) Explain using all the verbal and visual information given that $0 < k < 1$. [1]

(ii) Describe the relationship between the graphs $y = 2x - 1$ and $y = kx^2$ for $k = 1$. [1]

(iii) Hence find the range of values for p for which $\dfrac{p^4 - 2p^3 - 3p^2}{2p - 2p^2 - 1} > 0$. [2]

9 Two circles, C_1 and C_2, lie on the same plane.

A and B are the two points of intersection of C_1 and C_2.

C_1 has centre (2 , 5), B (5.2 , 2.6), and a point on C_2 is (4 , –1).

The two tangents to either circles at A intersect the other circles' centres.

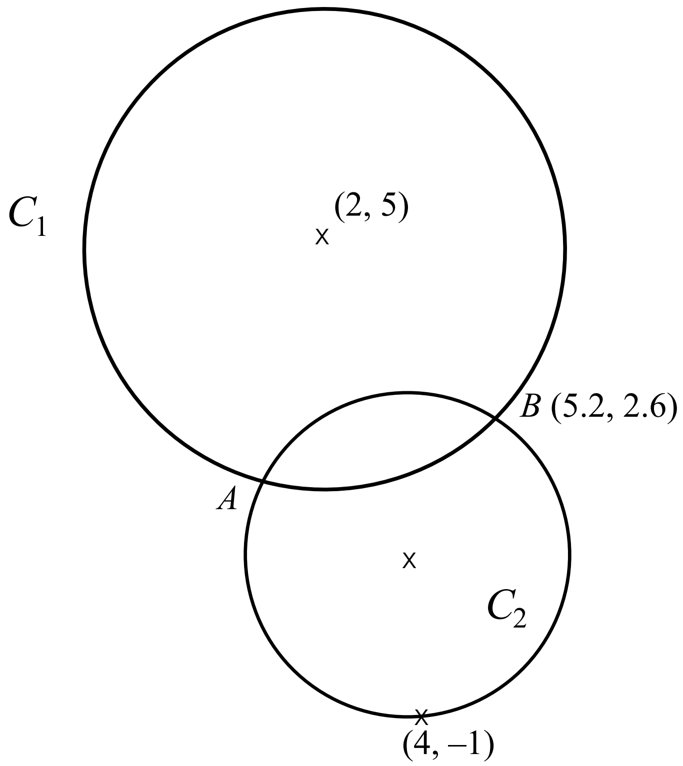

Diagram **9.1** (not drawn to scale)

(i) The radius connecting the centre C_2 and B has gradient $\frac{4}{3}$.

What can be deduced about the line connecting the centre of C_1 (2 , 5) and coordinate B in relation with the circle C_2? [1]

(ii) Find the equation of C_2. [3]

(iii) Find the exact equation(s) of the tangent(s) that are tangent to **both** circles C_1 and C_2. [4]

(iv) A line of symmetry is drawn passing through points A and B.
It reflects circle C_1 to form a new circle C_3 .

*Refer to Diagram **9.1** for the continuation of information*

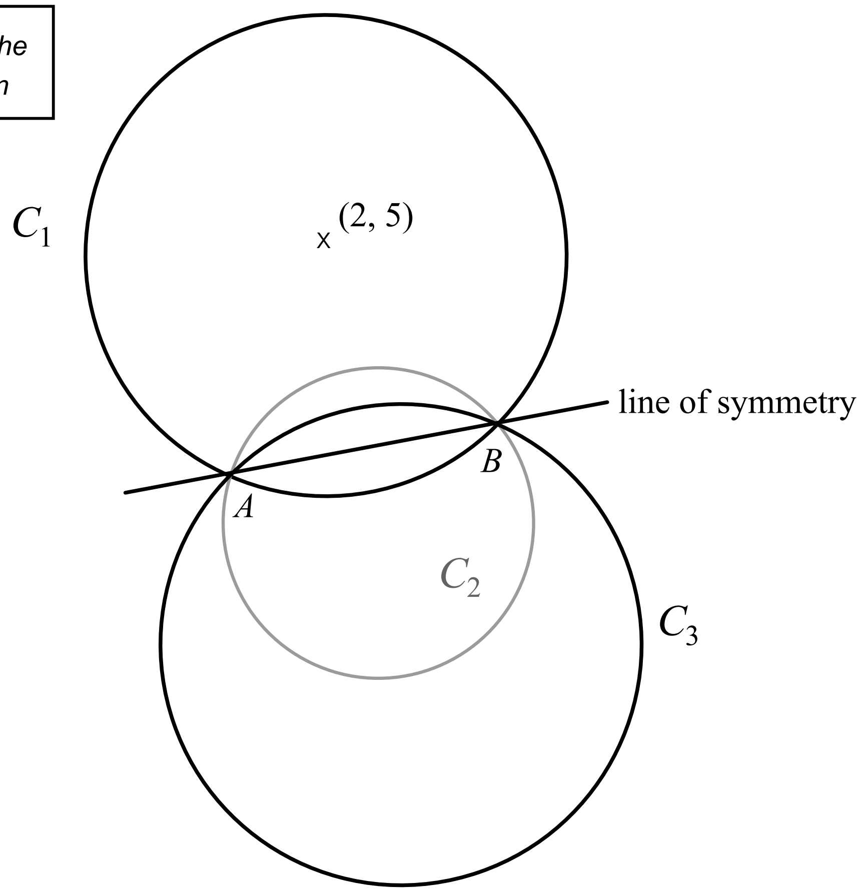

Diagram **9.2** (not drawn to scale)

Determine whether the centre of C_3 lies inside, outside, or on C_2 . [2]

10 (a) For the growth of bacteria, bacteria reproduces asexually via mitosis, through a process called binary fission. In this process, a regular bacterial cell grows to twice its starting size then splitting into two genetically identical bacterial cells.

(i) For a specific bacteria, the number of bacteria increases in such a way that it undergoes binary fission every 20 minutes.
A researcher stated that, "If the starting number of bacteria in a cultured plate is recorded, the new number of bacteria after an hour is 200% more than its initial value."
By forming an equation, determine whether the researcher's claim is true.

[1]

(ii) It is given that this bacteria spreads in a certain limited area such that the equation of the area of bacteria, A cm^2, growing after t hours, can be modelled as shown:

$$A = \frac{800}{k + 8e^{-rt}}$$

, where k and r are positive constants.

The initial area the bacteria covered is 80 cm^2, and amount of area the bacteria covered doubled after 3 hours and 30 minutes. Show $k = 2$, hence find r. [1]

(iii) Jon suggested another way of calculating the area the bacteria can cover using the model below. The rate of increase of A was also calculated differently.

$$A = \frac{400}{2 + 3(2)^{-0.2t}}$$

Explain why this model is unsuitable for the actual experiment. [1]

(b) A particle A moves from a fixed point O such that its acceleration, a ms^{-2}, $a = 3e^{-\frac{t}{3}} - 2$.
The particle also reaches zero acceleration after k seconds.
After **another** k seconds, Particle A reaches instantaneous rest.

(i) Show that the value of $k = m\,(\ln n - \ln p)$, where m, n, and p are positive integers and determine whether at $t = k$ s, the velocity is at a maximum or a minimum. [2]

(ii) Find and simplify an **exact** expression of the displacement (s_A) of Particle A. [2]

11 Look at Diagram **11** below.

It shows the line $l_1 : x^2 + axy = ay^2 + by$, intersecting the line $l_2 : \frac{by}{2x} - \frac{y^2}{ax} = x - \frac{y}{a}$ at (2 , 6).

A vertical line is drawn connecting both lines below. In addition, two points C and D are labelled.

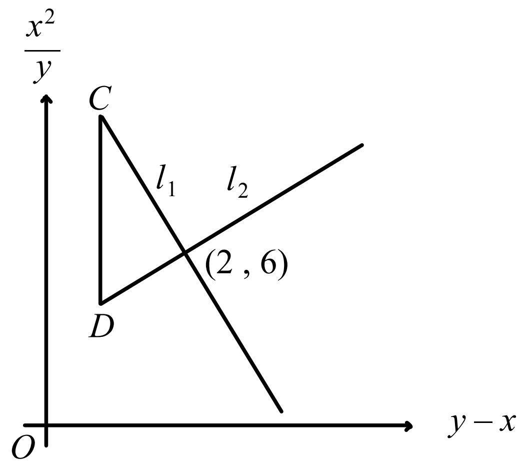

Diagram **11**

(i) Without finding for non-zero values a and b, Jon and Kate mention the following:

Jon: ‘ If a circle is drawn with CD as diameter, the intersection of lines l_1 and l_2 lie on the same circle.’

Kate: ‘I disagree, as without finding values a and b, that information cannot be determined.’

Determine whether Jon is correct, Kate is correct, or neither is correct.

Show relevant workings and explanation. [2]

....

(ii) Find values a and b, given that the area bounded by CD and the two other lines is 1.25 units^2, and CD has the equation $y = x + 1$. [2]

12 Belle runs in a straight running track. At time t seconds after leaving a fixed point O,the equation for Belle's velocity is given as $v = 3\pi \cos\left(\frac{\pi}{3}t - \frac{\pi}{2}\right)$, for $0 \leq t < n$. (v is in m/s)

She makes a first turn back to O when she reaches point P.

(i) Find the acceleration of the runner after 1 second, in terms of π. [1]

(ii) When Belle returns to O the fifth time from P, her subsequent velocity towards point P, in velocity v m/s, is given by $v = 0.2t - m$, for $t \geq n$, where m and n are constants.
Show that $n = 30$ and for Belle to never return to O subsequently, find the minimum value of m. [3]

(iii) Sketch out a velocity time graph for a minute of Belle's run.
Find the average speed of Belle's run in a minute. [4]

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13 **(a)** Look at the diagram below. DF and CH are straight lines.
G is the midpoint of DF, C splits BD such that $BC = DC$, and E is the midpoint of CH.

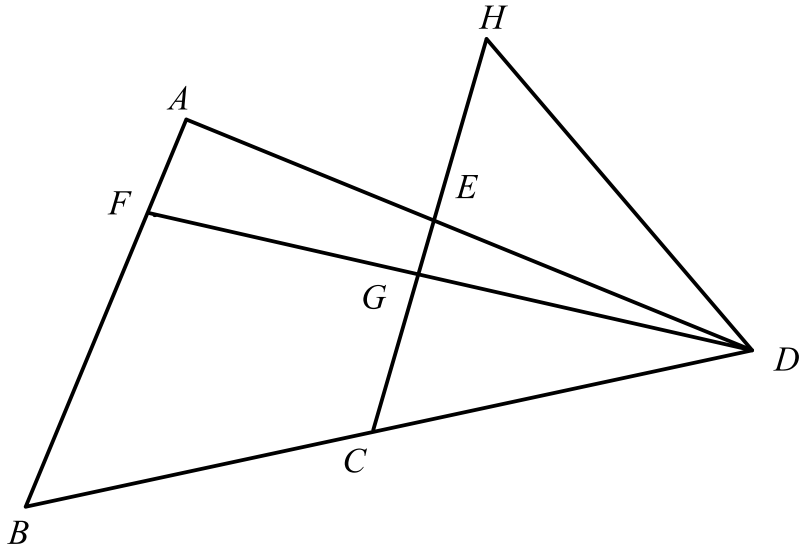

Diagram **13.1**

(i) Given that $\dfrac{AF}{BF} = x$, show that $AF = 2x\,(CG)$. [1]

(ii) It is further given that I lies on DH such that EI is parallel to FD.

Show that $\dfrac{IH}{DH} = \dfrac{x+1}{2x+1}$. [1]

(b) Look at another diagram below.

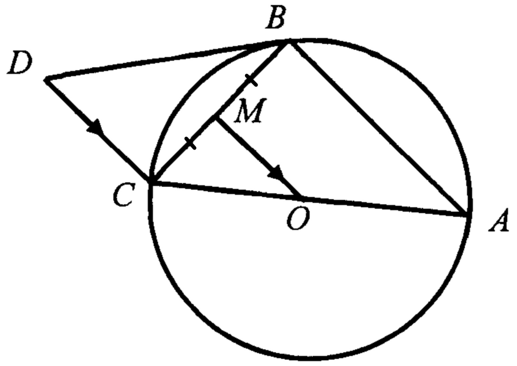

Diagram **13.2**

In the diagram, $CM = MB$ and $OM \parallel CD$. The line DB is tangent to the circle at B.

(i) Name a pair of similar triangles in the diagram above. [1]
Prove your answer.

(ii) Prove that $AC^2 - AB^2 = 2\,(OM)\,(CD)$. [3]

14 **(a)** Given that the term independent of x in the expansion of $\left(x^3 - \frac{q}{x^2}\right)^{10}\left(3 - \frac{5}{x^5}\right)^2$ is equivalent to $19005q^5$, determine the value(s) of q, given $q > 0$. [2]

(b) The expansion of the following expression $(2 - \frac{4}{x})^n(3 + \frac{24}{x})$ in the first two terms of the descending powers of x are $a + \frac{b}{x^2}$, where a, b, and n are non-zero constants and n is an integer greater than 2.

Find the values of a, b, and n. [3]

(c) The diagram shows part of the graph of $y = x^n$, where n is a positive integer greater than 2.

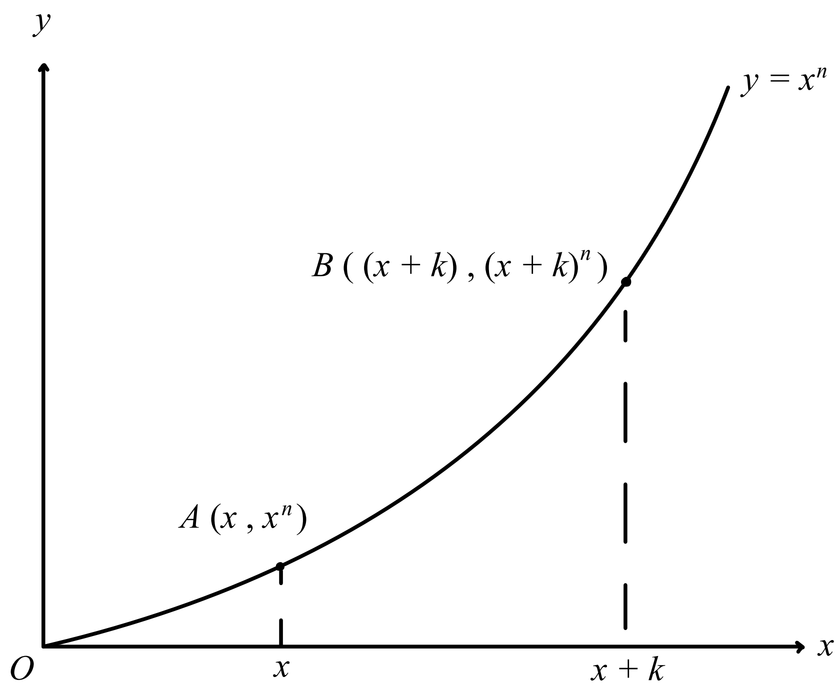

Diagram **14**

A and B are points in the graph, and k is a constant more than zero.
By considering the gradient of the line segment AB in its expanded form to a suitable precision, describe what happens to the gradient of AB when the value of k becomes a very small value. [2]

-End of paper **2**-
Do not give up, as this journey will be worthwhile!

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