Anglo - Chinese School

(Independent)

THE BEST IS YET TO BE

FINAL EXAMINATION 2021

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

XXXXXX **xxth October 2021** **1 hour 30 minutes**

ADDITIONAL MATERIALS:

Answer Paper (7 sheets)
Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS
The maximum mark for this paper is 80.

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This question paper consists of 4 printed pages.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1. *[Maximum mark: 12]*

(a) Evaluate $\dfrac{\ln(3.256)\times\sqrt{e^3+1.325}}{(0.25-2.38)^3}$, leaving your answer correct to 3 significant figures.

[2]

$$\frac{5.4623}{-9.6636}$$

$$= -0.565$$

Most students just use calculator to find the answer, a small group of students still give the wrong answer.

(b) Simplify $\dfrac{\left(\dfrac{3x}{y^2}\right)^3}{(3y^{\frac{1}{2}})^2\left(3^{-4}x^8\right)^{\frac{1}{4}}}$, expressing your answer in positive indices.

[3]

$$\frac{\left(\frac{3x}{y^2}\right)^3}{9y\left(\frac{1}{3}x^2\right)}$$

$$= \frac{\frac{27x^3}{y^6}}{3yx^2}$$

$$= \frac{27x^3}{3y^7x^2}$$

$$= \frac{9x}{y^7}$$

Careless in expanding the number 3 for both cube and square.

Some students did not leave it in positive indices.

(c) Simplify $\dfrac{27^{y+1}+2(3^{3y})}{3^{y+2}9^{y-1}}$.

[3]

$$\frac{(3^3)^{y+1}+2(3^{3y})}{3^{y+2}(3^2)^{y-1}}$$

$$=\frac{3^{3y+3}+2(3^{3y})}{3^{y+2}3^{2y-2}}$$

$$=\frac{27(3^{3y})+2(3^{3y})}{3^{3y}}$$

Let $x=3^{3y}$

$$=\frac{27x+2x}{x}$$

$$=\frac{29x}{x}$$

$$=29$$

Careless in factorising the 3 or expanding the power-power rule wrongly.

Some students see the question as 3^y instead of 3^{3y}

(d) Solve for x if $5^{x^2+x-6}=1$.

[4]

$5^{x^2+x-6}=5^0$

$x^2+x-6=0$

$(x-2)(x+3)=0$

$x=-3$ or $x=2$

Easy question to solve.

2 *[Maximum mark: 11]*

(a) Expand and simplify $(m+n)(n-m)\left(\frac{n^2p-n^2+m^2p-m^2}{p-1}\right)$.

[3]

$(m^2-n^2)\left(\frac{n^2(p-1)+m^2(p-1)}{p-1}\right)$

$= (m^2-n^2)\left(\frac{(n^2+m^2)(p-1)}{p-1}\right)$

$= (m^2-n^2)(n^2+m^2)$

$= n^4-m^4$

[Common mistake]
Did not simplify into the 2 terms answers.

[Reminder]
Always give the answer in simplest form. Simplest means shortest unless the question ask **Factorise completely!**

(b) Find the sum of $\frac{x}{3x-5}$ and $-\frac{6x+10}{9x^2-25}$, expressing your answer as a single fraction in its simplest form.

[3]

$\frac{x}{3x-5}-\frac{6x+10}{9x^2-25}$

$=\frac{x}{3x-5}-\frac{2(3x+5)}{(3x-5)(3x+5)}$

$=\frac{x}{(3x-5)}-\frac{2}{(3x-5)}$

$=\frac{x-2}{(3x-5)}$

Did not minus but sum up the 2 terms.

Did not factorise and express in simplest form by giving **$3x^2$-x-10** as the numerator.

(c) Solve the equation $\frac{3x^2+5}{(x-4)(x+2)}+\frac{4x}{4-x}=0$, **leaving your answers in <u>2 decimal places</u>.** [5]

$$\frac{3x^2+5}{(x-4)(x+2)}-\frac{4x}{x-4}=0$$

$$3x^2+5-4x(x+2)=0$$

$$3x^2+5-4x^2-8x=0$$

$$-x^2-8x+5=0$$

$$x=\frac{-(-8)\pm\sqrt{(-8)^2-4(-1)(5)}}{2(-1)}$$

$x=-8.58$ or 0.58

Many students see the question wrongly. ($4x$ became 4)

Did not leave the final answers in 2 d.p.

3 *[Maximum mark: 9]*

A circular cylinder container of base radius e^x cm, and height e^{2x} cm is fully filled with water.

(a) Given that the volume of water in the container is 12900 cm^3, find the value of x. [3]

Volume of container = $\pi(e^x)^2(e^{2x})=12900$

$$e^{4x}=\frac{12900}{\pi}$$

$$\ln e^{4x}=\ln\frac{12900}{\pi}$$

$$4x=8.32025$$

$$x=2.08$$

Wrong formula to find the Volume of container.

The water is poured into a rectangular tank of base area 850 cm^2 and height 60 cm.

(a) Find the depth of the water in the rectangular tank. [2]

Let h be the depth of the water in the rectangular tank,

$850(h) = 12900$

$h = 15.2$ cm

Did not give the final answer in 3 s.f. as instructed on the cover page.

(c) If a solid metal sphere of radius 14 cm is then put into the rectangular tank and the sphere is totally immersed in the water, will the water overflow? Explain your answer. [4]

Volume of sphere $= \frac{4}{3}\pi(14)^3$

Volume of sphere $= 11494$ cm^3

Total volume of water $= 11494 + 12900 = 24394$ cm^3

Total volume of vessel = $(850)(60) = 51000$ cm^3

Remaining space in the tank = 51000 – 24394 = 26606 cm^3

Since volume of water is less than total volume of vessel, water will not overflow.

Almost all students know how to explain their findings, but minority use the wrong formula to find the Volume of Sphere.

OR

Volume of sphere $= \frac{4}{3}\pi(14)^3$

Volume of sphere $= 11494$ cm^3

Height increased $= 11494/850 = 13.5$ cm

New height of water in the tank = $13.5 + 15.2 = 28.7$ cm

Since new height of water in the tank is less than the height of the tank, water will not overflow.

Almost all students know how to explain their findings, but minority use the wrong formula to find the Volume of Sphere.

4 *[Maximum mark: 7]*

(a) Solve $1-\log_3(1-2x)=\log_9(5-x)^2$.

[4]

$1=\log_3(5-x)+\log_3(1-2x)$

$\log_3[(5-x)(1-2x)]=1$

$(5-x)(1-2x)=3$

$5-10x-x+2x^2=3$

$2x^2-11x+2=0$

$x=0.188$ or $x=5.31$ (rejected)

Almost all students did it correctly, but did not check the final answer which need to reject the answer (x = 5.31)

Only minority applied the law of logarithms wrongly in combining the terms.

Did not check the equation properly end up **5 – 3 = – 2**

(b) Evaluate $\log_2 3\times\log_3 4\times\log_4 5\times\log_5 6\times\ldots\times\log_{63}(64)$, find the value of n.

[3]

$$\frac{\log 3}{\log 2}\times\frac{\log 4}{\log 3}\times\frac{\log 5}{\log 4}\times\frac{\log 6}{\log 5}\times\ldots\times\frac{\log(64)}{\log 63}$$

$$=\frac{\log 64}{\log 2}$$

$$=\frac{\log 2^6}{\log 2}$$

$$=6$$

Some students did not try at all, but some students applied the wrong law of logarithms as below.

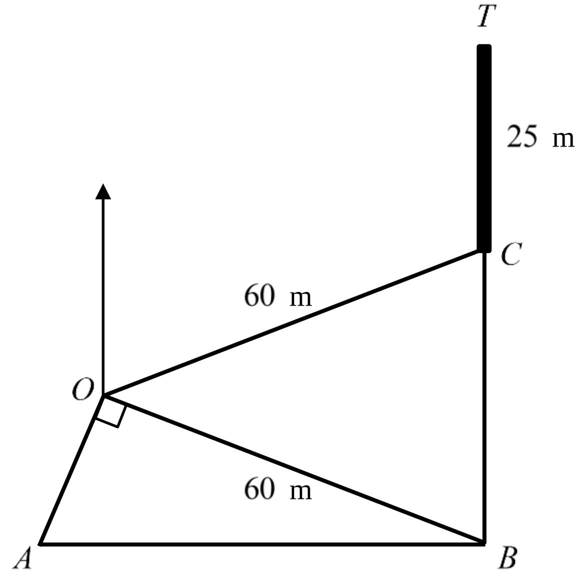
$\log_2(2+1)\times\log_3(3+1)$.....

$=(1+0)\times(1+0)$....

$=1$

5 *[Maximum mark: 15]*

O, *A*, *B* and *C* are four points on ground level. *C* is on a bearing of 50º from *O*. *B* is due east of *A* and south of *C*. OB = OC = 60 m and $\measuredangle AOB = 90^o$. A vertical flag pole, *TC*, 25 m high, is located at *C*. Find



(a) Bearing of *O* from *B*,

[2]

$\measuredangle OCB = 50^o$

$\measuredangle OBC = 50^o$

Bearing of O from B $= 360^o - 50^o = 310^o$

Well attempted

(b) the distance *OA*,

[3]

$\measuredangle OBA = 90 - 50^o = 40^o$

$\tan 40^o = \dfrac{OA}{60}$

$OA = 60 \times \tan 40^o$

$OA = 50.3$

Some students used the wrong formula for tangent. Note that:

$tangent = \dfrac{opposite}{adjacent}$

(c) the distance *BC*, [3]

$\measuredangle COB = 180 - 100 = 80^o$

Using Cosine Rule,

$BC^2 = 60^2 + 60^2 - 2(60)(60)\cos 80^o$

$BC^2 = 5949$

$BC = 77.1$

OR

$\measuredangle COB = 180 - 100 = 80^o$

Using Sine Rule,

$$\frac{BC}{\sin 80^o} = \frac{60}{\sin 50^o}$$

$BC = 77.1$

(d) the area of *OABC*, [3]

Area of COB = $\frac{1}{2}(60)(60)\sin 80$

Area of COB = 1772.65

Area of OAB = $\frac{1}{2}(50.3)(60)$

Area of OAB = 1509

Area of OABC = 1772.65 + 1509

Area of OABC = 3281.65

(e) the largest angle of elevation along *OB* to the top of the flag pole at *C*.

[4]

Let the point perpendicular from C to OB be P.

Area of triangle COB = $\frac{1}{2}(60)(60)\sin 80$

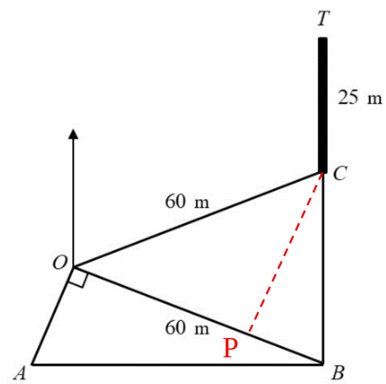
Area of triangle COB = 1772.65

$\frac{1}{2}(60)(CP) = 1772.65$

$CP = 59.088$

$\tan\theta = \frac{25}{59.088}$

$\theta = 22.9^o$



6 *[Maximum mark: 13]*

(a) Find the set of values of the constant m for which the line $y = 2x - m^2$ intersects the curve $y = x^2 - 3$ at two distinct points.

[4]

$2x - m^2 = x^2 - 3$

$x^2 - 2x + m^2 - 3 = 0$

$(-2)^2 - (4)(1)(m^2 - 3) > 0$

$4 - 4m^2 + 12 > 0$

$4m^2 - 16 < 0$

$m^2 - 4 < 0$

$(m-2)(m+2) <$

$-2 < m < 2$

Many students used the wrong method/working to obtain the correct answer. This is NOT acceptable. For example:

$m^2 - 4 < 0$
$m^2 < 4$
$m < \pm 2$
$-2 < m < 2$

The answers are contradicting and students are not allowed to make the examiner choose the correct answers for them.

It is important for students to learn the correct method and presentation to communicated their idea across to the examiner.

(b) Let $f(x)=2x^2-4kx+3k^2+5$.

(i) Prove that the equation $f(x)=0$ has no real roots.

[3]

$(-4k)^2-4(2)(3k^2+5)$

$=16k^2-24k^2-40$

$=-8k^2-40$

Since the discriminant is negative for all values of k, the graph of y does not cut the x −axis.

1) Students let the D < 0 in the first step. This is NOT correct as students have assumed D < 0 when the question is asking the student to prove that the equation has no real roots.
2) Students need to learn how to communicate effectively by writing a statement to explain why the expression $-8k^2-40$ is always negative for all values of k.
3) A good and clear way to explain:

   For all values of k,

   $$k^2 \geq 0$$
   $$-8k^2 \leq 0$$
   $$-8k^2-40 \leq -40$$

   Hence,

   $$D < 0$$

   Therefore, $f(x)=0$ has no real solutions.

(ii) Find the value(s) of k if the graph of $f(x)$ passes through the point (1, 6).

[4]

Sub $x = 1$ and $y = 6$ into $f(x)$

$6 = 2(1)^2 - 4k(1) + 3k^2 + 5$

$3k^2 - 4k + 1 = 0$

$(3k-1)(k-1) = 0$

$k = \frac{1}{3}$ or $k = 1$

7 *[Maximum mark: 15]*

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = \dfrac{5x(x-8)}{x+12}$. Some corresponding values of x and y are given in the following table.

x	–1	0	1	2	3	4	5	6
y	4.09	a	–2.69	–4.29	b	–5	–4.41	–3.33

(a) State the value of a and of b.

[2]

$$a = \frac{5(0)(0-8)}{0+12}$$

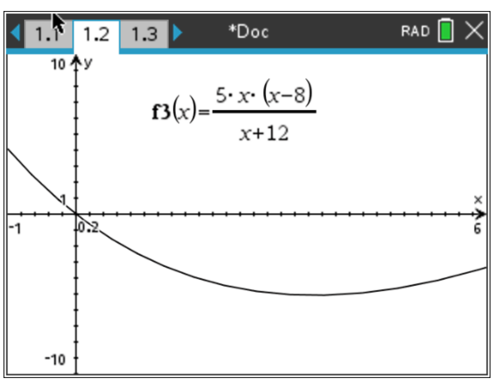
$$a = 0$$

$$b = \frac{5(3)(3-8)}{3+12}$$

$$b = -5$$

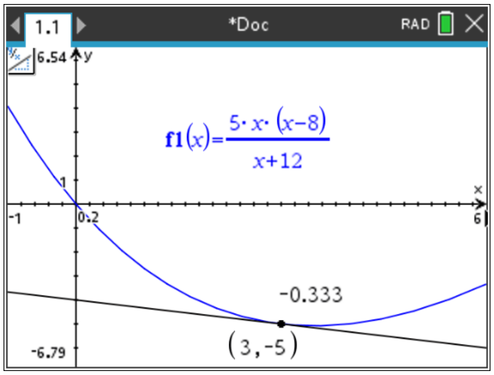
(b) Taking 2 cm to represent 1 unit on each axis, draw the graph of $y = \dfrac{5x(x-8)}{x+12}$ for $-1 \le x \le 6$.

[4]



(c) By drawing a tangent, find the gradient of the curve at the point where $x = 3$.

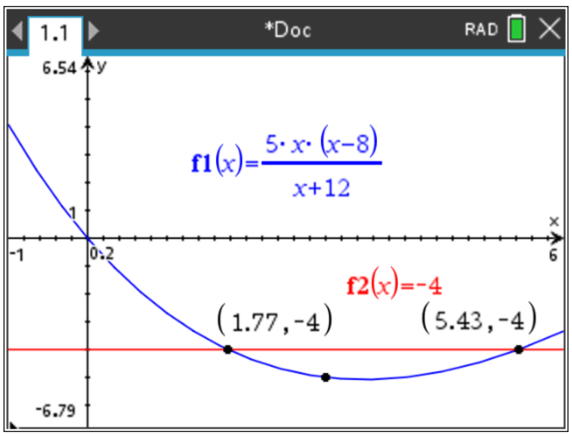
[3]



Gradient accept between -0.25 to -0.45

(d) Find the range of values of x for which $y \leq -4$.

[3]



(1.6 to 1.9) $\leq x \leq$ (5.4 to 5.6)

Students are supposed to draw a horizontal line at $y = -4$ and find the points of intersections. However, there are many students who attempted to solve the quadratic in equality.

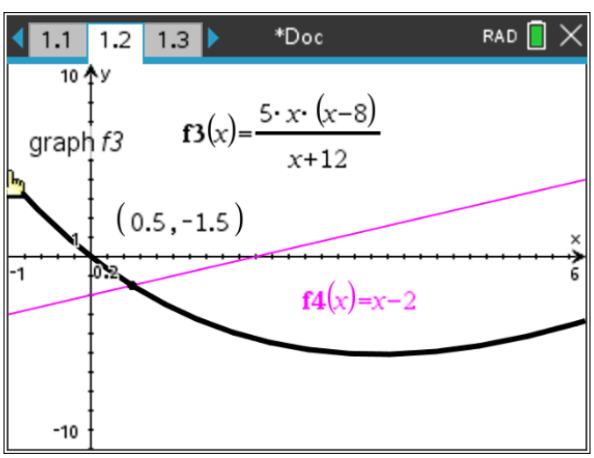
This is feasible but extremely time consuming.

(e) By drawing a suitable straight line on the graph, find the solution of the equation $5x(x-8) = (x-2)(x+12)$.

[3]

$$5x(x-8) = (x-2)(x+12)$$

$$\frac{5x(x-8)}{(x+12)} = x - 2$$



$x = 0.4 – 0.6$ as solution

Students should not write the solution as (0.5, -1.5). The question is asking the students to solve for the value of x.