1 The equation of a curve C is given by $4(x+y)^2+(x-y)^2=20$.

(i) Show that the gradient of C at the point (x,y) is given by

$$\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{5x+3y}{3x+5y}\ .$$ [3]

(ii) Find the equation(s) of the tangent(s) to the curve C which are perpendicular to the line $y=x$. [6]

2 The curves C_1 and C_2 are defined by the equations $y=\dfrac{6}{4-x^2}$ and $y=6-x^2$ respectively.

(i) On the same axes, sketch the graphs of the curves C_1 and C_2, stating the equations of any asymptotes, the exact coordinates of the turning point(s) and any points where the curve crosses the x- and y-axes. [5]

(ii) Solve the inequality $\dfrac{6}{4-x^2}<6-x^2$. [2]

(iii) The transformations A and B are given as follows:

A : Reflection about the y-axis;

B : Translation of 4 units in the negative x-direction.

The graphs C_1 and C_2 undergo in sequence, the transformations A and B. The resulting equations of the transformed graphs of C_1 and C_2 are $y=\mathrm{f}(x)$ and $y=\mathrm{g}(x)$ respectively.

Deduce the solution set of the inequality $\mathrm{f}(x)<\mathrm{g}(x)$. [2]

3 The function f is defined by $\mathrm{f} : x \mapsto ax^3 + bx^2 + cx + d$, where $x \in \mathbb{R}$ and a, b, c and d are constants.

The graph of f intersects the y-axis at $y = -3$ and passes through the points $(-1\,,0)$ and $(2\,,0)$.

(i) Explain why f does not have an inverse. [1]

(ii) Given also that the tangent to the graph of f at $x = 1$ is a horizontal line, find f(x). [3]

(iii) Sketch the graph of y = f(x), giving the coordinates of the turning points and the points which the graph intersects the axes. [2]

(iv) Given that the function f has an inverse if its domain is restricted to $x \geq k$, state the smallest possible value of k. [1]

For the rest of the question, use the domain given and value of *k* found in part (iv).

(v) Describe the relationship between the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$. [1]

(vi) Show that the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$ satisfies the equation $3x^3 - 11x - 6 = 0$. Hence, find the solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$. [3]

(vii) It is given that $\mathrm{g}(x) = \ln(x+5)$, where $x > -5$. A student attempts to find the composite function gf.
The student's solution is shown below:

$$\mathrm{g}(x) = \ln(x+5)$$
$$\mathrm{D_{gf}} = \mathrm{D_f} = [k\,,\infty)$$
$$\therefore \mathrm{gf}(x) = \ln\left(ax^3 + bx^2 + cx + d + 5\right),\ x \in \mathbb{R}\ ,\ x \geq k.$$

Comment on the validity of the student's solution. [1]

4 **(i)** By sketching the graph of $y=\dfrac{x+1}{2x-1}$, find the range of values of x for which $\dfrac{x+1}{2x-1}\geq 0$. [4]

(ii) Hence, without the use of a calculator, show that $\displaystyle\int_{-2}^{0}\left|\frac{x+1}{2x-1}\right|\,\mathrm{d}x=\frac{3}{2}\ln 3-\frac{3}{4}\ln 5$. [4]

5 The curve C is defined by the equation $y=\dfrac{1}{2}\tan^{-1}(2x)$ and the line L is defined by the equation $y=\dfrac{1}{2}x+\left(\dfrac{1}{4}-\dfrac{\pi}{8}\right)$. It is given that the line L intersects the y-axis at the point Q and is a tangent to the curve C at the point P where $x=-\dfrac{1}{2}$.

(i) Find the y-coordinates of P and Q. [2]

The region R is bounded by the line L, the curve C and the y-axis, for $x<0$.

(ii) Find the exact volume of the solid generated when R is rotated through 2π radians about the y-axis, giving your answer in the form $\dfrac{\pi}{8}(a-b)$ where a and b are positive constants to be found. [6]

6 With reference to the origin O, the points A and B have position vectors $\mathbf{a}$ and $\mathbf{b}$ respectively, where $\mathbf{a}$ and $\mathbf{b}$ are perpendicular. A point P lies on AB between A and B such that $AP:PB=\lambda:1-\lambda$, $0<\lambda<1$.

(i) Show that $\cos(\angle AOP)=\dfrac{(1-\lambda)|\mathbf{a}|}{|(1-\lambda)\mathbf{a}+\lambda\mathbf{b}|}$. [4]

(ii) Prove that $\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]\bullet\left[(1-\lambda)\mathbf{a}+\lambda\mathbf{b}\right]=(1-\lambda)^2|\mathbf{a}|^2+\lambda^2|\mathbf{b}|^2$. Hence, given also that OP bisects $\angle AOB$, find the ratio of $\dfrac{|\mathbf{a}|}{|\mathbf{b}|}$, leaving your answer in terms of λ. [6]

7 The rate of temperature loss of an animal corpse can be estimated using Newton's Law of Cooling, which states that the rate of change of temperature θ°C, t hours after death of an animal is proportional to the difference between its body temperature θ°C and the surrounding temperature θ_0°C, where $\theta > \theta_0$.

(i) Write down a differential equation for this situation. Solve this differential equation and show that the general solution of the above differential equation is given by $\theta = \theta_0 + Ae^{-kt}$, where A and k are positive constants. [3]

It is given that $\theta_0 = 24$, the initial value of θ is 36 and the initial rate of temperature loss is 2.5 °C per hour.

(ii) Calculate the exact values of A and k. [3]

(iii) Hence, sketch the graph of θ against t. [2]

(iv) Explain why the rate of change of temperature of an animal corpse cannot be modelled by a constant rate of decrease of 1.5°C. [1]

8 **(a)** It is given that the equation $3z^3 + az^2 + bz + c = 0$, where a, b and c are real numbers, has roots $\frac{5}{3} - \frac{\sqrt{11}}{3}\mathrm{i}$ and -2. Find the integer values of a, b and c. [4]

(b) It is known that a complex number $w = \dfrac{e^{i\theta} + e^{i\phi}}{e^{i\theta} - e^{i\phi}}$, where $\theta - \phi \neq 2n\pi$ and $\theta > \phi$ for any integer n.

(i) Show that $w = e^{-i\frac{\pi}{2}}\left(\cot \frac{1}{2}(\theta - \phi)\right)$. [3]

(ii) Hence, find $|w|$ and $\arg(w)$. [2]

9 The coach of Besto running club designed a training programme such that runners begin with a 400 m run on the first training session. On each subsequent session, the distance covered is 250 m more than the distance covered on the previous session.

(i) Find the minimum number of sessions required for runners from Besto on the training programme to run at least 20 km in a training session. [3]

For another group of runners in Besto, a circuit training exercise was designed to build up their stamina.

In this exercise, this group of runners from Besto run from a starting point O to and from a series of points, $P_1, P_2, P_3, \cdots$, increasingly far away in a straight line. In the exercise, they start at O and run stage 1 from O to P_1, and back to O, then stage 2 from O to P_2, and back to O, and so on.

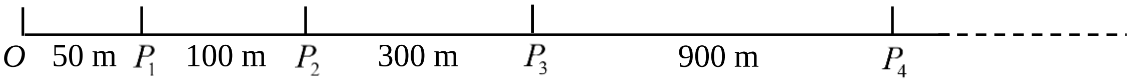

Fig. 1

The distances between the points are such that $OP_1 = 50\text{ m}$, $P_1P_2 = 100\text{ m}$, $P_2P_3 = 300\text{ m}$ and $P_nP_{n+1} = 3P_{n-1}P_n$ (see Fig. 1).

(ii) Find an expression for the distance run by a runner from Besto who completes n stages of the circuit training exercise. [3]

(iii) Hence, find the distance from O and the direction of travel, of a runner from Besto undergoing the circuit training exercise after he has run exactly 42 km. [3]

Another running club, Choco, designed a different training programme. The runners in Choco began with running 400 m on the 1st session. On each subsequent session, the distance covered was increased by 10% of the distance covered on the previous session. From the 11th session onwards and for all subsequent sessions, the distance covered was increased by r% of the distance covered on the previous session.

(iv) Given that the runners from Choco club covered at least 20 km on the 70th training session, find the range of values of r. [4]

10 The diagram below shows the floorplan of a square lawn with a circular pond with its centre coinciding with the centre of the square lawn (see Fig. 2).

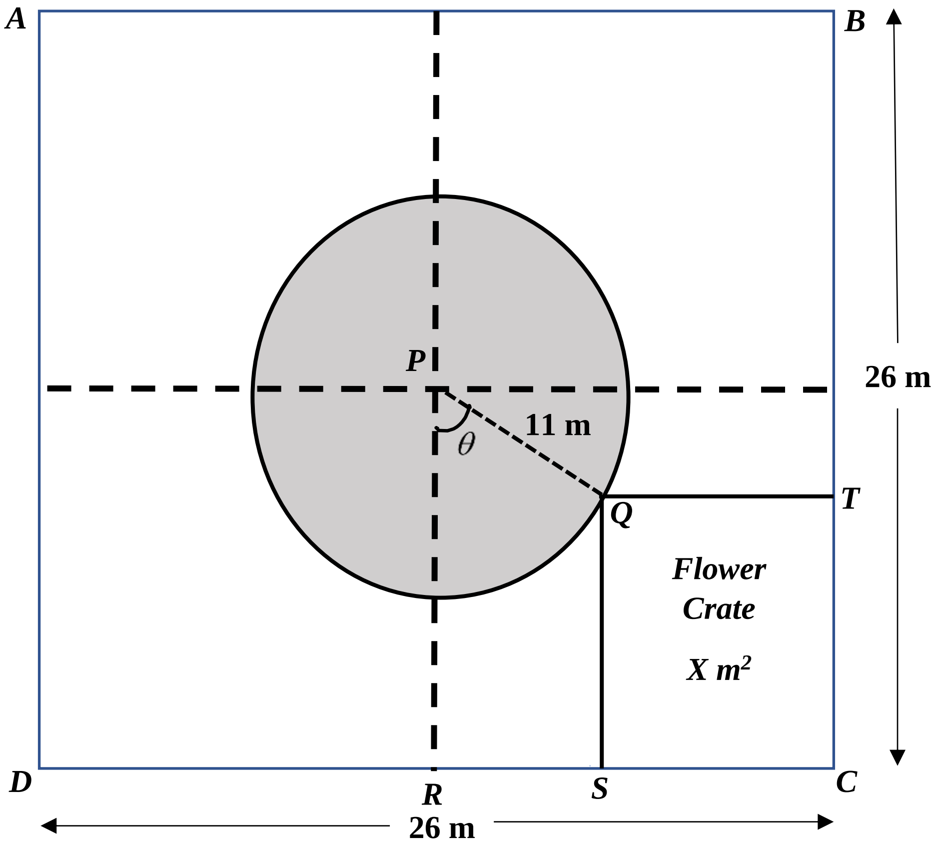

Fig. 2

The floorplan consists of a square lawn *ABCD* of side 26 m with a circular pond of radius 11 m built at the centre *P* of the square lawn. The owner intends to build a rectangular flower crate with base *QTCS*, with one corner of the base at *C*, and the opposite corner *Q* of the base, touching the circular pond, where angle *RPQ* = θ radians, measured from *RP* and $0 \le \theta \le \dfrac{\pi}{4}$. The base of the flower crate has area *X* m^2.

(i) By finding an expression of *X* in terms of θ, show that $\dfrac{\mathrm{d}X}{\mathrm{d}\theta} = 11(\sin\theta - \cos\theta)(13 - 11\sin\theta - 11\cos\theta)$. [3]

(ii) For stationary values of *X*, show that the corresponding values of θ, given by θ_1 and θ_2 satisfy the equations $\tan\theta_1 = 1$ and $\sin(\theta_2 + \alpha) = k$ respectively, where k and α are constants in exact form. Hence, find the values of θ_1 and θ_2. [4]

(iii) Determine which of the values of θ found in part **(ii)** give a minimum value of X and which give a maximum value of X, and find these values. [3]

(iv) The owner wishes to build 4 identical flower crates with corners at A, B, C and D respectively, and he intends to cover the rest with grass. Find the smallest area of the square lawn to be covered by grass, giving your answers to 3 significant figures. [3]