Gravitation Tutorial 7A Self Review Solution

S1 **C**

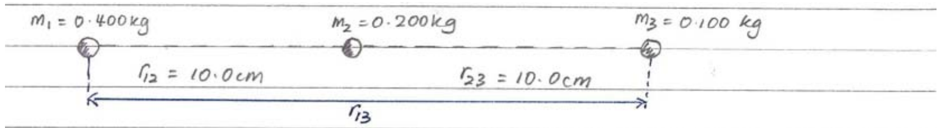
$$F = \frac{GMm}{r^2} \Rightarrow G = \frac{Fr^2}{Mm}$$

units of G = units of $\frac{Fr^2}{Mm} = \frac{kg\,m\,s^{-2}\,m^2}{kg^2} = m^3\,kg^{-1}\,s^{-2}$

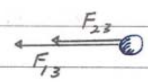
Strategy: always make the physical quantity or constant that you wish to find the subject first.

NOTE: pay attention to the presentation of answers

S2



(a) Consider the $m_3 = 0.100$ kg mass:



F_{23}: force by $m_2 = 0.200$ kg mass on m_3

F_{13}: force by m_1 mass on m_3

∴ Resultant force on m_3 (= 0.100 kg) mass

$= F_{23} + F_{13}$

$= \frac{G m_2 m_3}{r_{23}^2} + \frac{G m_1 m_3}{r_{13}^2}$

$= \frac{(6.67 \times 10^{-11})(0.200\,kg)(0.100\,kg)}{(10.0 \times 10^{-2})^2}$

$+ \frac{(6.67 \times 10^{-11})(0.400\,kg)(0.100\,kg)}{(20.0 \times 10^{-2})^2}$

$= 2.00 \times 10^{-10}$ N (to the left *

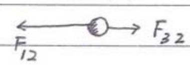
Strategy: vector sum of gravitational force acting on a mass due to the respective masses.

(b) Consider $m_2 = 0.200$ kg mass:

F_{12}: Force on m_2 by m_1

F_{32}: Force on m_2 by m_1

(→ +ve)



Resultant force on m_2 = $F_{32} - F_{12}$

$= G\frac{m_3 m_2}{r_{23}^2} - G\frac{m_1 m_2}{r_{12}^2}$

$= (6.67 \times 10^{-11})(0.200\,kg)\left[\frac{0.100}{(10 \times 10^{-2})^2} - \frac{(0.400)}{(10 \times 10^{-2})^2}\right]$

$= 4.01 \times 10^{-10}$ N towards the left *

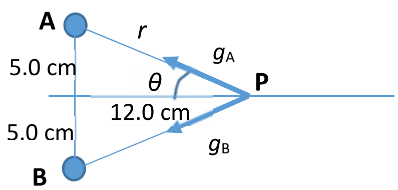
S3 **D**

$$W = \frac{GMm}{R^2}$$

$$\frac{GMm}{(7R)^2} = \frac{GMm}{49R^2} = \frac{W}{49}$$

The trickier questions often give the "height / altitude of the mass above surface of the Earth". When applying the formula for gravitational force, you should always consider the distance between the two masses i.e. (Earth radius + height).

S4



Strategy: vector sum of gravitational field strengths acting at the point due to the respective masses.

$\tan\theta = \frac{5.0}{12.0}$, $\theta = 22.6^\circ$

$r = \sqrt{5.0^2 + 12.0^2} = 13.0 \text{ cm}$

Field strength due to mass A, g_A = field strength due to mass B, $g_B = \frac{G(20.0)}{(13.0\times10^{-2})^2}$.

At P, the vertical components g_A cancels the vertical component of g_B.

Hence the resultant g is towards the left and its magnitude given by

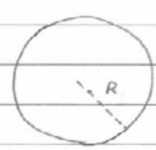
$g_R = 2\left(\frac{G(20.0)}{(13.0\times10^{-2})^2}\right)\cos 22.6^\circ = 1.46\times10^{-7}$ N kg^{-1}

Besides resolving forces in two perpendicular directions, another way is to draw a force vector triangle and use sine rule or cosine rule to obtain the resultant.

S5 **D**

$g = \frac{GM}{r^2}$, thus $g \propto \frac{M}{r^2}$ (since G is a constant)

S6



$\rho = 1.13\times10^4 \text{ kg/m}^3$

$$g = G\frac{M}{R^2}$$

$$= \frac{G}{\cancel{R^2}}\left(\frac{4}{3}\pi R^{\cancel{3}}\rho\right)$$

Since mass = density x volume

$$= \frac{4}{3}\pi G R \rho$$

$$\Rightarrow R = \frac{3}{4}\,\frac{g}{\pi G \rho}$$

$$= \frac{3}{4}\,\frac{(1.00 \text{ m/s}^2)}{\pi(6.67\times10^{-11})(1.13\times10^4 \text{ kg/m}^3)}$$

$$= 3.17\times10^5 \text{ m}$$ #

S7 **A**

$$mg = \frac{GMm}{R^2} \Rightarrow g = \frac{GM}{R^2} \Rightarrow g \propto \frac{1}{R^2}$$

$$\frac{g'}{g} = \frac{r^2}{(r+h)^2} \Rightarrow g' = \frac{gr^2}{(r+h)^2}$$

Another question similar to S3 involving the "height / altitude of the mass above surface of the Earth". When applying the formula for gravitational field strength, you should always consider the radius as (Earth radius + height).

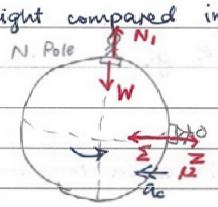
S8 **B**

A uniform gravitational field means that the gravitational field strength/force is constant.

S9 **D**

Similar to lecture example 3

The weight compared in this question is the apparent weight.



The apparent weight is the force that an object exerts on its support. (N)

For a person at the poles, it can be assumed that it is hardly rotating. $\therefore N_1 = W$ (Balance reflects gravitational force)

For a person at the equator, it is undergoing circular motion.

$$\therefore \quad W - N_2 = m a_c$$

$$\Rightarrow \quad N_2 = W - m a_c$$

$\therefore$ The balance reading (N_2) reflects the gravitational force less the centripetal force required to keep the object in circular motion

S10 **C**

The gravitational force exerted by the Earth on the moon is the real force that provides the Moon's centripetal acceleration (Newton's Second Law).

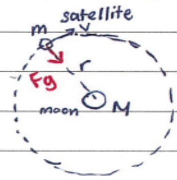
S11 **C**

A geostationary orbit must have the same angular velocity, hence the same period, as the Earth's rotation on its axis, so as to appear stationary to a point on Earth.

S12 **D**

The object in the space capsule orbiting the Earth seems to be floating suggests no normal contact force N acting on the object, hence weight is the only force acting on it. Both the space capsule and object experience the same centripetal acceleration towards the centre of the Earth, which is similar to the analogy of a man in a lift both free falling. In this case, the man will also experience apparent weightlessness as he falls as both him and lift are accelerating downwards at g = 9.81 m s^{-2}, hence no normal contact force acting on him.

S13



(a) The centripetal force is provided by the gravitational force.

$\therefore \quad F_g = F_c = ma_c$

$$\frac{GM\cancel{m}}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$r = 1.74 \times 10^6$ m

$M = 7.36 \times 10^{22}$ kg

$$\Rightarrow \quad v = \sqrt{\frac{GM}{r}}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11})(7.36 \times 10^{22}\,kg)}{(1.74 \times 10^6)}}$$

$$= 1.68 \times 10^3\ m/s$$

(b) The distance travelled by the satellite when it makes one complete orbit around the moon $= 2\pi r$

$$\therefore \quad v = \frac{2\pi r}{T} \quad \Rightarrow \quad T = \frac{2\pi r}{v}$$

$$= \frac{2\pi(1.74 \times 10^6\,m)}{1.68 \times 10^3\ m/s}$$

$$= 6.51 \times 10^3\ s$$

$$= 1.81\ h.$$