Chapter 9 Complex Numbers I Tutorial (A): Algebra of Complex Numbers Solutions

Basic Mastery Questions

1 i) $(3+2i)(3-2i) = 9-(2i)^2$

$= 9-4(-1)$

$= 13$ #

ii) $\dfrac{5+5i}{3-4i} + \dfrac{20}{4+3i} = \dfrac{5+5i}{3-4i}\cdot\dfrac{3+4i}{3+4i} + \dfrac{20}{4+3i}\cdot\dfrac{4-3i}{4-3i}$

$= \dfrac{15+35i-20}{9+16} + \dfrac{80-60i}{16+9}$

$= \dfrac{75-25i}{25}$

$= 3-i$ #

iii) $\dfrac{i^4+i^9+i^{16}}{2-i^5+i^{10}-i^{15}} = \dfrac{1+i+1}{2-i-1+i}$

$= 2+i$ #

2i) Let $\sqrt{-15-8i} = a+ib$

$\therefore -15-8i = (a+ib)^2$

$= (a^2-b^2) + 2abi$

$\therefore a^2-b^2 = -15$ —— (1)

$2ab = -8 \Rightarrow ab = -4$ —— (2)

From (2): $a = -\frac{4}{b}$ —— (3)

Sub (3) into (1): $\frac{16}{b^2} - b^2 = -15$

$b^4 - 15b^2 - 16 = 0$

$(b^2-16)(b^2+1) = 0$

$b^2 = 16$ or $b^2 = -1$ (NA) $\Rightarrow b = \pm 4$, $a = \mp 1$

$\therefore$ Square roots of $-15-8i$ are $1-4i$ and $-1+4i$ #

2ii) $2z^2+2z+13=0$

$$z=\frac{-2\pm\sqrt{4-4(2)(13)}}{2(2)}$$

$$=\frac{-2\pm\sqrt{-100}}{4}$$

$$=\frac{-2\pm10i}{4}=\frac{-1\pm5i}{2}$$ #

2iii) $z^4-16=0$

$$(z^2-4)(z^2+4)=0$$

$z^2=4$ or $z^2=-4$

$z=\pm2$ # or $z=\pm\sqrt{-4}$

$=\pm2i$ #

2iv) $z^2+(2i-3)z+5-i=0$

$$z=\frac{(3-2i)\pm\sqrt{(2i-3)^2-4(5-i)}}{2}$$

$$=\frac{(3-2i)\pm\sqrt{-4-12i+9-20+4i}}{2}$$

$$=\frac{(3-2i)\pm\sqrt{-15-8i}}{2}$$

$$=\frac{(3-2i)\pm(1-4i)}{2}\text{ or }\frac{(3-2i)\pm(-1+4i)}{2}$$ [from 2(i)]

$$=\frac{4-6i}{2},\ \frac{2+2i}{2},\ \frac{2+2i}{2},\ \frac{4-6i}{2}$$

$=1+i$ or $2-3i$ #

Concept

Why can we use quadratic formula here?

We can still complete the square with complex coefficients and make z the subject.

2v) $$\frac{z+2i}{z-(1-i)} = 3-i$$

$$(3-i)[z-(1-i)] = z+2i$$

$$(3-i)z - (3-i)(1-i) = z+2i$$

$$(3-i)z - (3-4i-1) = z+2i$$

$$(2-i)z = 2i+2-4i$$

$$z = \frac{2-2i}{2-i} \cdot \frac{2+i}{2+i}$$

$$= \frac{4-2i+2}{4+1}$$

$$= \frac{6-2i}{5} \quad \#$$