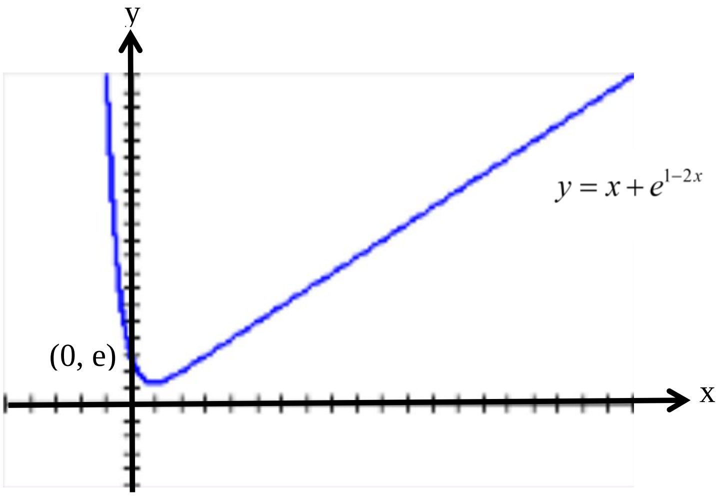
2018 H1 Mathematics A Level Paper

1	For $3kx^2+(k-6)x-2=0$ to have two distinct roots, Discriminant > 0 $(k-6)^2-4(3k)(-2)>0$ $k^2-12k+36+24k>0$ $k^2+12k+36>0$ $(k+6)^2>0$ $k\in\mathbb{R}, k\neq -6$
2(i)	Let $y=3\ln\left(4x^3+2\right)$ $\frac{\mathrm{d}y}{\mathrm{d}x}=3\left(\frac{12x^2}{4x^3+2}\right)$ $=\frac{36x^2}{4x^3+2}$
(ii)	$\int\frac{3x^2-2}{\sqrt{x}}dx=\int 3x^{\frac{3}{2}}-2x^{-\frac{1}{2}}dx$ $=\frac{3x^{\frac{5}{2}}}{5/2}-2\frac{x^{\frac{1}{2}}}{1/2}+C$ $=\frac{6}{5}x^{\frac{5}{2}}-4\sqrt{x}+C$ where C is an arbitrary constant
3(i)	Let x, y and z be the cost of training one accountant, one financial advisor and one consultant respectively $5x+12y+8z=294100$ $9x+7y+3z=270100$ $3x+6y+0z=122700$ Using GC to solve, $x=18500, y=11200, z=8400$ Therefore the cost of training one financial advisor is $11200 last year.
(ii)	Let n be the number of accounts that company S will be able to train this year. $n(1.1)(18500)+n(1.1)(11200)\leq 200000$ $32670n\leq 200000$ $n\leq 6.12$ The greatest possible number of accountants that company S will be able train is 6.

4	$y = x + e^{1-2x}$ $\dfrac{dy}{dx} = 1 - 2e^{1-2x}$ At stationary point, $\dfrac{dy}{dx} = 0$ $1 - 2e^{1-2x} = 0$ $2e^{1-2x} = 1$ $e^{1-2x} = \dfrac{1}{2}$ $(1-2x)\ln e = \ln\left(\dfrac{1}{2}\right) = \ln 1 - \ln 2$ $1 - 2x = -\ln 2$ $x = \dfrac{1+\ln 2}{2}$ When $x = \dfrac{1+\ln 2}{2}$, $y = \dfrac{1+\ln 2}{2} + e^{1-2\left(\frac{1+\ln 2}{2}\right)}$ $= \dfrac{1+\ln 2}{2} + e^{1-1-\ln 2}$ $= \dfrac{1+\ln 2}{2} + e^{-\ln 2}$ $= \dfrac{1+\ln 2}{2} + \dfrac{1}{2}$ $= 1 + \dfrac{\ln 2}{2}$ The exact coordinates of the turning point of C are $\left(\dfrac{1+\ln 2}{2}, 1+\dfrac{\ln 2}{2}\right)$.
(ii)	
(iii)	When $x = 2$, $y = 2 + e^{-3}$, $\dfrac{dy}{dx} = 1 - 2e^{-3}$

Equation of tangent when $x = 2$,

$$y - \left(2 + e^{-3}\right) = \left(1 - 2e^{-3}\right)\left(x - 2\right)$$

$$y = \left(1 - 2e^{-3}\right)x - 2 + 4e^{-3} + 2 + e^{-3}$$

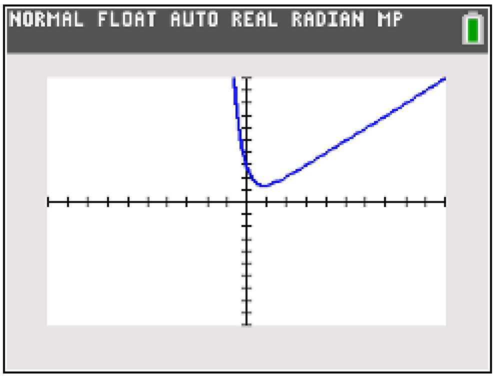
$$y = \left(1 - 2e^{-3}\right)x + 5e^{-3}$$

$$y = 0.900x + 0.249$$

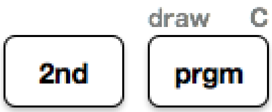
Alternatively: (better method since the question did not ask for Exact form)

Using GC, gradient of tangent when $x = 2$ is $y = 0.900x + 0.249$

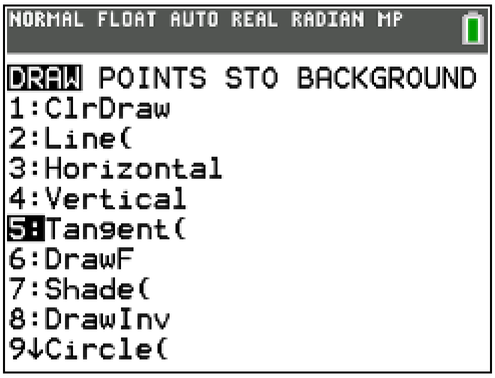
- Sketch $y = 2 + e^{-3}$



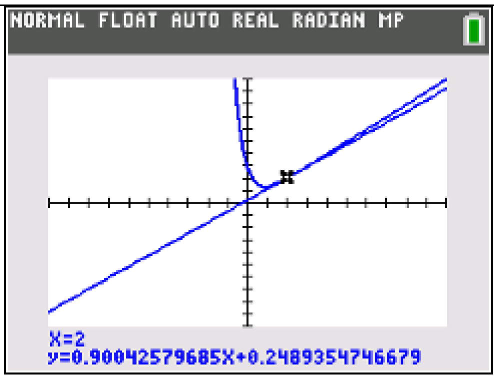
- Select Draw



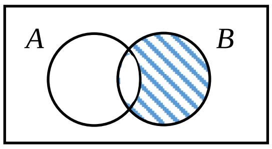
- followed by Tangent(

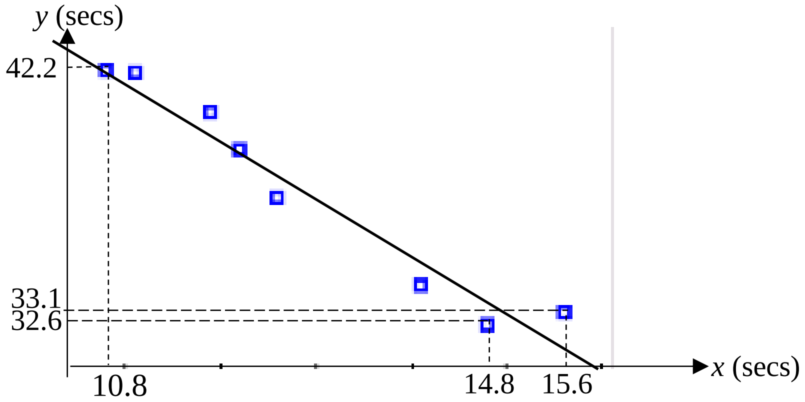


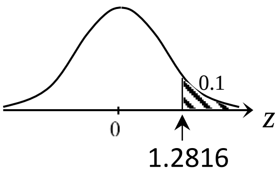
- Enter $x = 2$ and press Enter

	
(iv)	$\int x+e^{1-2x}\mathrm{d}x=\frac{x^2}{2}+\frac{e^{1-2x}}{-2}+C$ $=\frac{x^2}{2}-\frac{e^{1-2x}}{2}+C$ where C is an arbitrary constant Required Area $=\left[\frac{x^2}{2}-\frac{e^{1-2x}}{2}\right]_0^1$ $=\frac{1}{2}-\frac{e^{-1}}{2}-\left(0-\frac{e}{2}\right)$ $=\frac{1}{2}-\frac{1}{2e}+\frac{e}{2}$ units^2
5(i)	Number of times that the manager needs to order in a year $=\frac{1200}{x}$ Total ordering cost $=50\left(\frac{1200}{x}\right)=\frac{60000}{x}$ $C=\text{Storage Cost}+\text{Purchase Cost}+\text{Ordering Cost}$ $=6x+200(1200)+\frac{60000}{x}$
(ii)	$\frac{\mathrm{d}C}{\mathrm{d}x}=6-\frac{60000}{x^2}$ When C is stationary, $\frac{\mathrm{d}C}{\mathrm{d}x}=0$ $6-\frac{60000}{x^2}=0$ $x^2=10000$ $x=100$ or -100 (reject since $x>0$) $\frac{\mathrm{d}^2C}{\mathrm{d}x^2}=\frac{120000}{x^3}$ $\frac{\mathrm{d}^2C}{\mathrm{d}x^2}>0$ for all $x>0$

	Therefore minimum value of C $=6(100)+240000+\dfrac{60000}{100}=\$241200$
(iii)	This is not a reasonable model as various components of the cost differs depending on external factors such as the economical climate and real estate climate. For example the rental cost of warehouse will differs from year to year.
(iv)	Let *y* be the number of television sets sold. Gradient of the line $=\dfrac{1200-0}{300-700}=-3$ Equation of the line, $y-1200=-3(S-300)$ $y=-3S+900+1200$ $=-3S+2100$ $P=\text{Total Selling Price}-\text{Total Cost}$ $=yS-240000$ $=(-3S+2100)S-240000$ $=2100S-3S^2-240000$
(v)	$\dfrac{\mathrm{d}P}{\mathrm{d}S}=2100-6S$ When P is stationary, $\dfrac{\mathrm{d}P}{\mathrm{d}S}=0$ $2100-6S=0$ $S=350$ $\dfrac{\mathrm{d}^2P}{\mathrm{d}S^2}=-6<0$ Therefore when S is \$350, P will be maximum.
6i	Let *X* be the random variable the number of people who have blood group A, out of 6. $X \sim \mathrm{B}(6, 0.4)$ $\mathrm{P}(X=2)=0.31104\approx 0.311$ (3 s.f.)
ii	P(at least 4 people do not have blood group A) $=\mathrm{P}(X\le 2)=0.54432\approx 0.544$ (3 s.f.)
7i	No of selections = ${}^8C_3\times{}^4C_1\times{}^6C_2=3360$
ii	Case 1: no guitarist chosen No of selections = ${}^5C_3=10$ Case 2: 1 guitarist chosen No of selections = ${}^5C_2\times{}^3C_1=30$ Total number of selections = 30 + 10 = 40
iii	P(band contain none of the three guitarists) $=\dfrac{{}^5C_3}{{}^8C_3}=\dfrac{5}{28}$
8i	$\mathrm{P}(A\cup B)=\mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A\cap B)$

	$0.8 = p + 2p - \mathrm{P}(A \cap B)$ $\mathrm{P}(A \cap B) = 3p - 0.8$ Given $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$ Then $0.3 = \dfrac{3p - 0.8}{2p}$ $0.6p = 3p - 0.8$ $2.4p = 0.8$ $p = \dfrac{1}{3}$
ii	 $\mathrm{P}(A' \cap B)$ refers to the probability of event B occurring and A does not occur at all. $\mathrm{P}(A' \cap B) = \mathrm{P}(A \cup B) - \mathrm{P}(A)$ $= 0.8 - \dfrac{1}{3} = \dfrac{7}{15}$
9i	$\mathrm{P}(B) = P($ student studies Business$) = \dfrac{112}{300} = \dfrac{28}{75}$
ii	$\mathrm{P}(F \cup B) = P($ student is female or studies Business$)$ $= \dfrac{140}{300} + \dfrac{112}{300} - \dfrac{40}{300} = \dfrac{53}{75}$
iii	$\mathrm{P}(M \cap A) = P($ student is male and studies Art$)$ $= \dfrac{54}{300} = \dfrac{9}{50}$
iv	$\mathrm{P}(M) = P($ student is male$) = \dfrac{160}{300}$ $\mathrm{P}(A) = P($ student studies Art$) = \dfrac{90}{300}$ $\mathrm{P}(A) \times \mathrm{P}(M) = \dfrac{160}{300} \times \dfrac{90}{300} = \dfrac{4}{25}$ From (iii), $\mathrm{P}(A \cap M) = \dfrac{9}{50}$ Since $\mathrm{P}(A) \times \mathrm{P}(M) \neq \mathrm{P}(A \cap M)$ Events M and A are not independent.
v	$P($ exactly 2 students study Art out of 3$) = \dfrac{{}^{90}C_2 \times {}^{210}C_1}{{}^{300}C_3} = 0.189$ Or $= \dfrac{90}{300} \times \dfrac{89}{299} \times \dfrac{210}{298} \times 3 = 0.189$ (3 d.p.)

10i	
ii	Product moment correlation coefficient is –0.976 (3 s.f.), which is close to -1, suggesting a strong negative linear correlation between the times (in seconds) to run 100 metres and 10000 metres.
iii	Using GC, $y=-2.2181x+66.261$ $y=-2.22x+66.3$ (to 3 s.f) (see (10i) for line on the scatter diagram)
iv	When $x=13.1$, $y=-2.2181(13.1)+66.261$ $=37.204\approx 37.2$ The required time taken is 37.2 minutes.
v	Since $x=13.1$, is within data range and the product moment correlation coefficient is close to -1, this estimate is reliable.
11i	Unbiased estimate of the population mean, $\bar{x}=\dfrac{15}{100}+30=30.15$ Unbiased estimate of the population variance, $s^2=\dfrac{1}{99}\left[82-\dfrac{15^2}{100}\right]=\dfrac{29}{36}$
ii	Let X be the length, in cm, of a randomly chosen adult fish in a particular species of fish and μ be the population mean. Test $H_o:\mu=30$ against $H_1:\mu>30$ Under H_o, since sample size = 100 is large, by Central Limit Theorem, $\bar{X}\sim N\left(30,\dfrac{29/36}{100}\right)$ approximately. Use a one-tailed test at 2.5% level of significance. Using GC, with $\bar{x}=30.15$, $s=\sqrt{\dfrac{29}{36}}$, n =50, z value $=1.6712$ p value $=0.047335>0.025$ We do reject H_o and conclude that there is insufficient evidence at 2.5% significance level to conclude that mean length of an adult fish in a particular species of fish is greater than 30cm.

iii	Not necessary since the sample size of 100 is large. By Central Limit theorem, the sample mean lengths of adult fish will be approximately normal.
iv	Test $H_o : \mu = 30$ against $H_1 : \mu > 30$ Under H_o, since sample size = 100 is large, by Central Limit Theorem $\overline{X} \sim N\left(30, \frac{0.9}{100}\right)$ approximately. To support the scientist's claim that the mean length of the fish is greater than 30cm at 10% means sample mean lies in critical region. Using GC, $m > 30.12158$. Or use critical value method, $Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \sim N(0,1)$  $z_{calc} = \frac{m-30}{\sqrt{\frac{0.9}{100}}} > 1.2816$ $m - 30 > 1.2816\left(\sqrt{\frac{0.9}{100}}\right)$ $m > 0.12158 + 30$ $m > 30.12158$ $m \geq 30.2$
12i	Let A be the random variable "mass of Type A component in grams" $A \sim \mathrm{N}\left(250, 3^2\right)$ Let B be the random variable "mass of Type B component in grams" $B \sim \mathrm{N}\left(240, 4^2\right)$ $\mathrm{P}(A > 1.02(250)) = P(A > 255) = 0.047790 \approx 0.0478$
ii	$A - B \sim \mathrm{N}\left(10, 3^2 + 4^2\right)$ $A - B \sim \mathrm{N}(10, 25)$ $\mathrm{P}(A > B) = P(A - B > 0) = 0.97725 \approx 0.977$

iii	Let $T = A_1 + A_2 + \dots A_6 + B_1 + B_2 + B_3$ $E(T) = (6\times 250) + (3\times 240) = 2220$ $\text{Var}(T) = (6\times 9) + (3\times 16) = 102$ $T \sim \text{N}(2220, 102)$ $\text{P}(2190 < T < 2230) = 0.83746 \approx 0.837$
iv	Let $C = 0.03(B_1 + B_2 + \dots B_{10}) - 0.02(A_1 + A_2 + \dots A_{10})$ $E(C) = 0.03(10\times 240) - 0.02(10\times 250) = 22$ $\text{Var}(C) = 0.03^2(10\times 16) + 0.02^2(10\times 9) = 0.18$ $C \sim \text{N}(22, 0.18)$ $\text{P}(C > 22.50) = 0.119296 \approx 0.119$