# Anglo - Chinese School (Independent)



# FINAL YEAR EXAMINATION 2012 YEAR 3 INTEGRATED PROGRAMME CORE MATHEMATICS PAPER 2

Monday

8 October 2012

1 hour 30 minutes

Additional Materials

Answer Paper ( 6 sheets) Graph Paper ( 1 sheet)

# INSTRUCTIONS TO CANDIDATES

- Write your index number in the boxes on the cover page.
- Do not open this examination paper until instructed to do so.
- A calculator is required for this paper.
- Answer all the questions on the answer sheets provided.
- At the end of the examination, attach the answer sheets to the cover page provided.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- The maximum mark for this paper is 80.



This paper consists of 6 printed pages.

[Turn over

ACS(Independent)/Y3IPCoreMathsP2/2012/Endyear

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.



A triangular prism is shown in the diagram above. BCDE, ACDF and ABEF are rectangles. It is given that M is the midpoint of BC, AB = AC = 5 cm, BC = 8 cm and CD = 12 cm. (Volume of prism = Area of cross-section × Length)

i) Find 
$$\angle ADM$$
. [4 marks]

ii) Find the volume of the prism. [3 marks]

### 2. [Maximum mark: 7]

The two curves 
$$y = 3x^3$$
 and  $y = \frac{5}{x^2}$  intersect at the point A.

i) Find the coordinates of the point *A*. [3 marks]

ii) Hence sketch the graphs of  $y = 3x^3$  and  $y = \frac{5}{x^2}$  on the same diagram, indicating clearly the coordinates of A and where the graphs cut the axes (if any). [4 marks]

## 3. [Maximum mark: 8]



The diagram above shows the cross-section of a circular tube used in a wind instrument, with point *O* as the centre. The regular octagon *ABCDEFGH* is a hollow section which fits exactly in the tube with all 8 vertices on the circumference of the circle. It is given that the radius OB = x mm and the length of the chord AB = 7 mm.

i)	Find $\angle AOB$ .	[2 marks]
ii)	Show that $x = 9.146 \text{ mm}$ , correct to 3 decimal places.	[3 marks]
iii)	Find the area of the octagon ABCDEFGH.	[3 marks]

#### **4.** [Maximum mark: 10]



Diagram I shows a spherical steel ball of radius 3 cm. (Volume of sphere  $=\frac{4}{3}\pi r^3$ ; Volume of cylinder  $=\pi r^2 h$ ; Volume of cone  $=\frac{1}{3}\pi r^2 h$ )

i) Find the volume of one such spherical steel ball. [2 marks]

Diagram II shows the cross sectional view of an open cylindrical container filled with 7 such identical spherical steel balls touching one another and the side of the container. The height of the container is 6 cm.

ii)	Find the volume of the empty space in the container.	[4 marks]
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iii) These 7 spherical steel balls are melted and recast into the shape as shown in Diagram III. The new steel structure consists of a cylinder and a cone of same height and radius, r cm. Find the value of r. [4 marks]

## **5.** [Maximum mark: 13]

a) Find the range of values of *m* for which the line y = x - 2m intersects the curve  $y^2 + (x+m)^2 = 8$  at two distinct points. [5 marks]

**b**) The roots of the quadratic equation  $2x^2 + 9x + 5 = 0$  are  $\alpha$  and  $\beta$ .

- i) State the values of  $\alpha + \beta$  and  $\alpha\beta$ . [2 marks]
- ii) Find the quadratic equation whose roots are  $1-3\alpha^2$  and  $1-3\beta^2$ . [6 marks]

### 6. [Maximum mark: 10] Solution to this question by accurate drawing will not be accepted.



The diagram above, not drawn to scale, shows a parallelogram *ABCD* where the coordinates of the points *A*, *B* and *C* are (-3, -8), (3, -4) and (-6, 5) respectively. *E* is a point with coordinates (-10, 9) and *BCE* is a straight line.

i) Find the coordinates of the point D. [1 mark]

ii) Find the gradient of AB. [1 mark]

iii) Find the equation of the perpendicular bisector of AB and show that the point E lies on it. [5 marks]

iv) Find the area of  $\triangle ABE$ . [3 marks]

## **7.** [Maximum mark: 12]

Solve the following equations

**a**) 
$$2e^{-2x} - 5e^{-x} - 3 = 0;$$
 [4 marks]

**b**) 
$$3\log_2 y + \log_{\sqrt{2}} y = 12;$$
 [4 marks]

c)  $5^{7-3z} = e^{12z+17}$ . [4 marks]

# **8.** [Maximum mark: 13]

The table shows experimental values of two variables x and y, which are connected by an equation of the form  $y = Ax^n$ , where A and n are constants.

x	2	3	4	5	6
У	9.9	18.2	28.0	39.1	51.4

- i) Using a scale of 2cm to 0.1 unit on the  $\lg x$  -axis and 2cm to 0.2 units on the  $\lg y$  axis, plot  $\lg y$  against  $\lg x$  and draw a straight line graph. [4 marks]
- ii) Use your graph to estimate the value of A and of n. [4 marks]
- iii) On the same diagram, draw the line representing the equation  $y = x^{10}$ . [2 marks]
- iv) Hence find the range of values of x for which  $Ax^n \le x^{10}$ . [3 marks]

### End of Paper 2

# **ANSWERS:**

1(i)
 
$$\angle ADM = 13.3^{\circ} (3sf)$$
 5(a)
  $-\frac{4}{3} < m < \frac{4}{3}$ 

 1(ii)
 144 cm<sup>3</sup>
 5(b)(i)
  $\alpha + \beta = -\frac{9}{2}$  and  $\alpha \beta = \frac{5}{2}$ 

 2(i)
  $A = (1.11, 4.08)$ 
 5(b)(ii)
  $x^2 + \frac{175}{4}x + \frac{23}{2} = 0$  or  $4x^2 + 175x + 46 = 0$ 

 2(ii)
  $\int (y = \frac{5}{2})^{1/2}$ 
 6(i)
  $D = (-12, 1)$ 

 6(ii)
  $2$ 
 $3$ 

 6(iii)
  $2$ 
 $3$ 
 $y = -\frac{3}{2}x - 6$ 
 6(iv)
 65 units<sup>2</sup>

 3(i)
  $\angle AOB = 45^{\circ}$ 
 7(a)
  $x = -\ln 3 = -1.10$ 

 3(ii)
  $237 \text{ mm}^2$ 
 7(b)
  $y = 2^{\frac{15}{5}} = 5.28$ 

 4(i)
 113 cm<sup>3</sup>
 7(c)
  $z = -0.341$ 

 4(ii)
 735 cm<sup>3</sup>
 8(ii)
  $A = 3.47$  Accept  $3.24 \le A \le 3.80$ 
 $n = 1.51$  Accept  $1.30 \le n \le 1.70$ 
 $4(iii)$ 
 $x \ge 1.16$