RAFFLES INSTITUTION
H2 Mathematics 9758
2023 Year 6 Term 3 Revision 16

Topic: Review the Use of Graphing Calculators

In this revision, we will review the *use of graphing calculators* for the following topics. There may be tricks that you may not be aware of.

1: Graphing – Functions
2: Graphing – Parametric Equations
3: Equations & Inequalities
4: Complex Numbers
5: Discrete Random Variables

1: Graphing – Functions

Skills Required: By using the **FUNCTION** mode,

1. Sketch the graphs of functions and composite functions [Y=, WINDOW, ZOOM].
2. Determine the y-value for a given x-value [[CALC] value].
3. Determine the x-intercepts [[CALC] zero].
4. Determine the coordinates of the minimum & maximum points [[CALC] minimum/maximum]
5. Determine the points of intersection of 2 graphs [[CALC] intersect].
6. Determine the gradient of the function [[CALC] $\frac{\mathrm{d}y}{\mathrm{d}x}$ OR nDeriv].
7. Determine the area bounded by the function with the x-axis [[CALC] $\int \mathrm{f}(x)\,\mathrm{d}x$ OR fnInt].
8. Sketch piecewise functions (OS 5.3.0 and 5.3.1) [MATH B: piecewise]

Exercise 1

(a) Functions f and g are defined as follows:

$$\mathrm{f}(x) = \frac{x^2 - 2x - 1}{x + 2}, \qquad \text{for } x \in \mathbb{R},\ x \neq -2,$$

$$\mathrm{g}(x) = x - 3 - \mathrm{e}^{-x} - \ln(x+1), \qquad \text{for } x > -1.$$

Sketch, on separate diagrams, showing clearly any asymptotes, intersections with the axes and coordinates of stationary points, the graphs of

(i) $y = \mathrm{f}(x)$, (ii) $y = \mathrm{g}(x)$ [*Note for Teachers: May need to guide students for (ii)*].

The function h is defined as

$$\mathrm{h}: x \mapsto \mathrm{f}(x), \quad \text{for } x \in \mathbb{R},\ x > -2.$$

(iii) Find the equation of the tangent to $y = \mathrm{gh}(x)$ at the point where $x = 0$.

(iv) Find the area bounded by the graph of $y = \mathrm{gh}(x)$ with the x-axis.

(v) Solve the inequality $\mathrm{gh}(x) < 2\ln(x+1)$.

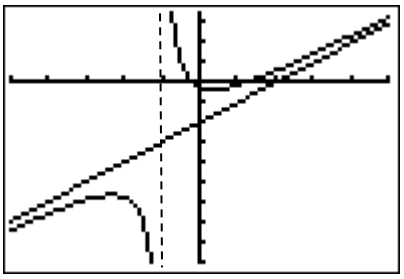
(b) Sketch the curve $y = 1 + \dfrac{1-10x}{x(x+1)}$, $x \in \mathbb{R}$, $x \neq -1$, $x \neq 0$, stating clearly equations of any asymptotes, intersections with the axes and coordinates of turning points.

(c) Sketch the graph of $y = \mathrm{f}(x)$ where $\mathrm{f}(x) = \begin{cases} \sin x + \cos x - 1 & \text{for } -\frac{7}{12}\pi \le x < 0, \\ -\sqrt{1 - \dfrac{(x-2)^2}{4}} & \text{for } 0 \le x \le 2. \end{cases}$

State clearly the coordinates of the endpoints.

Answers to Exercise 1(a)

(i) Use $Y_1 = \dfrac{x^2 - 2x - 1}{x+2}$ to sketch $y = \mathrm{f}(x)$.

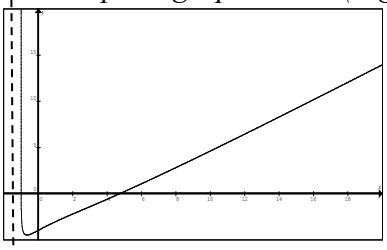


Asymptotes: $y = x - 4$, $x = -2$

Intercepts: (–0.414, 0), (2.41, 0), (0, –0.5)

Stationary Points: (0.646, –0.708), (–4.65, –11.3)

(ii) Use $Y_2 = x - 3 - \mathrm{e}^{-x} - \ln(x+1)$ to sketch $y = \mathrm{g}(x)$. [*Note for Teachers: GC will not show the complete graph below (regardless of zoom)*]

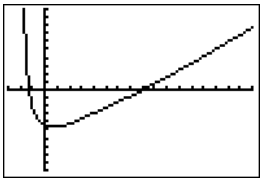


Asymptote: $x = -1$

Intercepts: (–0.999, 0), (4.76, 0), (0, –4)

Stationary Point: (–0.659, –4.52)

(iii) Use $Y_3 = Y_2(Y_1)$ to sketch $y = \mathrm{gh}(x)$



Use [[CALC] value] to obtain $y = -4.46$ at $x = 0$

OR Type $\boxed{\mathrm{Y_3(0)}}$ on the home screen.

Use [[CALC] $\dfrac{\mathrm{d}y}{\mathrm{d}x}$] to obtain $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -0.487$ at $x = 0$

OR Type $\boxed{\dfrac{\mathrm{d}}{\mathrm{dX}}(\mathrm{Y_3})\Big|_{\mathrm{X=0}}}$ on the home screen.

Equation of tangent at $x = 0$ is

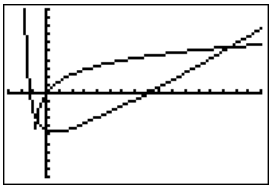
$y = -0.487x - 4.46$

(iv) Use [[CALC] zero] to obtain $x = -1.3044$ and $x = 8.0638$ when $y = 0$.

Use [[CALC] $\int \mathrm{f}(x)\,\mathrm{d}x$] to obtain area = 26.3 units^2

OR Type $\boxed{\int_{-1.3044}^{8.0638} (\mathrm{Y_3})\,\mathrm{dX}}$ on the home screen.

(v) Sketch $Y_3 = Y_2(Y_1)$ and $y = 2\ln(x+1)$

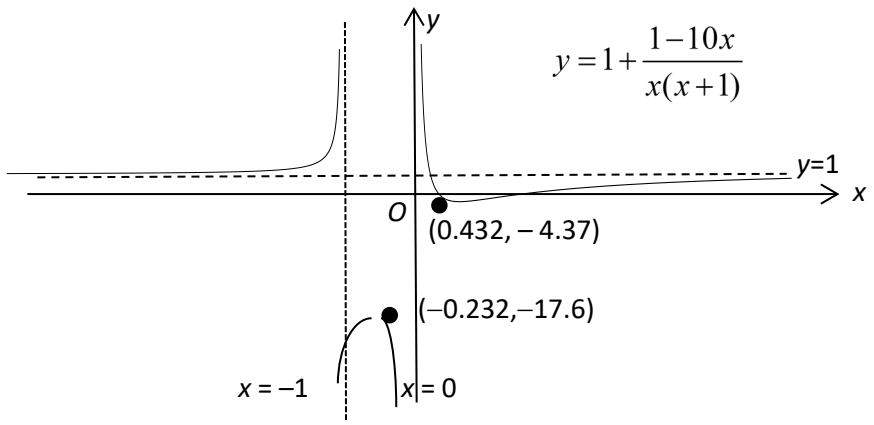


Use [[CALC] intersect] to obtain intersections at $x = -0.786$ and $x = 14.5$

$\mathrm{gh}(x) < 2\ln(x+1) \Rightarrow -0.786 < x < 14.5$

Answers to Exercise 1(b) & 1(c)

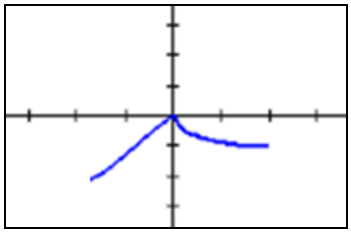
1(b) Intercepts: (0.113, 0) and (8.89, 0)



1(c)

Use MATH → B: piecewise to sketch graph

Endpoints: (–1.83, –2.22) and (2, –1)



2: Graphing – Parametric Equations

Skills Required: By using the **PARAMETRIC** mode,

1. Sketch the curve for the given domain [Y=, WINDOW].
2. Determine the coordinates of a point on the curve [TRACE, [CALC] value].
3. Determine the gradient of the curve at a point [[CALC] $\frac{dy}{dx}$ OR nDeriv].
4. Obtain the value of an integral with parametric equations [fnInt].

Exercise 2

(a) A curve has parametric equations $x = \sin 2\theta + \sin\theta$, $y = \cos\theta$, where $0 \le \theta \le \frac{2\pi}{3}$.

(i) Find the equation of the tangent to the curve at the point where $\theta = \frac{\pi}{2}$.

(ii) Sketch the curve, giving the coordinates of any intercepts.

(b) A curve has parametric equations $x = \cos^2 t$, $y = \sin^3 t$, for $0 \le t \le \frac{\pi}{2}$.

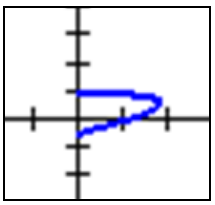
(i) Sketch the curve, giving the coordinates of any intercepts.
(ii) Find the area bounded by the curve and the axes.

Answers to Exercise 2(a)

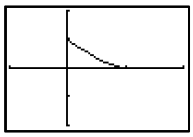
(i) Use X_{1T} and Y_{1T} to sketch the curve. Use [[CALC] value] to obtain $x = 1$ and $y = 0$ at $\theta = \frac{\pi}{2}$. Use [[CALC] $\frac{dy}{dx}$] to obtain $\frac{dy}{dx} = \frac{1}{2}$ at $\theta = \frac{\pi}{2}$. [OR Type $X_{1T}(\frac{\pi}{2})$, $Y_{1T}(\frac{\pi}{2})$, etc.]

Equation of tangent at $\theta = \frac{\pi}{2}$ is $y = \frac{1}{2}(x-1)$.

(ii) Intercepts: (1, 0), (0, 1), (0, –0.5)



Answers to Exercise 2(b)

(i) Use X_{2T} and Y_{2T} to sketch the curve.  Intercepts: (0, 1) and (1, 0)	**(ii)** Type $\int_{\pi/2}^{0}(Y_{2T}\times\frac{d}{dT}(X_{2T})\Big\|_{T=T})\,dT$ or $\int_{0}^{\pi/2}(X_{2T}\times\frac{d}{dT}(Y_{2T})\Big\|_{T=T})\,dT$ on the home screen to get required area = 0.4 units^2.

3: Equations & Inequalities

Skills Required:

1. Solve equations & inequalities.

- By using APPS → **PlySmlt2** → SIMULTANEOUS EQN SOLVER,

2. Solve system of linear equations.

Exercise 3

(a) Find the smallest positive integer n such that $10000\left(1-0.9^n\right)<25n\left(n+3\right)$.

(b) Find, for $-2\le x\le 2$, the set of values of x such that $\left|e^x\sin x-\left(x+x^2+\frac{x^3}{3}\right)\right|<0.05$.

(c) Use your calculator to find a vector equation of the line of intersection between

$\pi_1:\ \mathbf{r}\cdot(\mathbf{i}+3\mathbf{j}+2\mathbf{k})=4$ and $\pi_2:\ \mathbf{r}\cdot(\mathbf{i}-\mathbf{j}-\mathbf{k})=4$.

(d) A finite arithmetic progression $u_1, u_2, u_3,..., u_{2n}$ has first term a and common difference d.

It is given that $\frac{u_{n+1}}{u_n}=\frac{4027}{4025}$, $u_n+u_{2n}=36228$ and the average of all the odd-numbered terms is 12 075. By formulating three equations involving a, d and nd, find the values of a, d and n.

Answers to Exercise 3

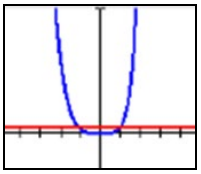
(a) $Y_1=10000\left(1-0.9^n\right)$ and $Y_2=25n\left(n+3\right)$

X	Y1	Y2
10	6513.2	3250
11	6861.9	3850
12	7175.7	4500
13	7458.1	5200
14	7712.3	5950
15	7941.1	6750
16	8147	7600
17	8332.3	8500
18	8499.1	9450
19	8649.1	10450
20	8784.2	11500

X=17

Least $n=17$

(b) $Y_1=\left|e^x\sin x-\left(x+x^2+\frac{x^3}{3}\right)\right|$ and $Y_2=0.05$

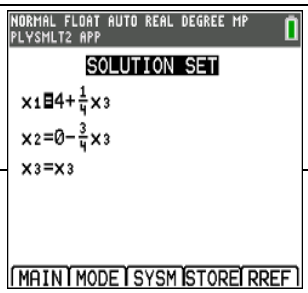


The set of values of x is $(-1.17,1.02)$.

(c) $\pi_1:\ x+3y+2z=4$

$\pi_2:\ x-y-z=4$

An equation of the intersecting line is



(d)

$\mathbf{r} = \begin{pmatrix} 4 \\ 0 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ -3 \\ 4 \end{pmatrix}$, $t \in \mathbb{R}$.	$2a - 4027d + 2nd = 0$ $2a - 2d + 3nd = 36228$ $a - d + nd = 12075$ From the GC, $a = 3$, $d = 6$ and $nd = 12078 \Rightarrow n = 2013$

4: Complex Numbers

Skills Required:

1. Perform operations on complex numbers.

- By using APPS → **PlySmlt2** → POLYNOMIAL ROOT FINDER,

2. Find the roots of a polynomial equation with real coefficients.

- By using MATH → **CMPLX**,

3. Find the conjugate, modulus and argument of a complex number.
4. Convert a complex number from the Cartesian form to the Polar form and vice versa.

Exercise 4

(a) Evaluate the following: (i) $\dfrac{(3+4\mathrm{i})^3}{(1-\mathrm{i})^5}$, (ii) $\sqrt{5-12\mathrm{i}}$. [(i) $\dfrac{161}{8}+\dfrac{73}{8}\mathrm{i}$; (ii) $3-2\mathrm{i}$]

(b) Find the roots of the following equations:
(i) $z^2+(-1+4\mathrm{i})z+(-5+\mathrm{i})=0$, (ii) $z^4-2z^3+8z-16=0$.

(c) Find the conjugate, modulus and argument of
(i) $(3-\mathrm{i})(\mathrm{i}-3)$, (ii) $\dfrac{(5-\mathrm{i})}{(3+2\mathrm{i})}$. [(i) $-8-6\mathrm{i}$, 10 , 2.50; (ii) $1+\mathrm{i}$, $\sqrt{2}$, $-\dfrac{\pi}{4}$]

(d) Express the following complex numbers in cartesian form.
(i) $\sqrt{2}e^{\left(\frac{\pi}{4}\right)\mathrm{i}}$, (ii) $2e^{\left(\frac{\pi}{3}\right)\mathrm{i}}$. [(i) $1+\mathrm{i}$; (ii) $1+\sqrt{3}\mathrm{i}$]

(e) Express the following complex numbers in polar form.
(i) $\dfrac{(2+2\mathrm{i})}{(\mathrm{i}-1)}$, (ii) $\left(-\dfrac{1}{2}-\dfrac{\sqrt{3}}{2}\mathrm{i}\right)(\mathrm{i}-\sqrt{3})$. [(i) $2e^{-\left(\frac{\pi}{2}\right)\mathrm{i}}$; (ii) $2e^{\left(\frac{\pi}{6}\right)\mathrm{i}}$]

Answers to Exercise 4(b)

(a)(i) Note: GCs with older OS cannot perform division in Math Print, i.e. fraction.	
(b)(i) [Use Quadratic Formula & GC] $z = 2-3\mathrm{i}$ or $-1-\mathrm{i}$	**(b)(ii)** [Use APPS] $z = \pm 2,\ 1 \pm \sqrt{3}\mathrm{i}$

5: Discrete Random Variables

Skills Required: Use of STAT, summation, [TABLE].

Exercise 5

(a) Find the mean and standard deviation of the random variable X with the following probability distribution:

X	-4	-3	-2	-1	0	1	2	3
$\mathrm{P}(X=x)$	0.04	0.16	0.24	0.16	0.15	0.1	0.1	0.05

(b) A committee of 12 people is chosen at random from a group consisting of 24 women and 16 men. The number of women on the committee is denoted by R.

Use your calculator to find

(i) $\mathrm{P}(R \geq 6)$,

(ii) the probability that R is divisible by 3,

(iii) $\mathrm{E}(R)$,

(iv) $\mathrm{Var}(R)$,

(v) the most probable number of women on the committee.

Answers to Exercise 5

(a) Enter x in L1 and $\mathrm{P}(X=x)$ in L2.
Use STAT → CALC → 1-Var Stats to get mean = –0.83 and standard deviation = 1.86

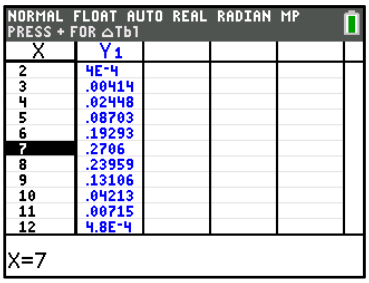
(b)(i) $\displaystyle\sum_{X=6}^{12}\left(\frac{{}^{24}C_X \times {}^{16}C_{12-X}}{{}^{40}C_{12}}\right) = 0.884$

(b)(ii) $\displaystyle\sum_{X=0}^{4}\left(\frac{{}^{24}C_{3X} \times {}^{16}C_{12-3X}}{{}^{40}C_{12}}\right) = 0.329$

(b)(iii) $\displaystyle\sum_{X=0}^{12}\left(X \times \frac{{}^{24}C_X \times {}^{16}C_{12-X}}{{}^{40}C_{12}}\right) = 7.2$

(b)(iv) $\displaystyle\sum_{X=0}^{12}\left(X^2 \times \frac{{}^{24}C_X \times {}^{16}C_{12-X}}{{}^{40}C_{12}}\right) - (7.2)^2 = 2.07$

(b)(v) $Y_1 = \dfrac{{}^{24}C_X \times {}^{16}C_{12-X}}{{}^{40}C_{12}}$



Most probable number of women = 7