H

millennia institute

CANDIDATE NAME

CLASS

ADMISSION NUMBER

2024 Preliminary Examination
Pre-University 3

MATHEMATICS **9758/01**

Paper 1 **9 September 2024**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your admission number, name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Qn No.	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	*	Total
Score													
Max Score	**5**	**4**	**7**	**8**	**9**	**9**	**11**	**11**	**11**	**13**	**12**		**100**

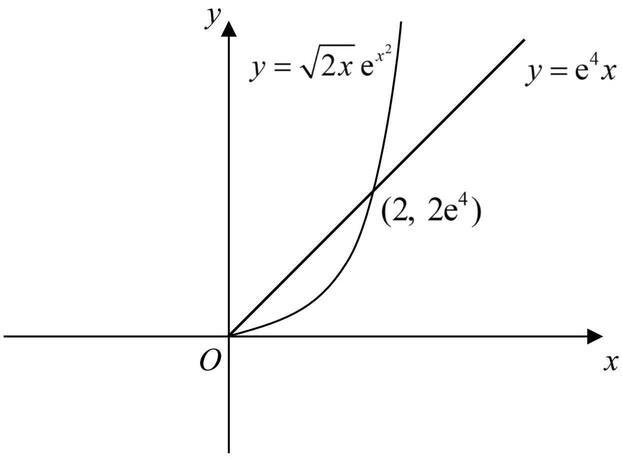
This document consists of **7** printed pages.

1 **(i)** A quadratic curve passes through the point $(-1,-4)$ and has its turning point at $(2,5)$. Find the equation of the curve. [4]

(ii) Given instead that a cubic curve passes through the same point $(-1,-4)$ and has the same turning point as stated in part **(i)**. Explain whether it is possible to obtain a unique equation of the curve based on given information. [1]

2 Use the substitution $u=5^x$ to find $\int 5^x \sin^2\left(5^x\right) \mathrm{d}x$. [4]

3 A line l has equation $y=\mathrm{e}^4 x$, $x \geq 0$ and curve C has equation $y=\sqrt{2x}\,\mathrm{e}^{x^2}$, $x \geq 0$. Both l and C intersect at the points with coordinates (0, 0) and (2, 2e^4) as shown in the diagram below.



(a) The region R is bounded by the line l, curve C, and the lines $x=1$ and $x=3$. Find, correct to 4 significant figures, the area of region R. [3]

(b) The region S is bounded by the line l and curve C. Show that the volume V of the solid formed when S is rotated 2π radians about the x-axis is $V=\frac{\pi}{6}\left(A\mathrm{e}^8+B\right)$, where A and B are exact constants to be determined.

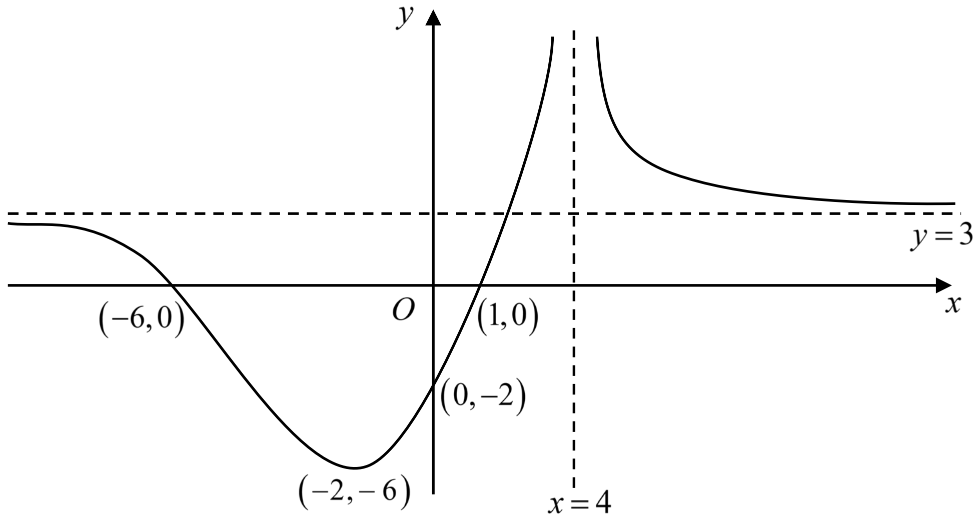
[4]

4 The equation of a curve is $2xy + (1+y)^2 = x$.

(i) Find the equations of the two tangents which are parallel to the y-axis. [4]

(ii) The normal to the curve at the point $A\ (-4,\ 5)$ meets the curve again at the point B. Find the coordinates of point B. [4]

5 The diagram below shows the curve of $y = \mathrm{f}(x)$. The curve cuts the axes at $(-6,0)$, $(1,0)$, and $(0,-2)$. It has a minimum point at $(-2,-6)$. There is a horizontal asymptote at $y=3$ and a vertical asymptote at $x=4$.



On separate diagrams, sketch the following graphs, stating the equations of any asymptotes, the coordinates of any turning points and axial intercepts.

(i) $y = \mathrm{f}(-|x|)$ [3]

(ii) $y = \mathrm{f}'(x)$ [3]

(iii) $y = \dfrac{1}{\mathrm{f}(x)}$ [3]

6 **(a)** The sum, S_n, of the first n terms of a sequence $u_1, u_2, u_3, \ldots$ is given by $S_n = -2n^2 + An$ for $n \geq 1$, where A is a non-zero constant.

(i) Find an expression for u_n in terms of n and A. [2]

(ii) Hence, determine if the sequence is an arithmetic progression. [2]

(iii) Describe how the sequence of sums $S_1, S_2, S_3, \ldots$ behaves when $A = 20$. [1]

(b) A geometric progression has first term 7 and common ratio r. The sum of the first 15 terms of the progression is 28.

(i) Show that $r^{15} - 4r + 3 = 0$. Explain why the common ratio cannot be 1 even though $r = 1$ is a root of this equation. [2]

(ii) Given that $|r| < 1$, find the sum to infinity, giving your answer correct to 2 decimal places. [2]

7 Functions f and g are defined respectively by

$$\mathrm{f}: x \mapsto \frac{(x-2)^2}{x^2-1},\ x \in \mathbb{R},\ x \neq -1,\ x \neq 1$$
$$\mathrm{g}: x \mapsto 1 - \sqrt{x},\ x \in \mathbb{R},\ x > 0,\ x \neq 4$$

(i) Show that the composite function fg exists. [2]

(ii) Find the range of fg. [2]

(iii) Explain why f does not have an inverse. [2]

(iv) If the domain of f is further restricted to $x \geq k$, state the least value of k for which the function f^{-1} exists. [1]

(v) For this restricted domain, find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [4]

8 **Do not use a calculator in answering this question.**

(a) The complex numbers z and w satisfy the following equations. It is known that w is not purely imaginary.

$$4z+1=|w|+6\mathrm{i}$$
$$w^*-2z=3-8\mathrm{i}$$

Find z and w, giving your answers in the form $a+\mathrm{i}b$, where a and b are real numbers. [5]

(b) Two complex numbers are $z_1=-2\sqrt{3}+6\mathrm{i}$ and $z_2=8\mathrm{e}^{-\mathrm{i}\frac{\pi}{3}}$.

(i) Find $\dfrac{z_1}{\sqrt[3]{z_2}}$ in the form $r(\cos\theta+\mathrm{i}\sin\theta)$, where $r>0$ and $-\pi<\theta\leq\pi$. [4]

(ii) It is known that z_1, z_2 and $(z_2)^3$ are roots of a polynomial equation of degree n with real coefficients. Explain why the smallest possible value of n is 5. [2]

9 **(a)** **(i)** Verify that $\dfrac{1}{r!}-\dfrac{1}{(r+1)!}=\dfrac{1}{(r-1)!\,(r+1)}$. [1]

(ii) Hence find an expression for $\displaystyle\sum_{r=1}^{n}\frac{1}{(r-1)!\,(r+1)}$. [3]

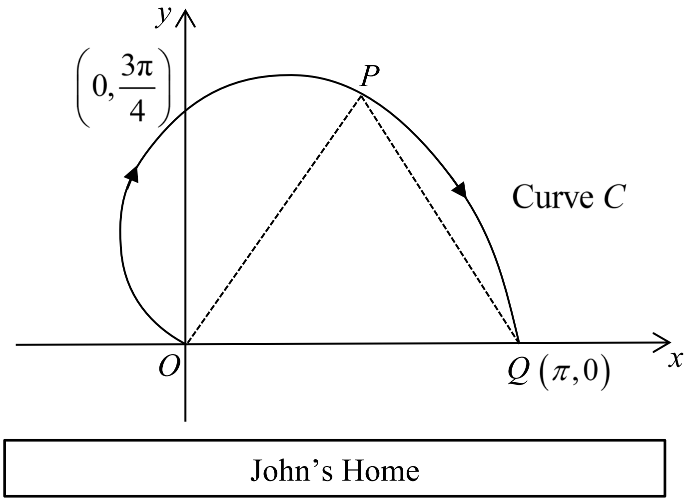
(b) It is given that $\displaystyle\sum_{r=1}^{n}r^2=\frac{n(n+1)(2n+1)}{6}$.

(i) Show that $\displaystyle\sum_{r=1}^{n}(2r-1)^2=\frac{n\left(4n^2-1\right)}{3}$. [3]

(ii) Hence find an expression for $13^2+15^2+\ldots+(4m-1)^2$ in terms of m. [4]

10 The diagram below shows a curve C with parametric equations given by

$$x = 2\theta \cos 2\theta, \quad y = 3\theta \sin 2\theta, \qquad \text{for } -\frac{\pi}{2} \le \theta \le 0.$$



The area bounded by curve C and the x-axis is a vineyard owned by John in front of his house where he used to grow grapes. He decided to install a Wi-Fi-enabled surveillance camera which moves automatically along the boundary of the vineyard in a clockwise direction along the curve C starting from point O and ending at point Q before moving in an anti-clockwise direction along the curve C back to point O.

At any point, the camera is located at a point P with parameter θ on the curve C. The camera is orientated such that the field of view spans from point P to points O and Q exactly as shown. It is assumed that the camera is at O initially.

(i) Show that the area of triangle OPQ, A units^2, is given by

$$A = \frac{3\pi}{2}\left(\theta \sin 2\theta\right).$$ [1]

(ii) Using differentiation, find the value of θ for $-\frac{\pi}{2} \le \theta \le 0$ that would maximise A and explain why A is a maximum for that value of θ.
Hence find this value of A. [5]

(iii) The image captured shows a good view of the vineyard when the camera is positioned such that $OP = PQ$. Find the coordinates of the position of the camera at this instant. [3]

(iv) John decides to apply fertilisers to a certain area of the vineyard to observe its effectiveness. This area is enclosed by the curve C, the line $x = 1$ and the x-axis, where $x \ge 1$. Find the approximate value of this area. [4]

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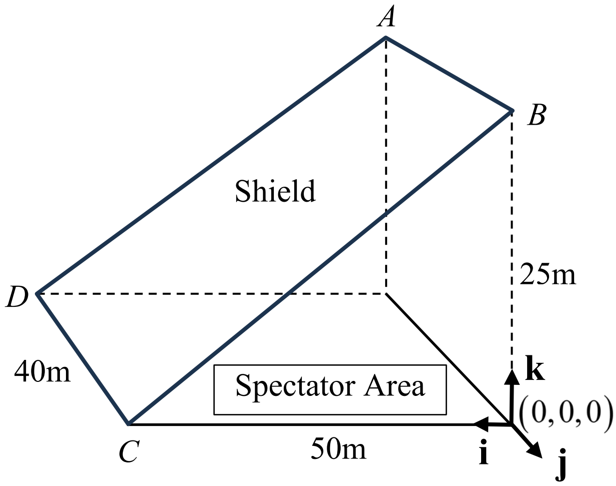

Organisers of an airshow are setting up the venue and performing safety checks before the event. Points (x, y, z) are defined relative to the entrance at $(0,0,0)$, where units are in metres. A spectator area of length 50 metres and width 40 metres is created on the horizontal ground, with a transparent rectangular flat shield erected to protect the spectators. You may assume that the shield is of negligible thickness. Support poles measuring 25 metres are attached at points A and B and anchored directly to the ground.

(i) Show that the cartesian equation of the shield $ABCD$ can be written as

$$x + 2z = 50.$$ [3]

A helicopter follows a flight path described by the line with equation

$$\mathbf{r} = \begin{pmatrix} 60 \\ 0 \\ 100 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 5 \\ 0 \end{pmatrix}, \ \lambda \in \mathbb{R}.$$

To check for potential interference by the crowd, a technician shoots a laser beam from the point $P(30,\ 10,\ 1)$ in the direction that is perpendicular to the shield $ABCD$.

(ii) Assuming the laser beam travels in a straight line, determine if it would intersect with the flight path of the helicopter. [3]

As part of the marketing campaign, the organiser wants to advertise about the proximity between the spectators and the helicopter.

(iii) Find the shortest possible distance from a spectator standing at the point $P(30,\ 10,\ 1)$ to the flight path of the helicopter, giving your answer to the nearest metre. [3]

For additional safety, a second protective shield, parallel to shield $ABCD$, is to be erected such that the perpendicular distance between them is 3 metres.

(iv) Given that the second protective shield is to be placed further away from the spectators, find the cartesian equation of the second protective shield. [3]

END OF PAPER

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CANDIDATE NAME

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ADMISSION NUMBER

2024 Preliminary Examination
Pre-University 3

MATHEMATICS **9758/02**

Paper 2 **12 September 2024**

3 hours

Candidates answer on the Question Paper.

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This document consists of **6** printed pages.

Section A: Pure Mathematics [40 marks]

1 Without the use of a calculator, solve the inequality $\dfrac{x+5}{(x-3)^2} \geq \dfrac{x-7}{x(x-3)}$. [4]

Hence solve $\dfrac{x-5}{(x+3)^2} \leq \dfrac{x+7}{x(x+3)}$. [2]

2 **(i)** Find $\int x^2 e^{3x} \, dx$. [5]

(ii) Hence evaluate $\int_0^k x^2 e^{3x} \, dx$. [2]

3 **(i)** Use the substitution $w = x + y$, show that the differential equation $\dfrac{dy}{dx} = \dfrac{3-x-y}{1+x+y}$ can be reduced into $\dfrac{dw}{dx} = \dfrac{4}{1+w}$. Solve this differential equation and deduce that the general solution can be written as $(x+y)^2 = 2(3x-y) + A$, where A is an arbitrary constant. [5]

(ii) Find the particular solution for the above differential equation, given that the curve passes through y-intercept $(0, 5)$. Hence or otherwise, find the coordinates of the other y-intercept. [2]

4 Referred to the origin O, the points A and B are such that $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. The point C lies on OB such that $\overrightarrow{OC} = p\overrightarrow{OB}$, where p is a constant. The point D is on AB such that $AD : DB = 1 : 3$ and the point E is on AC such that $AE = \dfrac{2}{5} AC$.

(i) Find $\overrightarrow{OE}$ in terms of p, $\mathbf{a}$ and $\mathbf{b}$. [1]

(ii) If $|\mathbf{b}| = 5$, show that the shortest distance from D to OB can be expressed as $k|\mathbf{a} \times \mathbf{b}|$, where k is a constant to be determined. [4]

(iii) Hence show that $\dfrac{\text{Area of } \Delta OCD}{\text{Area of } \Delta OCE} = \dfrac{5}{4}$. [4]

5 **(a)** Find the series expansion for $\dfrac{\cos 2x}{(1+\tan x)^2}$ up to and including the term in x^2, given that x is a sufficiently small angle. [3]

(b) It is given that $\mathrm{f}(x) = (1-x)^{\frac{1}{2}} \ln \sqrt{1+x}$.

(i) Find the numerical value of $\mathrm{f}'(x)$ when $x = 0.01$, leaving the answer in 5 decimal places. [1]

(ii) Using the standard series from the List of Formulae (MF26), expand $\mathrm{f}(x)$ as far as the term in x^3. State the range of values of x for which the expansion is valid. [4]

(iii) Denote the answer in part **(b)(ii)** as $\mathrm{g}(x)$. Hence find $\mathrm{g}'(x)$ when $x = 0.01$, leaving the answer in 5 decimal places. [2]

(iv) Comparing your answer in part **(b)(i)** against **(b)(iii),** comment on the validity of this answer. [1]

Section B: Probability and Statistics [60 marks]

6 A group consists of 3 female students, 5 male students, 1 female teacher and 1 male teacher.

(i) Find the number of ways the group can stand in a row for photo-taking if the 2 teachers are to stand at the two ends. [2]

State, with a reason, whether your answer will be different if the 2 teachers occupy the two positions at the centre instead. [1]

A team of 4 is to be formed from the group with at least 1 teacher.

(ii) Explain why the solution "Number of ways to form the team = ${}^2C_1 \times {}^9C_3 = 168$." is incorrect. [1]

(iii) Find the probability that the above team is formed if 4 people are randomly chosen from the group. [3]

7 A biased 6-sided die gives the score 1, 2, 3, 4, 5 and 8 with probabilities shown in the table below, where p and q are positive constants such that $p \neq q$.

Score	1	2	3	4	5	8
Probability	p	q	0.1	$p-q$	0.2	$p+q$

(i) Explain why a score of 1 is more likely to happen than a score of 2. [1]

(ii) Show that $3p+q=0.7$. [1]

(iii) Given that the mean score is 4.6, find the variance of the score. [5]

8 **(a)** Let X denotes the number of the chess puzzles that James attempts in a month. Assume that X follows a binomial distribution. It is given that James attempts 30 chess puzzles in a month and the probability of him solving each chess puzzle is 0.8.

(i) Find the probability that James solves at least 25 chess puzzles in a month. [2]

(ii) The probability of James solves at least 25 chess puzzles per month in exactly n months over a period of 3 years exceeds 13%. Find the two possible values of n. [3]

(b) **(i)** The number of residents in a village who are infected by a type of skin disease is denoted by Y. State, in context, two assumptions required for Y to be well modelled by a binomial distribution. [2]

Do not use a calculator in answering this part.

(ii) Assume Y follows a binomial distribution and the probability, p, of a resident in the village being infected by the skin disease is 0.7.
15 residents from the village are randomly selected. By considering the inequality of $\dfrac{\mathrm{P}(Y=y+1)}{\mathrm{P}(Y=y)}>1$, find the most likely number of residents to be infected with the skin disease. [3]

[You may use the result of $\dfrac{\mathrm{P}(Y=y+1)}{\mathrm{P}(Y=y)}=\left(\dfrac{n-y}{y+1}\right)\left(\dfrac{p}{1-p}\right)$ without any proof]

9 The average daily number of people hospitalised in a week, y, and the estimated number of COVID-19 infections in a week, x thousands, over a period of 8 weeks in a certain country are given in the following table.

x	10.7	22.1	32.0	56.0	58.3	39.1	21.2	19.8
y	120.2	435.7	525.3	549.6	559.9	540.0	375.3	208.0

(i) Draw a scatter diagram for these values. [1]

(ii) Calculate the product moment correlation coefficient between x and y, giving your answer correct to 4 decimal places. [1]

Ian claimed that the model $y = c\ln x + d$ is a better fit to the data as compared to the model $y = ax + b$.

(iii) Explain, with appropriate calculations, whether Ian's claim is valid. [2]

(iv) Find the values of a, b, c and d. [2]

(v) Use the better model to estimate the average daily number of people hospitalised for the week where the estimated number of COVID-19 infections in that week is 10000, giving your answer to 1 decimal place. [3]

(vi) Comment on the reliability of the estimate in part **(v)**. [1]

10 A dispensing machine is set to dispense 330ml of soft drink into cans. The company manager wishes to check if the machine is set correctly by performing a hypothesis test.

(i) State null and alternative hypotheses for the manager's test, defining any symbols that you use. [2]

The manager asks his assistant to collect 8 readings from the day's production for him to perform the test. The assistant decided to spread out the collection and took the 8 readings from the can that was produced at 9am, 10am, 11am, … , 4pm.

(ii) Give **one** reason why this sample is not suitable for a z-test. [1]

The manager decided to collect a suitable sample of 80 readings. The volumes, in ml, of soft drink dispensed into cans are summarized as follows.

$$\sum(x-330) = -60 \qquad \sum(x-330)^2 = 450$$

(iii) Carry out the manager's test in part **(i)**, at 1% level of significance, giving your conclusion in context. [5]

The company bought a new dispensing machine which is also set to dispense 330ml of soft drink into cans. The volume of soft drink dispensed into cans by the new machine is now normally distributed with a population variance of $3\,\text{ml}^2$. The manager suspects that the volume dispensed by the new machine is more than 330 ml. The manager decides to collect n suitable readings to test his suspicion at the 3% level of significance.

(iv) Given that the mean volume based on the n readings is 330.33 ml, find the least value of n for which the above test will confirm the manager's suspicion. [4]

11 **In this question you should state the parameters of any distributions you use.**

A factory produces its own brand of instant coffee. Their 2-in-1 instant coffee comes in the form of sticks. Each 2-in-1 stick contains creamer and coffee powder. The masses, in grams, of creamer and coffee powder in a randomly chosen 2-in-1 stick of instant coffee, are denoted by X and Y respectively. X and Y can be modelled using normal distributions with the means and standard deviations as follows.

	Mean (gram)	Standard deviation (gram)
Mass of Creamer, X	7.8	0.25
Mass of Coffee Powder, Y	1.9	0.04

(i) Find the probability that the contents in a randomly chosen 2-in-1 stick of instant coffee is more than 10 grams. [2]

(ii) Find the probability that the amount of creamer weighs more than 4 times that of the coffee powder in a randomly chosen 2-in-1 stick of instant coffee. [3]

(iii) State one assumption needed for the calculations in parts **(i)** and **(ii)** to be valid. [1]

Assume that the assumption in part **(iii)** holds. The 2-in-1 sticks of instant coffee are sold in packets of 50 and the mass of sticks of instant coffee are independent of each other within each packet.

(iv) Find the probability that a randomly chosen packet of 2-in-1 instant coffee weighs less than 488 grams. [3]

(v) Is your calculation in part **(iv)** still valid if X and Y are not normally distributed? Justify your answer. [1]

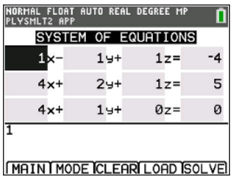
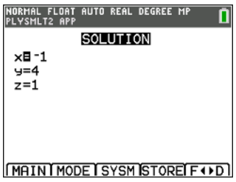
The factory also produces 3-in-1 instant coffee sticks. Each 3-in-1 stick contains creamer, coffee powder and sugar. The masses, in grams, of creamer and coffee powder in a randomly chosen 3-in-1 stick have the same distribution as that in a randomly chosen 2-in-1 stick. The mass, in grams, of sugar in a 3-in-1 stick follow the distribution $\text{N}(\mu, 0.36)$ and it is independent of X and Y.

(vi) Find the value of μ if 30% of the 3-in-1 sticks of instant coffee weighs less than 20 grams. [4]

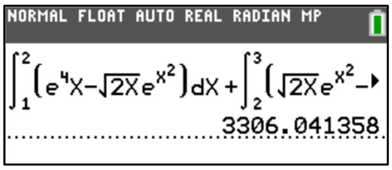
End of Paper

2024 PU3 H2 MATHEMATICS
PRELIM PAPER 1 Solutions

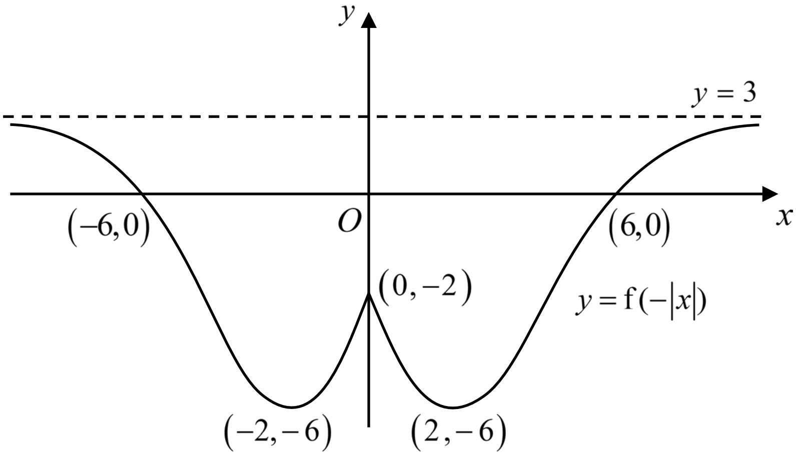
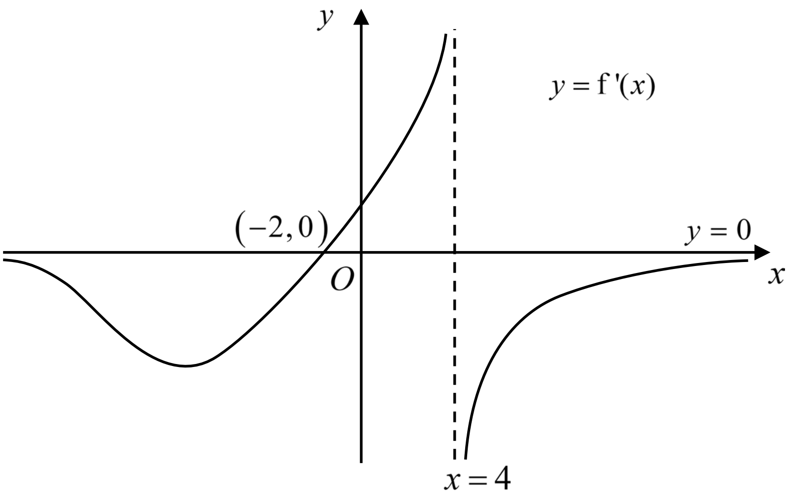
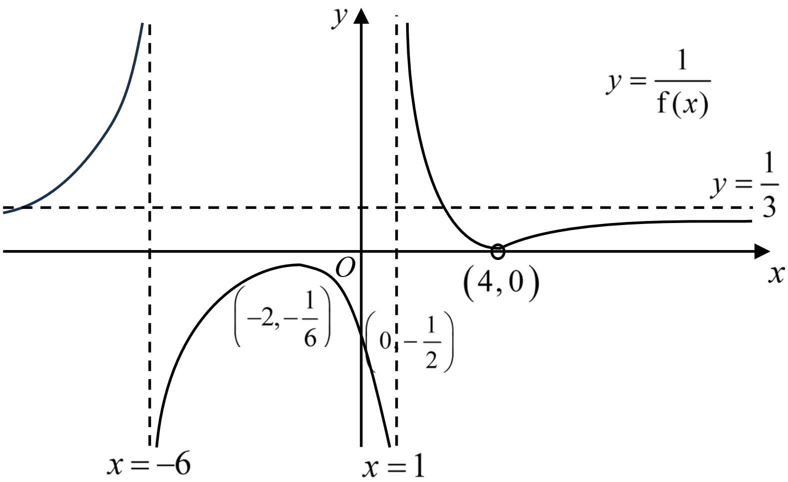
Paper 9758/01

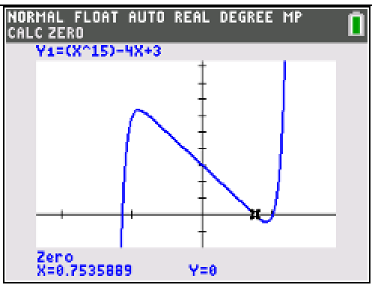
Qn	Solution
1(i) **[4]**	Let $y = ax^2 + bx + c$. For point $(-1,-4)$: $a(-1)^2 + b(-1) + c = -4$ $a - b + c = -4$ ---- (1) For point $(2,5)$: $a(2)^2 + b(2) + c = 5$ $4a + 2b + c = 5$ ---- (2) $\frac{dy}{dx} = 2ax + b.$ At turning point, when $x = 2$, $\frac{dy}{dx} = 0$. $2a(2) + b = 0$ $4a + b = 0$ ---- (3)  From GC, $a = -1,\ b = 4,\ c = 1$. Hence, equation of the curve is $y = -x^2 + 4x + 1$.
1(ii) **[1]**	Not possible. For a general cubic equation $y = ax^3 + bx^2 + cx + d$, there are 4 unknowns to solve for. But we can only form 3 equations from the given information. Therefore, we will obtain infinitely many solutions.

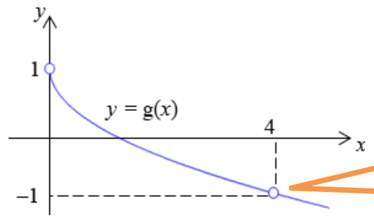
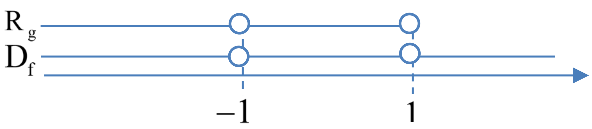
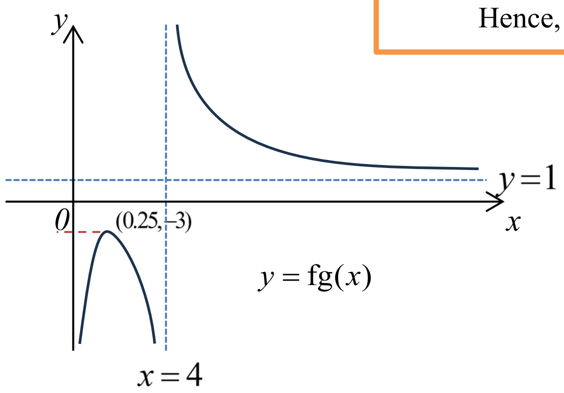
Qn	Solution
2 **[4]**	$\int 5^x \sin^2(5^x)\,\mathrm{d}x$ $=\int u\sin^2 u \cdot \frac{1}{u\ln 5}\,\mathrm{d}u$ $=\frac{1}{\ln 5}\int \sin^2 u\,\mathrm{d}u$ $=\frac{1}{2\ln 5}\int (1-\cos 2u)\,\mathrm{d}u$ $=\frac{1}{2\ln 5}\left[u-\frac{\sin 2u}{2}\right]+C$ $=\frac{1}{2\ln 5}\left[5^x-\frac{\sin 2(5^x)}{2}\right]+C$

Qn	Solution
3(a) **[3]**	Required area $=\int_1^2\left(\mathrm{e}^4x-\sqrt{2x}\,\mathrm{e}^{x^2}\right)\mathrm{d}x+\int_2^3\left(\sqrt{2x}\,\mathrm{e}^{x^2}-\mathrm{e}^4x\right)\mathrm{d}x$ $=3306$ (4 s.f.) 
3(b) **[4]**	**<u>Method 1</u>** Required vol. $=\frac{1}{3}\pi\left(2\mathrm{e}^4\right)^2(2)-\int_0^2\pi\left(\sqrt{2x}\mathrm{e}^{x^2}\right)^2\mathrm{d}x$ $=\frac{8}{3}\pi\mathrm{e}^8-\int_0^2\pi\left(2x\mathrm{e}^{2x^2}\right)\mathrm{d}x$ $=\frac{8}{3}\pi\mathrm{e}^8-\frac{1}{2}\pi\int_0^2\left(4x\mathrm{e}^{2x^2}\right)\mathrm{d}x$ $=\frac{8}{3}\pi\mathrm{e}^8-\frac{1}{2}\pi\left[\mathrm{e}^{2x^2}\right]_0^2$ $=\frac{8}{3}\pi\mathrm{e}^8-\frac{1}{2}\pi\left(\mathrm{e}^8-1\right)$ $=\frac{13}{6}\pi\mathrm{e}^8+\frac{1}{2}\pi$ $=\frac{\pi}{6}\left(13\mathrm{e}^8+3\right)$ $\therefore\ A=13,\ B=3$ **<u>Method 2</u>** Required vol. $=\int_0^2\pi\left[\left(\mathrm{e}^4x\right)^2-\left(\sqrt{2x}\mathrm{e}^{x^2}\right)^2\right]\mathrm{d}x$ $=\pi\int_0^2\left[\mathrm{e}^8x^2-2x\mathrm{e}^{2x^2}\right]\mathrm{d}x$ $=\pi\int_0^2\left[\mathrm{e}^8x^2\right]\mathrm{d}x-\frac{1}{2}\pi\int_0^2 4x\mathrm{e}^{2x^2}\,\mathrm{d}x$ $=\pi\mathrm{e}^8\left[\frac{x^3}{3}\right]_0^2-\frac{1}{2}\pi\left[\mathrm{e}^{2x^2}\right]_0^2$ $=\frac{8}{3}\pi\mathrm{e}^8-\frac{1}{2}\pi\left(\mathrm{e}^8-1\right)$ $=\frac{13}{6}\pi\mathrm{e}^8+\frac{1}{2}\pi$ $=\frac{\pi}{6}\left(13\mathrm{e}^8+3\right)$

Qn	Solution
4(i) **[4]**	$2xy+(1+y)^2=x$ Differentiating wrt x, $2y+2x\dfrac{\mathrm{d}y}{\mathrm{d}x}+2(1+y)\dfrac{\mathrm{d}y}{\mathrm{d}x}=1$ $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{1-2y}{2(x+y+1)}$ When the tangent is parallel to the y-axis, $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ is undefined. $\therefore 2(x+y+1)=0$ $y=-1-x$, substitute this into the equation of the curve, $2x(-1-x)+(1-1-x)^2=x$ $x^2+3x=0$ $x=0$ or $x=-3$
4(ii) **[4]**	Gradient of normal at $A=\dfrac{-1}{\left(\dfrac{1-2(5)}{2(-4+5+1)}\right)}=\dfrac{4}{9}$ Equation of normal at A: $y-5=\dfrac{4}{9}(x-(-4))$ $y=\dfrac{4}{9}x+\dfrac{61}{9}$ To find B, $2x\left(\dfrac{4}{9}x+\dfrac{61}{9}\right)+\left(1+\left(\dfrac{4}{9}x+\dfrac{61}{9}\right)\right)^2=x$ $x=-13.920=-13.9\text{ (3 s.f.)}$ or $x=-4$ (given point) $x=-13.920,\ y=0.591$ => Coordinates of B are $(-13.9, 0.591)$.

Qn	Solution
5(i) **[3]**	 $y=\mathrm{f}(x) \xrightarrow[\text{the } y\text{-axis}]{\text{reflect in}} y=\mathrm{f}(-x) \xrightarrow[\text{keep } x\geq 0 \text{ part of graph and replace } x<0 \text{ part with mirror image of } x\geq 0]{} y=\mathrm{f}(-\lvert x\rvert)$
5(ii) **[3]**	
5(iii) **[3]**	

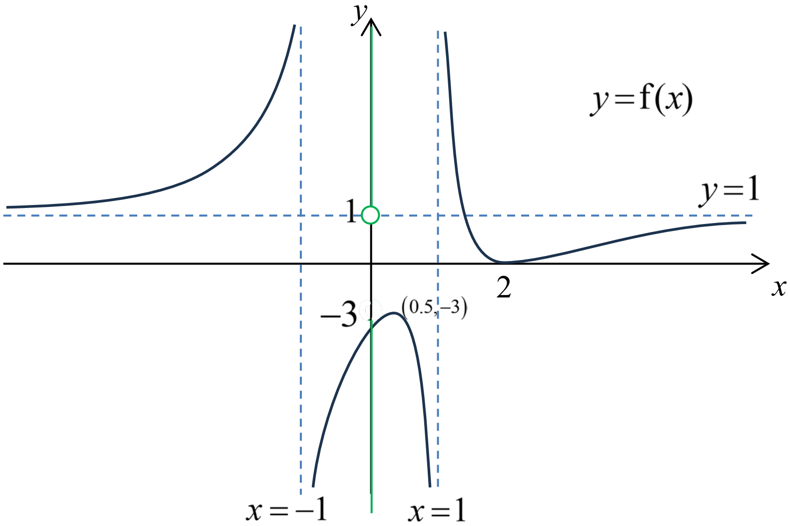
Qn	Solution
6(a)(i) [2]	$u_n = S_n - S_{n-1}$ $= -2n^2 + An - \left[-2(n-1)^2 + A(n-1)\right]$ $= -2n^2 + An - \left[-2(n^2 - 2n + 1) + An - A\right]$ $= -2n^2 + An - \left[-2n^2 + 4n - 2 + An - A\right]$ $= -4n + 2 + A$
6(a) (ii) [2]	$u_n - u_{n-1} = -4n + 2 + A - \left[-4(n-1) + 2 + A\right]$ $= -4n + 2 + A - [-4n + 4 + 2 + A]$ $= -4n + 2 + A + 4n - 6 - A$ $= -4$ Since $u_n - u_{n-1} = -4$ is a constant independent of n, the sequence is an arithmetic progression.
6(a) (iii) [1]	When $A = 20$, the sequence increases to 50 (or S_5) and then decreases indefinitely towards $-\infty$.
6(b) (i) [2]	$\frac{7\left(1 - r^{15}\right)}{1 - r} = 28$ $1 - r^{15} = 4(1 - r)$ $r^{15} - 4r + 3 = 0$ If the common ratio is 1, each of the terms in the geometric progression is 7 (made up of constant terms). Thus, the sum of the first 15 terms is 7(15) = 105 and not 28.
6(b) (ii) [2]	 From GC, since $\|r\| < 1$, $r = 0.75359$ (5 s.f.) or: [$r = -1.1445$ (rejected) or $r = 1$ (rejected)] $S_\infty = \frac{7}{1 - 0.75359}$ $= 28.41$ (2 d.p.)

7(i) **[2]**	$\mathrm{f}: x \mapsto \dfrac{(x-2)^2}{x^2-1},\ x \in \mathbb{R},\ x \neq -1,\ x \neq 1$ $\mathrm{g}: x \mapsto 1-\sqrt{x},\ x \in \mathbb{R},\ x > 0,\ x \neq 4$  Note this **excluded** point, to exclude from From the graph, $\mathrm{R_g} = (-\infty, -1) \cup (-1, 1)$  Since $\mathrm{R_g} = (-\infty, -1) \cup (-1, 1) \subseteq \mathbb{R} \setminus \{-1, 1\} = \mathrm{D_f}$, then fg exists.
7(ii) **[2]**	<u>**Method 1: Direct**</u> $\mathrm{D_{fg}} = \mathrm{D_g} = (0,4) \cup (4,\infty)$ $\mathrm{fg}(x) = \dfrac{\left[\left(1-\sqrt{x}\right)-2\right]^2}{\left(1-\sqrt{x}\right)^2-1}$ $\mathrm{fg}(x) = \dfrac{\left[\left(1-\sqrt{x}\right)-2\right]^2}{\left(1-\sqrt{x}\right)^2-1} = \dfrac{1+2\sqrt{x}+x}{-2\sqrt{x}+x}$ How to obtain horizontal asymptote? $\mathrm{fg}(x) = \dfrac{\frac{1+2\sqrt{x}+x}{x}}{\frac{-2\sqrt{x}+x}{x}} = \dfrac{\frac{1}{x}+\frac{2}{\sqrt{x}}+1}{-\frac{2}{\sqrt{x}}+1}$ As $x \to \infty$, $\frac{1}{x} \to 0$, $\frac{2}{\sqrt{x}} \to 0$. So, $\mathrm{fg}(x) \to \frac{1}{1} = 1$. Hence, $y = 1$ is a horizontal asymptote.  From the graph, $\mathrm{R_{fg}} = (-\infty, -3] \cup (1, \infty)$

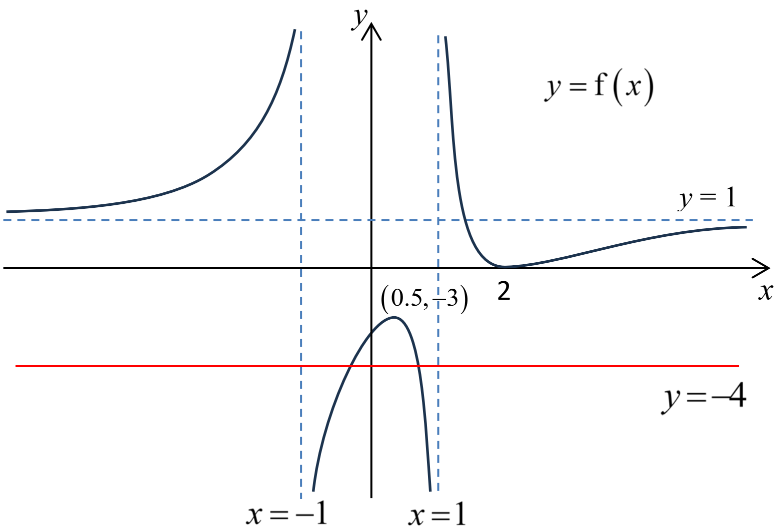
<table>
<tr><td></td><td>

<u>Method 2: Mapping</u>

$$\underbrace{(0,4)\cup(4,\infty)}_{D_{fg}=D_g} \xrightarrow{g} \underbrace{(-\infty,-1)\cup(-1,1)}_{\text{Range of g = NEW domain of f}} \xrightarrow{f} \underbrace{(-\infty,-3]\cup(1,\infty)}_{R_{fg}}$$



</td></tr>
<tr><td>7(iii)
[2]</td><td>



Since the line $y=-4$ intersects the graph of $y=\mathrm{f}(x)$ more than once, f is not a 1-1 function.
$\therefore$ f does not have an inverse (shown).

</td></tr>
<tr><td>7(iv)
[1]</td><td>

From the diagram, f is 1-1 $\Rightarrow x\ge 2$.

$\therefore$ least $k=2$.

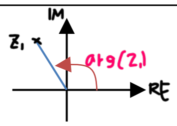
</td></tr>
</table>

7(v) **[4]**	Let $y=\dfrac{(x-2)^2}{x^2-1}$ $y(x^2-1)=(x-2)^2$ $yx^2-y=x^2-4x+4$ $(1-y)x^2-4x+(y+4)=0$ Use quadratic formula: $x=\dfrac{4\pm\sqrt{(-4)^2-4(1-y)(y+4)}}{2(1-y)}$ $=\dfrac{4\pm\sqrt{16-4(4-3y-y^2)}}{2(1-y)}$ $=\dfrac{4\pm\sqrt{4\left[4-(4-3y-y^2)\right]}}{2(1-y)}$ $=\dfrac{4\pm2\sqrt{3y+y^2}}{2(1-y)}$ $=\dfrac{2\pm\sqrt{y^2+3y}}{1-y}$ $x=\dfrac{2+\sqrt{y^2+3y}}{1-y}$ or $\underbrace{\dfrac{2-\sqrt{y^2+3y}}{1-y}}_{(\text{Reject since } x\geq 2 \text{ and } 0\leq y<1)}$ $\therefore\ \mathrm{f}^{-1}(x)=\dfrac{2+\sqrt{x^2+3x}}{1-x}$ and under the restricted domain of f which is $[2,\infty)$, the corresponding range of f is $[0,\ 1)$. $\mathrm{D}_{\mathrm{f}^{-1}}=\mathrm{R}_{\mathrm{f}}=[0,\ 1)$

Qn	Solution
8(a) **[5]**	$4z+1=\|w\|+6\text{i}$ ---- (1) $w^*-2z=3-8\text{i}$ ---- (2) From (2): $z=\dfrac{w^*-3+8\text{i}}{2}$ ---- (3) Sub (3) into (1): $4\left(\dfrac{w^*-3+8\text{i}}{2}\right)+1=\|w\|+6\text{i}$ $2w^*-6+16\text{i}+1-6\text{i}=\|w\|$ $2w^*-5+10\text{i}=\|w\|$ Let $w=a+b\text{i}$. $2a-2b\text{i}-5+10\text{i}=\sqrt{a^2+b^2}$ $(2a-5)+(10-2b)\text{i}=\sqrt{a^2+b^2}$ Comparing imaginary parts: $10-2b=0$ $\Rightarrow b=5$ Comparing real parts: $2a-5=\sqrt{a^2+5^2}$ $(2a-5)^2=a^2+25$ $4a^2-20a+25=a^2+25$ $3a^2-20a=0$ $a(3a-20)=0$ $a=0$ (rejected since w is not purely imag) or $a=\dfrac{20}{3}$ $\therefore w=\dfrac{20}{3}+5\text{i}$ Sub back into (3): $z=\dfrac{1}{2}\left(\dfrac{20}{3}-5\text{i}-3+8\text{i}\right)$ $=\dfrac{11}{6}+\dfrac{3}{2}\text{i}$ Ans: $w=\dfrac{20}{3}+5\text{i},\ z=\dfrac{11}{6}+\dfrac{3}{2}\text{i}$

8(b) (i) [4]

$z_1 = -2\sqrt{3} + 6\mathrm{i}$



$|z_1| = \sqrt{\left(-2\sqrt{3}\right)^2 + 6^2} = \sqrt{48}$

$\arg(z_1) = \pi - \tan^{-1}\left(\dfrac{6}{2\sqrt{3}}\right) = \pi - \dfrac{\pi}{3} = \dfrac{2\pi}{3}$

<u>Method 1 (Use exponential form)</u>

$z_1 = \sqrt{48}\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}$, $\quad \sqrt[3]{z_2} = \left(8\mathrm{e}^{-\mathrm{i}\frac{\pi}{3}}\right)^{\frac{1}{3}} = 2\mathrm{e}^{-\mathrm{i}\frac{\pi}{9}}$

$$\frac{z_1}{\sqrt[3]{z_2}} = \frac{\sqrt{48}\mathrm{e}^{\mathrm{i}\frac{2\pi}{3}}}{2\mathrm{e}^{-\mathrm{i}\frac{\pi}{9}}}$$

$$= \frac{\sqrt{48}}{2}\mathrm{e}^{\mathrm{i}\frac{2\pi}{3} - \left(-\mathrm{i}\frac{\pi}{9}\right)}$$

$$= \sqrt{12}\mathrm{e}^{\mathrm{i}\frac{7\pi}{9}}$$

$$\therefore \frac{z_1}{\sqrt[3]{z_2}} = \sqrt{12}\left(\cos\frac{7\pi}{9} + \mathrm{i}\sin\frac{7\pi}{9}\right)$$

$$\text{or } 2\sqrt{3}\left(\cos\frac{7\pi}{9} + \mathrm{i}\sin\frac{7\pi}{9}\right)$$

<u>Method 2 (finding mod / arg)</u>

From question, $|z_2| = 8$, $\arg(z_2) = -\dfrac{\pi}{3}$

$\left|\sqrt[3]{z_2}\right| = \sqrt[3]{|z_2|} = \sqrt[3]{8} = 2$

$\arg\left(\sqrt[3]{z_2}\right) = \arg\left(z_2^{\frac{1}{3}}\right) = \dfrac{1}{3}\arg(z_2) = \dfrac{1}{3}\left(-\dfrac{\pi}{3}\right) = -\dfrac{\pi}{9}$

$$\left|\frac{z_1}{\sqrt[3]{z_2}}\right| = \frac{|z_1|}{\left|\sqrt[3]{z_2}\right|} = \frac{\sqrt{48}}{2} = \sqrt{12}$$

$$\arg\left(\frac{z_1}{\sqrt[3]{z_2}}\right) = \arg(z_1) - \arg\left(\sqrt[3]{z_2}\right)$$

$$= \frac{2\pi}{3} - \left(-\frac{\pi}{9}\right)$$

$$= \frac{7\pi}{9}$$

$$\therefore \frac{z_1}{\sqrt[3]{z_2}} = \sqrt{12}\left(\cos\frac{7\pi}{9} + \mathrm{i}\sin\frac{7\pi}{9}\right)$$

$$\text{or } 2\sqrt{3}\left(\cos\frac{7\pi}{9} + \mathrm{i}\sin\frac{7\pi}{9}\right)$$

8(b) (ii) [2]	Since the coefficients of the polynomial are real, and z_1 and z_2 are roots, then z_1^* and z_2^* are roots too.
	Method 1 $\arg\left[(z_2)^3\right] = 3\arg(z_2) = 3\left(-\frac{\pi}{3}\right) = -\pi$ This means that $(z_2)^3$ lies on the negative real axis.
	Method 2 $z_2{}^3 = \left(8e^{-i\frac{\pi}{3}}\right)^3 = 512e^{-i\pi}$ $= 512\left[\cos(-\pi) + i\sin(-\pi)\right]$ $= -512$
	This implies that $z_2{}^3$ is a real number. There are at least 5 roots of the polynomial equation. Therefore, the smallest value of n is 5.

Qn	Solution
9(a)(i) **[1]**	$\text{LHS} = \frac{1}{r!} - \frac{1}{(r+1)!}$ $= \frac{1}{r!} - \frac{1}{r!(r+1)}$ $= \frac{(r+1)}{r!(r+1)} - \frac{1}{r!(r+1)}$ $= \frac{r}{r!(r+1)}$ $= \frac{r}{(r-1)!r \times (r+1)}$ $= \frac{1}{(r-1)!(r+1)}$ $= \text{LHS (verified)}$
9(a) **(ii)** **[3]**	$\sum_{r=1}^{n} \frac{1}{(r-1)!(r+1)} = \sum_{r=1}^{n}\left(\frac{1}{r!} - \frac{1}{(r+1)!}\right)$ $= \frac{1}{1} - \cancel{\frac{1}{2!}}$ $+ \cancel{\frac{1}{2!}} - \cancel{\frac{1}{3!}}$ $+ \cancel{\frac{1}{3!}} - \cancel{\frac{1}{4!}}$ $+ \cancel{\ldots}$ $+ \cancel{\frac{1}{(n-1)!}} - \cancel{\frac{1}{n!}}$ $+ \cancel{\frac{1}{n!}} - \frac{1}{(n+1)!}$ $= 1 - \frac{1}{(n+1)!}$

Qn	Solution
9(b) (i) [3]	$\sum_{r=1}^{n}(2r-1)^2$ $=\sum_{r=1}^{n}(4r^2-4r+1)$ EXPAND! $=4\sum_{r=1}^{n}r^2-4\sum_{r=1}^{n}r+\sum_{r=1}^{n}1$ $=4\sum_{r=1}^{n}r^2-4\underbrace{[1+2+3+...+n]}_{\text{Sum of AP with }a=1,d=1,\ n\text{ terms}}+\underbrace{(1+1+1+...+1)}_{n\text{ times}}$ $=4\left[\frac{n(n+1)(2n+1)}{6}\right]-4\left[\frac{n(n+1)}{2}\right]+n\times 1$ $=\frac{2n(n+1)(2n+1)}{3}-2n(n+1)+n$ $=\frac{2n(n+1)(2n+1)-6n(n+1)+3n}{3}$ $=\frac{n[2(n+1)(2n+1)-6(n+1)+3]}{3}$ $=\frac{n(4n^2+6n+2-6n-6+3)}{3}$ $=\frac{n(4n^2-1)}{3}$
9(b) (ii) [4]	Identifying the lower and upper limit of the series: $2r-1=13\Rightarrow r=7$ $2r-1=4m-1\Rightarrow r=2m$ $13^2+15^2+...+(4m-1)^2=\sum_{r=7}^{2m}(2r-1)^2$ $=\sum_{r=1}^{2m}(2r-1)^2-\sum_{r=1}^{6}(2r-1)^2$ $=\frac{2m\left[4(2m)^2-1\right]}{3}-\frac{6\left[4(6)^2-1\right]}{3}$ $=\frac{2m\left[16m^2-1\right]}{3}-\frac{6\left[4(6)^2-1\right]}{3}$ $=\frac{2m\left[16m^2-1\right]}{3}-286$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>10(i)
[1]</td><td>
Let area of triangle OPQ be A.

$$A=\frac{1}{2}(\pi)(3\theta\sin 2\theta)=\frac{3\pi}{2}(\theta\sin 2\theta) \quad \text{(shown)}$$
</td></tr>
<tr><td>10(ii)
[5]</td><td>

$$\frac{\mathrm{d}A}{\mathrm{d}\theta}=\frac{3\pi}{2}(2\theta\cos 2\theta+\sin 2\theta)$$

When $\frac{\mathrm{d}A}{\mathrm{d}\theta}=0$,

$$\frac{3\pi}{2}(2\theta\cos 2\theta+\sin 2\theta)=0$$

Since $\frac{3\pi}{2}\neq 0$,

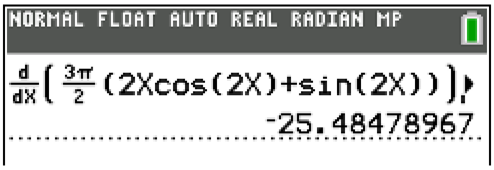
$$2\theta\cos 2\theta+\sin 2\theta=0$$

Using GC,

$\theta=-1.0144$ (5 s.f.)

$\quad=-1.01$ (3 s.f.)

<u>Method 1: Second Derivative Test</u>



OR

$$\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2}=\frac{3\pi}{2}(-4\theta\sin 2\theta+4\cos 2\theta)$$

When $\theta=-1.0144$,

$$\frac{\mathrm{d}^2A}{\mathrm{d}\theta^2}=-25.484=-25.5<0$$

$\therefore \theta=-1.0144$ will result in maximum A.

****

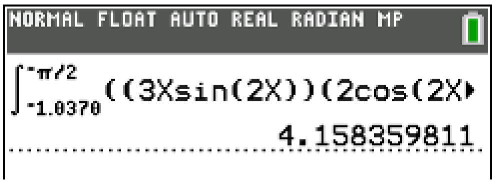
<u>Method 2: First Derivative Test</u>

<table>
<tr><td>A</td><td>$(-1.01)^-$
(–1.02)</td><td>-1.01</td><td>$(-1.01)^+$
(–1.00)</td></tr>
<tr><td>$\frac{\mathrm{d}A}{\mathrm{d}\theta}$</td><td>$>0$
(0.144)</td><td>0</td><td><0
(–0.363)</td></tr>
<tr><td>Slope</td><td>/</td><td>–</td><td>\</td></tr>
</table>

$\therefore \theta=-1.0144$ will result in maximum A.

****
</td></tr>
</table>

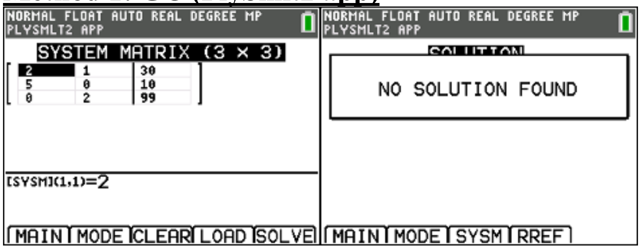
Qn	Solution
	When $\theta=-1.0144$, $A=\frac{3\pi}{2}(-1.0144)\times\sin\left[2\times(-1.0144)\right]$ $=4.29\text{ units}^2\ (3\text{ s.f.})$
10(iii) **[3]**	For $OP=PQ$, triangle OPQ is an isosceles triangle. **<u>Method 1</u>** $x=\pi\div 2=\frac{\pi}{2}$ $2\theta\cos 2\theta=\frac{\pi}{2}$ Using GC, $\theta=-1.1581$ ** **<u>Method 2</u>** $OP=PQ$ $\sqrt{x^2+y^2}=\sqrt{(x-\pi)^2+y^2}$ $x^2+y^2=(x-\pi)^2+y^2$ $x^2=x^2-2x\pi+\pi^2$ $2x\pi=\pi^2$ $2x=\pi$ $x=\frac{\pi}{2}$ $\therefore\ 2\theta\cos 2\theta=\frac{\pi}{2}$ Using GC, $\theta=-1.1581$ ** $y=3(-1.1581)\sin(2\times-1.1581)=2.55$ $\therefore$ coordinates of camera position $(1.57, 2.55)$
10(iv) **[4]**	$x=2\theta\cos 2\theta\ \Rightarrow\ \frac{\mathrm{d}x}{\mathrm{d}\theta}=2\cos 2\theta-4\theta\sin 2\theta$ When $x=1$, $2\theta\cos 2\theta=1$ Solving for θ, $\theta=-1.0370$ (5 s.f.) When $x=\pi$, $2\theta\cos 2\theta=\pi$

Qn	Solution
	Solving for θ, $\theta = -\dfrac{\pi}{2}$  Required area $= \int_{1}^{\pi} y\,dx$ $= \int_{-1.0370}^{-\pi/2} (3\theta \sin 2\theta)(2\cos 2\theta - 4\theta \sin 2\theta)\,d\theta$ $= 4.1584$ $= 4.16$ (3 s.f.)

Qn	Solution
11(i) **[3]**	$\overrightarrow{OC}=\begin{pmatrix}50\\0\\0\end{pmatrix},\overrightarrow{OD}=\begin{pmatrix}50\\-40\\0\end{pmatrix},\overrightarrow{OB}=\begin{pmatrix}0\\0\\25\end{pmatrix},\overrightarrow{OA}=\begin{pmatrix}0\\-40\\25\end{pmatrix}$ $\overrightarrow{CD}=\begin{pmatrix}50\\-40\\0\end{pmatrix}-\begin{pmatrix}50\\0\\0\end{pmatrix}=\begin{pmatrix}0\\-40\\0\end{pmatrix}$ $\overrightarrow{CB}=\begin{pmatrix}0\\0\\25\end{pmatrix}-\begin{pmatrix}50\\0\\0\end{pmatrix}=\begin{pmatrix}-50\\0\\25\end{pmatrix}$ A normal to plane $ABCD=\overrightarrow{CD}\times\overrightarrow{CB}$ $=\begin{pmatrix}0\\-40\\0\end{pmatrix}\times\begin{pmatrix}-50\\0\\25\end{pmatrix}$ $=\begin{pmatrix}-1000\\0\\-2000\end{pmatrix}=-1000\begin{pmatrix}1\\0\\2\end{pmatrix}$ $\mathbf{r}\bullet\mathbf{n}=\mathbf{a}\bullet\mathbf{n}$ $\mathbf{r}\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}=\begin{pmatrix}50\\0\\0\end{pmatrix}\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}\Rightarrow\mathbf{r}\bullet\begin{pmatrix}1\\0\\2\end{pmatrix}=50$ $\therefore x+2z=50$
11(ii) **[3]**	Flight path: $\mathbf{r}=\begin{pmatrix}60\\0\\100\end{pmatrix}+\lambda\begin{pmatrix}-2\\5\\0\end{pmatrix},\lambda\in\mathbb{R}$ Laser beam: $\mathbf{r}=\begin{pmatrix}30\\10\\1\end{pmatrix}+\mu\begin{pmatrix}1\\0\\2\end{pmatrix},\mu\in\mathbb{R}$ $\begin{pmatrix}60\\0\\100\end{pmatrix}+\lambda\begin{pmatrix}-2\\5\\0\end{pmatrix}=\begin{pmatrix}30\\10\\1\end{pmatrix}+\mu\begin{pmatrix}1\\0\\2\end{pmatrix}$ $2\lambda+\mu=30$ $5\lambda=10$ $2\mu=99$

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td></td><td>

<u>Method 1: GC (PlySmlt2 app)</u>



Using GC, there is no unique solution for λ and μ. Therefore, the laser beam **<u>does not intersect</u>** the flight path of the helicopter.

<u>Method 2: Algebraic</u>

$2\lambda + \mu = 30$ ---(1)

$5\lambda = 10$ ---(2)

$2\mu = 99$ ---(3)

From (2): $\lambda = 2$

From (3): $\mu = 49.5$

Sub into (1): $2(2) + 49.5 = 53.5 \neq 30$

There is no unique values for λ and μ that satisfies the equations. Therefore, the laser beam **<u>does not intersect</u>** the flight path of the helicopter.

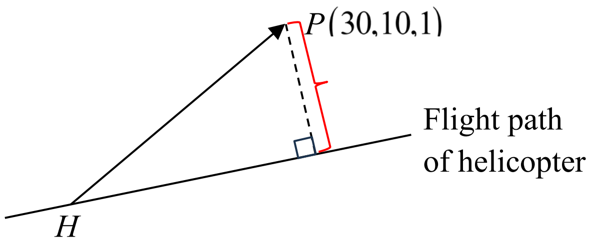
</td></tr>
<tr><td>

11(iii)
[3]

</td><td>

<u>Method 1: Direct</u>

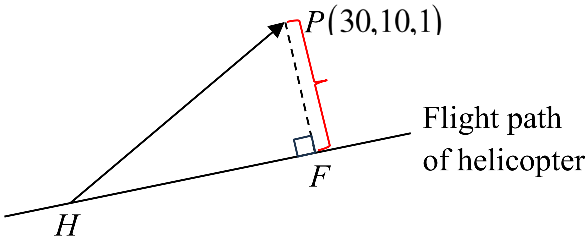
The shortest possible distance is the perpendicular distance from point P to the line (flight path of helicopter).



Let H be at $(60,\ 0,\ 100)$.

$$\overrightarrow{HP} = \begin{pmatrix} 30 \\ 10 \\ 1 \end{pmatrix} - \begin{pmatrix} 60 \\ 0 \\ 100 \end{pmatrix} = \begin{pmatrix} -30 \\ 10 \\ -99 \end{pmatrix}$$

</td></tr>
</table>

Qn	Solution
	Shortest distance from P to flight path $=\frac{1}{\sqrt{(-2)^2+5^2}}\left\|\begin{pmatrix}-30\\10\\-99\end{pmatrix}\times\begin{pmatrix}-2\\5\\0\end{pmatrix}\right\|$ $=\frac{1}{\sqrt{29}}\left\|\begin{pmatrix}495\\198\\-130\end{pmatrix}\right\|$ $=101.90$ (5 s.f.) ≈ 102 (nearest metre)
	<u>Method 2: Find foot of perpendicular</u>  Let F be the foot of perpendicular from P to the flight path. $\overrightarrow{OF}=\begin{pmatrix}60\\0\\100\end{pmatrix}+\lambda\begin{pmatrix}-2\\5\\0\end{pmatrix}$ for some $\lambda\in\mathbb{R}$ $\overrightarrow{PF}=\begin{pmatrix}60\\0\\100\end{pmatrix}+\lambda\begin{pmatrix}-2\\5\\0\end{pmatrix}-\begin{pmatrix}30\\10\\1\end{pmatrix}=\begin{pmatrix}30-2\lambda\\5\lambda-10\\99\end{pmatrix}$ $\overrightarrow{PF}\bullet\underset{\sim}{d}=0$ $\begin{pmatrix}30-2\lambda\\5\lambda-10\\99\end{pmatrix}\bullet\begin{pmatrix}-2\\5\\0\end{pmatrix}=0$ $-60+4\lambda+25\lambda-50=0$ $29\lambda=110$ $\lambda=3.7931$ (5 s.f.)

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td></td><td>

$\left|\overrightarrow{PF}\right| = \left|\begin{pmatrix} 30-2(3.7931) \\ 5(3.7931)-10 \\ 99 \end{pmatrix}\right|$

$= \left|\begin{pmatrix} 22.4138 \\ 8.9655 \\ 99 \end{pmatrix}\right|$

$= \sqrt{22.4138^2 + 8.9655^2 + 99^2}$

$= 101.90$ (5 s.f.)

=102 (nearest metre)

</td></tr>
<tr><td>10(iv)
[3]</td><td>

<u>Method 1 (Using perp distance formula)</u>

Let the equation of the second shield be $\mathbf{r}\bullet\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = D$

$\dfrac{\left|D-(50)\right|}{\sqrt{1^2+2^2}} = 3$

$\left|D-(50)\right| = 3\sqrt{5}$

$D-50 = 3\sqrt{5}$ or $D-50 = -3\sqrt{5}$

$D = 50+3\sqrt{5}$ or $D = 50-3\sqrt{5}$

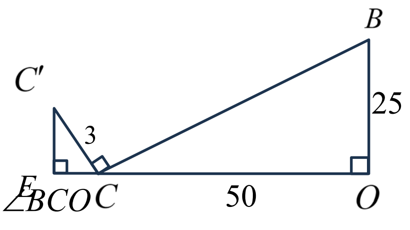
Since the second shield is to be placed further away, then $D = 50+3\sqrt{5}$.

Cartesian equation is: $x+2z = 50+3\sqrt{5}$

<hr>

<u>Method 2 (Form $\overrightarrow{OC'}$ using trigo)</u>

Let $A'B'C'D'$ be the 2nd protective shield, where the points A', B', C', D' are 3m from the points *A, B, C* and *D* respectively.



$\angle C'CE = \pi - \dfrac{\pi}{2} - \angle BCO$

$= \dfrac{\pi}{2} - \tan^{-1}\left(\dfrac{25}{50}\right)$

$= 1.1071$

</td></tr>
</table>

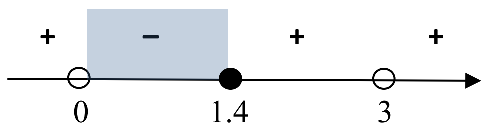
Qn	Solution
	$\sin\angle C'CE = \frac{C'E}{3} \Rightarrow C'E = 3\sin(1.1071) = 2.6833$ $\cos\angle C'CE = \frac{CE}{3} \Rightarrow CE = 3\cos(1.1071) = 1.3416$ $\overrightarrow{OC'} = \begin{pmatrix} 50+CE \\ 0 \\ C'E \end{pmatrix} = \begin{pmatrix} 50+1.3416 \\ 0 \\ 2.6833 \end{pmatrix} = \begin{pmatrix} 51.3416 \\ 0 \\ 2.6833 \end{pmatrix}$ $\mathbf{r}\bullet\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 51.3416 \\ 0 \\ 2.6833 \end{pmatrix}\bullet\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ $= 51.3416 + 5.3666$ $= 56.7082$ $= 56.7$ (3 s.f.) Cartesian equation is: $x + 2z = 56.7$ **Method 3 (Form $\overrightarrow{OC'}$ by unit normal vector)** Let $A'B'C'D'$ be the 2nd protective shield, where the points A', B', C', D' are 3m from the points *A, B, C* and *D* respectively. $\overrightarrow{OC'} = \overrightarrow{OC} + 3\underset{\sim}{\hat{n}} = \begin{pmatrix} 50 \\ 0 \\ 0 \end{pmatrix} + 3\left[\frac{1}{\sqrt{1^2+2^2}}\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}\right] = \begin{pmatrix} 50+\frac{3}{\sqrt{5}} \\ 0 \\ \frac{6}{\sqrt{5}} \end{pmatrix}$ $\mathbf{r}\bullet\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 50+\frac{3}{\sqrt{5}} \\ 0 \\ \frac{6}{\sqrt{5}} \end{pmatrix}\bullet\begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$ $= 50 + \frac{3}{\sqrt{5}} + \frac{12}{\sqrt{5}}$ $= 50 + \frac{15}{\sqrt{5}}$ $= 50 + 3\sqrt{5}$ Cartesian equation is: $x + 2z = 56.7$

End of Paper

2024 PU3 H2 MATHEMATICS
PRELIM PAPER 2 Solutions

Paper 9758/02

Section A: Pure Mathematics

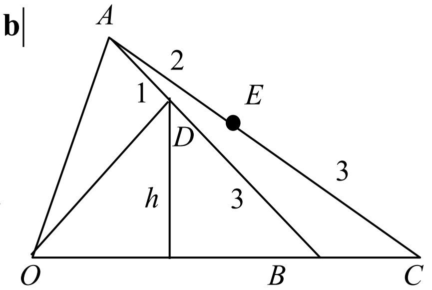
Qn	Solution
1 **[4]**	$\dfrac{x+5}{(x-3)^2} \geq \dfrac{x-7}{x(x-3)}$ $\dfrac{x(x+5)-(x-7)(x-3)}{x(x-3)^2} \geq 0$ $\dfrac{15x-21}{x(x-3)^2} \geq 0$ $\dfrac{5x-7}{x} \geq 0 \quad \left(\because (x-3)^2 \geq 0\right)$ $x<0$ or $x \geq \dfrac{7}{5}, x \neq 3$ 
1 **Hence** **[2]**	$\dfrac{x+5}{(x-3)^2} \geq \dfrac{x-7}{x(x-3)}$ Replace x by $-x$ $\dfrac{(-x)+5}{(-x-3)^2} \geq \dfrac{(-x)-7}{-x(-x-3)}$ $\dfrac{5-x}{(x+3)^2} \geq \dfrac{-x-7}{x(x+3)}$ $\dfrac{x-5}{(x+3)^2} \leq \dfrac{x+7}{x(x+3)}$ Hence, $-x<0$ or $-x \geq \dfrac{7}{5}, x \neq 3$ $x>0$ or $x \leq -\dfrac{7}{5}, x \neq -3$

Qn	Solution
2 (i) **[5]**	$\int x^2 e^{3x}\ dx$ $= x^2\left(\frac{1}{3}e^{3x}\right) - \int 2x\left(\frac{1}{3}e^{3x}\right) dx$ $= \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}\ dx + c_1,$ c_1 is an arbitary constant Let $u = x^2$, $\frac{dv}{dx} = e^{3x}$ $\frac{du}{dx} = 2x$, $v = \frac{1}{3}e^{3x}$ $\frac{2}{3}\int xe^{3x}\ dx$ $= \frac{2}{3}\left[x\left(\frac{1}{3}e^{3x}\right) - \int(1)\left(\frac{1}{3}e^{3x}\right) dx\right]$ $= \frac{2}{3}\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\right) + c_2,$ c_2 is an arbitary constant $= \frac{2}{9}xe^{3x} - \frac{2}{27}e^{3x} + c_2$ Let $u = x$, $\frac{dv}{dx} = e^{3x}$ $\frac{du}{dx} = 1$, $v = \frac{1}{3}e^{3x}$ $\int x^2e^{3x}\ dx$ $= \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + c, c$ is an arbitary constant
2 (ii) **[2]**	$\int_0^k x^2e^{3x}\ dx$ $= \left[\frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x}\right]_0^k$ $= \frac{1}{27}\left[e^{3x}\left(9x^2 - 6x + 2\right)\right]_0^k$ $= \frac{1}{27}\left[e^{3k}\left(9k^2 - 6k + 2\right) - e^0\left(0 - 0 + 2\right)\right]$ $= \frac{1}{27}\left[e^{3k}\left(9k^2 - 6k + 2\right) - 2\right]$ or $e^{3k}\left(\frac{1}{3}k^2 - \frac{2}{9}k + \frac{2}{27}\right) - \frac{2}{27}$

Qn	Solution
3(i) **[5]**	$w=x+y$ $y=w-x---(1)$ Diff y wrt to x $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\mathrm{d}w}{\mathrm{d}x}-1---(2)$ Given $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{3-(x+y)}{1+x+y}---(3)$ Sub (1) and (2) into (3) $\frac{\mathrm{d}w}{\mathrm{d}x}-1=\frac{3-w}{1+w}$ $\frac{\mathrm{d}w}{\mathrm{d}x}=\frac{3-w}{1+w}+1$ $\frac{\mathrm{d}w}{\mathrm{d}x}=\frac{3-w+1+w}{1+w}$ $\frac{\mathrm{d}w}{\mathrm{d}x}=\frac{4}{1+w}$ (shown) $\int 1+w\ \mathrm{d}w=\int 4\ \mathrm{d}x$ $w+\frac{w^2}{2}=4x+c$, where c is an arbitary constant Sub $w=x+y$ $(x+y)+\frac{(x+y)^2}{2}=4x+c$ $2x+2y+(x+y)^2=8x+2c$ $(x+y)^2=2(3x-y)+A$, where $A=2c$
3 (ii) **[2]**	The curves passes through $(0,5)$, $(0+5)^2=2(0-5)+A$ $A=35$ The particular solution: $\therefore (x+y)^2=2(3x-y)+35$

	Let the other y-intercept be $(0,k)$ $(0+k)^2=2(0-k)+35$ $k^2+2k-35=0$ $(k+7)(k-5)=0$ $k=-7$ or 5 (rej) Coordinates of the other y-intercept: $(0,-7)$

Qn	Solution
4(i) **[1]**	Using ratio theorem, $\overrightarrow{OE}=\dfrac{3\overrightarrow{OA}+2\overrightarrow{OC}}{5}=\dfrac{3\overrightarrow{OA}+2p\overrightarrow{OB}}{5}=\dfrac{3\mathbf{a}+2p\mathbf{b}}{5}$
4(ii) **[4]**	<u>Method 1</u> Using ratio theorem, $\overrightarrow{OD}=\dfrac{3\overrightarrow{OA}+\overrightarrow{OB}}{4}=\dfrac{3\mathbf{a}+\mathbf{b}}{4}$ The shortest distance from D to OB $\left\lvert\overrightarrow{OD}\times\dfrac{\overrightarrow{OB}}{\lvert\overrightarrow{OB}\rvert}\right\rvert=\left\lvert\left(\dfrac{3\mathbf{a}+\mathbf{b}}{4}\right)\times\dfrac{\mathbf{b}}{5}\right\rvert$ $=\dfrac{1}{20}\lvert(3\mathbf{a}+\mathbf{b})\times\mathbf{b}\rvert$ $=\dfrac{1}{20}\lvert(3\mathbf{a}\times\mathbf{b})+(\mathbf{b}\times\mathbf{b})\rvert$ $=\dfrac{1}{20}\lvert 3\mathbf{a}\times\mathbf{b}\rvert\quad(\because\ \mathbf{b}\times\mathbf{b}=\mathbf{0})$ $=\dfrac{3}{20}\lvert\mathbf{a}\times\mathbf{b}\rvert$ [AG1, show $\mathbf{b}\times\mathbf{b}=\mathbf{0}$]

OR 4(ii) **[4]**	Method 2 Area of $\Delta OAB = \frac{1}{2}\lvert \mathbf{a}\times\mathbf{b}\rvert$ Area of $\Delta ODB = \frac{3}{4}\times$ Area of ΔOAB $= \frac{3}{4}\times\frac{1}{2}\lvert \mathbf{a}\times\mathbf{b}\rvert$ $= \frac{3}{8}\lvert \mathbf{a}\times\mathbf{b}\rvert$  Area of $\Delta ODB = \frac{1}{2}\times\lvert\mathbf{b}\rvert\times h$ $\frac{3}{8}\lvert \mathbf{a}\times\mathbf{b}\rvert = \frac{1}{2}\times 5\times h$ $h = \frac{2}{5}\times\frac{3}{8}\lvert \mathbf{a}\times\mathbf{b}\rvert$ $h = \frac{3}{20}\lvert \mathbf{a}\times\mathbf{b}\rvert$ $\therefore k = \frac{3}{20}$
4(iii) **[4]**	Method 1 Area $\Delta OCD = \frac{1}{2}\left\lvert \overrightarrow{OC}\times\overrightarrow{OD}\right\rvert$ $= \frac{1}{2}\left\lvert p\mathbf{b}\times\left(\frac{3\mathbf{a}+\mathbf{b}}{4}\right)\right\rvert$ $= \frac{1}{8}\left\lvert (p\mathbf{b}\times 3\mathbf{a}) + (p\mathbf{b}\times p\mathbf{b})\right\rvert$ $= \frac{1}{8}\left\lvert 3p(\mathbf{b}\times\mathbf{a}) + p^2(\mathbf{b}\times\mathbf{b})\right\rvert$ $= \frac{3p}{8}\lvert \mathbf{a}\times\mathbf{b}\rvert \quad (\because \mathbf{b}\times\mathbf{b}=\mathbf{0})$ Area $\Delta OCE = \frac{1}{2}\left\lvert \overrightarrow{OC}\times\overrightarrow{OE}\right\rvert$ $= \frac{1}{2}\left\lvert p\mathbf{b}\times\left(\frac{3\mathbf{a}+2p\mathbf{b}}{5}\right)\right\rvert$ $= \frac{1}{10}\left\lvert (p\mathbf{b}\times 3\mathbf{a}) + (p\mathbf{b}\times 2p\mathbf{b})\right\rvert$ $= \frac{1}{10}\left\lvert 3p(\mathbf{b}\times\mathbf{a}) + 2p^2(\mathbf{b}\times\mathbf{b})\right\rvert$ $= \frac{3p}{10}\lvert \mathbf{a}\times\mathbf{b}\rvert \quad (\because \mathbf{b}\times\mathbf{b}=\mathbf{0})$

	$\dfrac{\text{Area of } \Delta OCD}{\text{Area of } \Delta OCE}=\dfrac{\frac{3p}{8}\lvert\mathbf{a}\times\mathbf{b}\rvert}{\frac{3p}{10}\lvert\mathbf{a}\times\mathbf{b}\rvert}=\dfrac{5}{4}$ (shown)
OR 4(iii) [4]	<u>Method 2</u> $\text{Area of } \Delta OCE=\dfrac{3}{5}\times\text{Area of } \Delta OAC$ $=\dfrac{3}{5}\times\dfrac{1}{2}\lvert\mathbf{a}\times\mathbf{c}\rvert$ $=\dfrac{3}{5}\times\dfrac{1}{2}\lvert\mathbf{a}\times p\mathbf{b}\rvert$ $=\dfrac{3p}{10}\lvert\mathbf{a}\times\mathbf{b}\rvert$ $\dfrac{\text{Area of } \Delta OCD}{\text{Area of } \Delta OCE}=\dfrac{\frac{1}{2}\times\lvert\overrightarrow{OC}\rvert\times h}{\frac{3p}{10}\lvert\mathbf{a}\times\mathbf{b}\rvert}$ $=\dfrac{\frac{1}{2}\times p\lvert\mathbf{b}\rvert\times\frac{3}{20}\lvert\mathbf{a}\times\mathbf{b}\rvert}{\frac{3p}{10}\lvert\mathbf{a}\times\mathbf{b}\rvert}$ $=\dfrac{\frac{1}{2}\times 5p\times\frac{3}{20}\lvert\mathbf{a}\times\mathbf{b}\rvert}{\frac{3p}{10}\lvert\mathbf{a}\times\mathbf{b}\rvert}$ $=\dfrac{5}{4}$

Qn	Solution
5(a) **[3]**	$\dfrac{\cos 2x}{(1+\tan x)^2}$ $\approx \left(\dfrac{1-\dfrac{(2x)^2}{2!}}{(1+x)^2}\right)$ $=(1-2x^2)(1+x)^{-2}$ $=(1-2x^2)\left(1+(-2)(x)+\dfrac{(-2)(-3)}{2!}(x)^2+\dots\right)$ $=(1-2x^2)(1-2x+3x^2+\dots)$ $=1-2x+x^2+\dots$
5(b) **(i)** **[1]**	By GC: $\mathrm{f}'(0.01)=\dfrac{\mathrm{d}}{\mathrm{d}x}\left[(1-x)^{\frac{1}{2}}\cdot\ln\sqrt{1+x}\right]\Big\|_{x=0.01}$ $=0.490068=\ 0.49007$ (5 dp)
5(b) **(ii)** **[4]**	$\mathrm{f}(x)=(1-x)^{\frac{1}{2}}\left(\ln\sqrt{1+x}\right)$ $=(1-x)^{\frac{1}{2}}\left(\dfrac{1}{2}\ln(1+x)\right)$ $=\left(1+\dfrac{1}{2}(-x)+\dfrac{\left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)}{2}(-x)^2\right)\left(\dfrac{1}{2}\right)\left(x-\dfrac{1}{2}x^2+\dfrac{1}{3}x^3+\dots\right)$ $=\dfrac{1}{2}\left(1-\dfrac{1}{2}x-\dfrac{1}{8}x^2+\dots\right)\left(x-\dfrac{1}{2}x^2+\dfrac{1}{3}x^3+\dots\right)$ $=\dfrac{1}{2}x-\dfrac{1}{4}x^2+\dfrac{1}{6}x^3-\dfrac{1}{4}x^2+\dfrac{1}{8}x^3-\dfrac{1}{16}x^3+\dots$ $=\dfrac{1}{2}x-\dfrac{1}{2}x^2+\dfrac{11}{48}x^3+\dots$ Valid range of expansion $\|x\|<1$ and $-1<x\le 1$ $-1<x<1$

5(b) (iii) [2]	Let $g(x)=\frac{1}{2}x-\frac{1}{2}x^2+\frac{11}{48}x^3+...$ $g'(x)=\frac{1}{2}-x+\frac{11}{16}x^2+...$ When $x=0.01$ $g'(0.01)=\frac{1}{2}-(0.01)+\frac{11}{16}(0.01)^2$ $g'(0.01)=0.490069=$ 0.49007 (5 dp)
5(b) (iv) [1]	The answer is valid as the $x=0.01$ is small and **within the valid range** of expansion.

Section B: Probability and Statistics

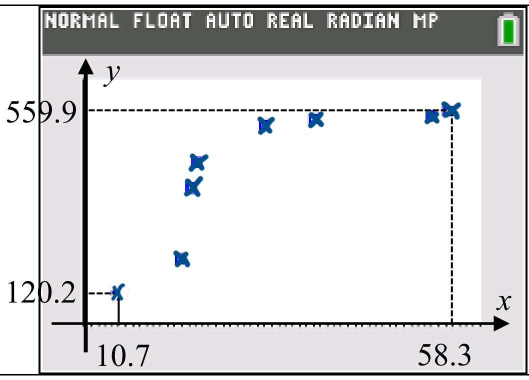
Qn	Solution
6(i) [2] [1]	Number of ways $=2!\times 8!=80640$. No. (i.e. my answer will not be different OR my answer is still the same) Possible justification: • Same working (mention or calculate) • Redefine at the ends and at the centre • Renumbering of the 10 positions
6(ii) [1]	Possible explanation: • The event of choosing the one teacher from 2 is not independent to that of choosing 3 from the remaining 9 teacher and students. • This will lead to overcounting in cases where the second teacher is one of the 3 people chosen from the remaining 9. • Give specific example of overcounting: Choosing T1, followed by T2, M1, M2 gives the same team as choosing T2, followed by T1, M1, M2. • Give the correct solution: Answer should be 140.
6(iii) [3]	Number of ways without restriction = ${}^{10}C_4=210$. **<u>Method 1: Complement</u>** Number of ways with no teachers = ${}^{8}C_4=70$. Required probability $=\frac{210-70}{210}=\frac{140}{210}=\frac{2}{3}$.

	Method 2: Direct cases Number of ways with 1 teacher = ${}^{2}C_{1} \times {}^{8}C_{3} = 112$. Number of ways with 2 teachers = ${}^{2}C_{2} \times {}^{8}C_{2} = 28$. Required probability $= \dfrac{112+28}{210} = \dfrac{140}{210} = \dfrac{2}{3}$.

Qn	Solution
7(i) **[1]**	Let X be the score. $\mathrm{P}(X = 4) = p - q > 0$ (since $p \neq q$) Hence, $\mathrm{P}(X = 1) = p > q = \mathrm{P}(X = 2)$.
7(ii) **[1]**	Total probability = $1\,(p)+(q)+0.1+(p-q)+0.2+(p+q)=1$ Hence, $3p+q=0.7$ (shown)
7(iii) **[5]**	$\mathrm{E}(X) = 4.6$ $(1\times p) + (2\times q) + 3(0.1) + 4(p - q) + 5(0.2) + 8(p + q) = 4.6$ $p + 2q + 0.3 + 4(p - q) + 1 + 8(p + q) = 4.6$ $13p + 6q + 1.3 = 4.6$ $13p + 6q = 3.3$ … (1) $3p+q=0.7$ … (2) From (ii) From graphing calculator, $p = 0.18$, $q = 0.16$. $\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$ $= [(1^2\times p) + (2^2\times q) + 3^2(0.1) + 4^2(p - q) + 5^2(0.2)$ $+ 8^2(p + q)] - (4.6)^2 = 7.64$

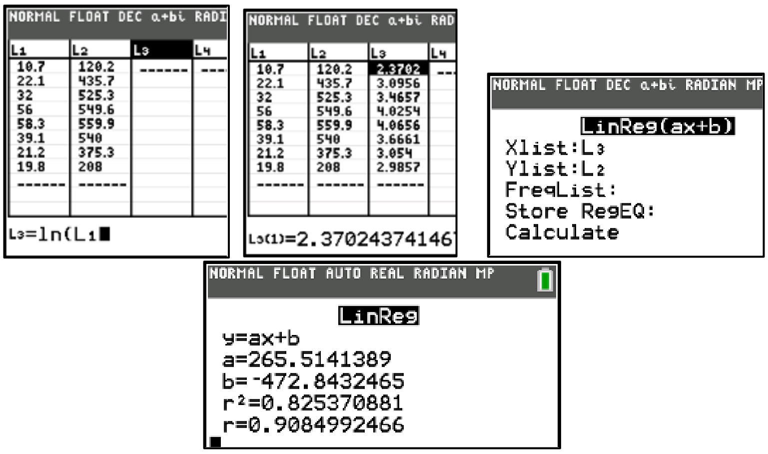
<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>8(a)(i)
[2]</td><td>$X \sim \mathrm{B}(30, 0.8)$
$\mathrm{P}(X \geq 25) = 1 - \mathrm{P}(X \leq 24)$
$= 1 - 0.57249$
$= 0.42751$
$= 0.428$ (3 s.f.)</td></tr>
<tr><td>8(a)(ii)
[3]</td><td>Let W be the number of months which James solves at least 25 chess puzzles per month out of 36 months.
$W \sim \mathrm{B}(36\ ,\ 0.42751)$
$\mathrm{P}(W = n) > 0.13$
By GC:
<table><tr><th>n</th><th>$\mathrm{P}(W = n)$</th></tr><tr><td>14</td><td>0.1212</td></tr><tr><td>15</td><td>0.1327 > 0.13</td></tr><tr><td>16</td><td>0.1301 > 0.13</td></tr><tr><td>17</td><td>0.1143</td></tr></table>
$n = 15$ or 16</td></tr>
<tr><td>8(b)(i)
[2]</td><td>Two assumptions:
1. The event that a resident in the village is infected by a skin disease is independent of other residents.
2. The probability that a resident in the village is infected by a skin disease is the same for each resident.</td></tr>
<tr><td>8(b)(ii)
[3]</td><td>If $\dfrac{\mathrm{P}(Y = y+1)}{\mathrm{P}(Y = y)} > 1$
$\left(\dfrac{n-y}{y+1}\right)\left(\dfrac{p}{1-p}\right) > 1$
Given that $n = 15,\ p = 0.7$
$\left(\dfrac{15-y}{y+1}\right)\left(\dfrac{0.7}{1-0.7}\right) > 1$
$(15-y)(0.7) > (y+1)(0.3) \quad \left[\because (y+1) > 0\right]$
$7(15-y) > 3(y+1)$
$105 - 7y > 3y + 3$
$y < 10.2$</td></tr>
</table>

	When $y=10$, $\frac{\mathrm{P}(Y=10+1)}{\mathrm{P}(Y=10)}=\left(\frac{15-10}{10+1}\right)\left(\frac{0.7}{1-0.7}\right)=1.0606>1$ $\mathrm{P}(Y=11)>\mathrm{P}(Y=10)>...>\mathrm{P}(Y=0)$ When $y=11$, $\frac{\mathrm{P}(Y=11+1)}{\mathrm{P}(Y=11)}=\left(\frac{15-11}{11+1}\right)\left(\frac{0.7}{1-0.7}\right)=0.777<1$ $\frac{\mathrm{P}(Y=11+1)}{\mathrm{P}(Y=11)}<1$ $\mathrm{P}(Y=11)>\mathrm{P}(Y=12)>...>\mathrm{P}(Y=15)$ $\mathrm{P}(Y=11)$ has the highest probability. Hence, the most likely number of residents infected with the skin disease is 11.

<table>
<tr><th>Qn</th><th>Solution</th></tr>
<tr><td>9(i)
[1]</td><td>


</td></tr>
<tr><td>9(ii)
[1]</td><td>
From graphing calculator,
product moment correlation coefficient (for $y = ax + b$)
$\approx 0.823690 = 0.8237$. (4 d.p.)

</td></tr>
<tr><td>9(iii)
[2]</td><td>
From graphing calculator,
product moment correlation coefficient (for $y = c\ln x + d$)
$\approx 0.908499 = 0.9085$. (4 d.p.)



Since $r = 0.9085$ is closer to 1 compared to $r = 0.8237$, this indicates a stronger linear correlation between the variables. Hence Ian's claim is valid.
</td></tr>
<tr><td>9(iv)
[2]</td><td>
From graphing calculator,
$a \approx 7.9473 = 7.95$. (3 s.f.)
$b \approx 156.76 = 157$. (3 s.f.)
$c \approx 265.51 = 266$. (3 s.f.)
$d \approx -472.84 = -473$. (3 s.f.)
</td></tr>
<tr><td>9(v)</td><td>From graphing calculator, when $x = 10$,</td></tr>
</table>

[3]	$y \approx 265.51\ln(10) - 472.84 \approx 138.526 = 138.5$ (1 d.p) Hence, the required estimated average daily number of people that are hospitalised is 138.5.
9(vi) [1]	Not reliable as $x = 10$ is outside the data range of x.

Qn	Solution
10(i) [2]	Let X be the volume, in ml, of soft drink dispensed by the machine in a randomly chosen can and μ be the population mean volume, in ml, of soft drink in the cans that is dispensed by the machine. Null Hypothesis, $H_0: \mu = 330$ Alternative Hypothesis, $H_1: \mu \neq 330$
10(ii) [1]	Suggested reasons: The sample that is obtained this way is not random. The sample size of 8 is not large. Other acceptable reasons: The population variance is unknown. The population is not normal. The distribution is unknown.
10(iii) [5]	Unbiased estimate of population mean, $\bar{x} = \frac{-60}{80} + 330 = \frac{1317}{4} = 329.25.$ Unbiased estimate of population variance, $s^2 = \frac{1}{79}\left[450 - \frac{(-60)^2}{80}\right] = \frac{405}{79} \approx 5.1266 = 5.13.$ (3 s.f.) Under H_0, since $n = 80$ is large, by Central Limit Theorem, $\bar{X} \sim \mathrm{N}\left(330, \frac{81}{1264}\right)$ or $\mathrm{N}\left(330, \frac{5.1266}{80}\right)$ approximately. Use a z-test at $\alpha = 0.01$. From graphing calculator, p-value $\approx 0.0030494 = 0.00305$. (3 s.f.) Since p-value $= 0.00305 < 0.01$, we reject H_0 and conclude that there is sufficient evidence, at the 1% level of significance that the dispensing machine is not set correctly.
10(iv) [4]	Let Y be the volume, in ml, of soft drink dispensed by the new dispensing machine in a randomly chosen can and μ be the population mean volume, in ml, of soft drink in the cans that is dispensed by the new dispensing machine. $H_0: \mu = 330$ $H_1: \mu > 330$ Under H_0, $\bar{Y} \sim \mathrm{N}\left(330, \frac{3}{n}\right)$. Use a z-test at $\alpha = 0.05$. --

Method 1: Critical region method

Under H_0, test statistics $Z = \dfrac{\overline{Y} - 330}{\sqrt{\dfrac{3}{n}}} \sim \mathrm{N}\left(0,1\right)$.

Value of test statistics, $z = \dfrac{330.33 - 330}{\sqrt{\dfrac{3}{n}}} = \dfrac{0.33}{\sqrt{\dfrac{3}{n}}} = 0.33\sqrt{\dfrac{n}{3}}$.

Critical region: $z \geq 1.8808$ (5 s.f.)

Since we reject H_0 at 5% level of significance,

$0.33\sqrt{\dfrac{n}{3}} \geq 1.8808 \Rightarrow n \geq 97.449$

OR using table from graphing calculator, we have

n	$0.33\sqrt{\dfrac{n}{3}}$
97	1.8765 < 1.8808
98	1.8861 > 1.8808
99	1.8957 > 1.8808

Hence, least value of n is 98.

Method 2: p-value method

Since we reject H_0 at 5% level of significance,

p-value = $\mathrm{P}(\overline{Y} > 330.33) \leq 0.03$.

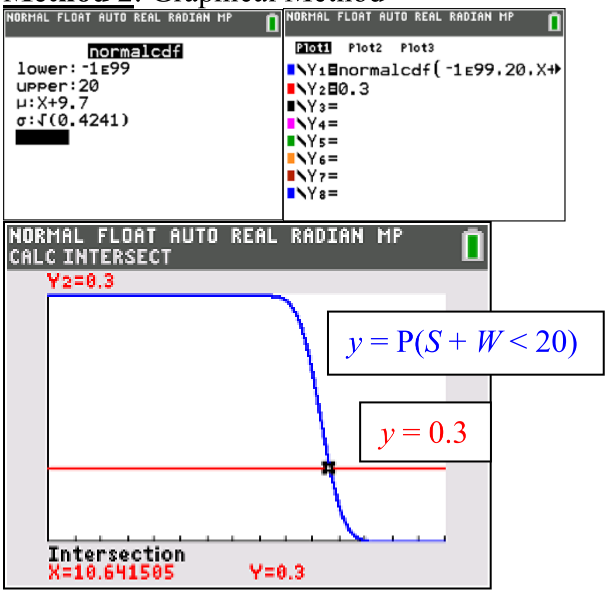
From graphing calculator,

n	$\mathrm{P}(\overline{Y} > 330.33)$
97	0.0303 > 0.03
98	0.0296 < 0.03
99	0.0290 < 0.03

Hence, least value of n is 98.

Qn	Solution
11(i) **[2]**	$X \sim \mathrm{N}(7.8, 0.25^2)$, $Y \sim \mathrm{N}(1.9, 0.04^2)$. Let $W = X + Y$. $\mathrm{E}(W) = \mathrm{E}(X) + \mathrm{E}(Y) = 7.8 + 1.9 = 9.7$. $\mathrm{Var}(W) = \mathrm{Var}(X) + \mathrm{Var}(Y) = 0.25^2 + 0.04^2 = 0.0641$. Then $W \sim \mathrm{N}(9.7, 0.0641)$ Required probability = $\mathrm{P}(W > 10) \approx 0.11802 = 0.118$. (3 s.f.)
11(ii) **[3]**	Required probability = $\mathrm{P}(X > 4Y) = \mathrm{P}(X - 4Y > 0)$. $\mathrm{E}(X - 4Y) = \mathrm{E}(X) - 4\mathrm{E}(Y) = 7.8 - 4(1.9) = 0.2$. $\mathrm{Var}(X - 4Y) = \mathrm{Var}(X) + 4^2\mathrm{Var}(Y)$ $= 0.25^2 + 16(0.04^2) = 0.0881$. $X - 4Y \sim \mathrm{N}(0.2, 0.0881)$ Required probability = $\mathrm{P}(X - 4Y > 0)$ $\approx 0.74979 = 0.750$. (3 s.f.)
11(iii) **[1]**	Assume that the mass of creamer is independent of the mass of coffee powder in a randomly chosen 2-in-1 stick of instant coffee OR Assume that X and Y are independent.
11(iv) **[3]**	Let $T = W_1 + W_2 + \ldots + W_{50}$. $\mathrm{E}(T) = \mathrm{E}(W_1 + W_2 + \ldots + W_{50}) = 50\mathrm{E}(W) = 50(9.7) = 485$. $\mathrm{Var}(T) = \mathrm{Var}(W_1 + W_2 + \ldots + W_{50})$ $= 50\mathrm{Var}(W) = 50(0.0641) = 3.205$. $T \sim \mathrm{N}(485, 3.205)$ Required probability = $\mathrm{P}(T < 488)$ $\approx 0.95311 = 0.953$. (3 s.f.)
11(v) **[1]**	The calculations in part **(iv)** is still valid because 50 is large, so by Central Limit Theorem, T is approximately normal.
11(vi) **[4]**	Let S be the mass, in grams, of sugar in a randomly chosen 3-in-1 stick of instant coffee. Then $S \sim \mathrm{N}(\mu, 0.36)$. $\mathrm{E}(S + W) = \mathrm{E}(S) + \mathrm{E}(W) = \mu + 9.7$. $\mathrm{Var}(S + W) = \mathrm{Var}(S) + \mathrm{Var}(W) = 0.36 + 0.0641 = 0.4241$. $S + W \sim \mathrm{N}(\mu + 9.7, 0.4241)$ Now, $\mathrm{P}(S + W < 20) = 0.3$ -- **<u>Method 1</u>**: Standardisation $\Rightarrow \mathrm{P}\left(Z < \dfrac{20-(\mu+9.7)}{\sqrt{0.4241}}\right) = 0.3$, where $Z \sim \mathrm{N}(0, 1)$ From graphing calculator, $\dfrac{10.3-\mu}{\sqrt{0.4241}} \approx -0.52440 \Rightarrow \mu \approx 10.3 + 0.5244\sqrt{0.4241}$ $\approx 10.642 = 10.6$. (3 s.f.) --

<u>Method 2</u>: Graphical Method



From graphing calculator,

$\mu \approx 10.642 = 10.6$. (3 s.f.)

THE END