MERIDIAN JUNIOR COLLEGE

JC2 Preliminary Examination

Higher 2

H2 Mathematics 9740/02

Paper 2 **17 September 2012**

3 Hours

Additional Materials: Writing paper
List of Formulae (MF 15)

READ THESE INSTRUCTIONS FIRST

Write your name and civics group on all the work you hand in.
Write in dark blue or black pen on both sides of the paper.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use a graphic calculator.
Unsupported answers from a graphic calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphic calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

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Section A: Pure Mathematics [40 marks]

1 A lake in Pangaea has a population of 600 000 adult fish at the start of 2012 and it is increasing at a rate of approximately 5% per year. It is proposed that at the end of every year 40 000 adult fish should be harvested.

Let u_n denote the size of the population (in thousands) at the start of n years after 2012.

(i) Write down a recurrence relation for u_n, and show that $u_n = 800 - 200(1.05)^n$. [4]

(ii) Hence find the predicted population of the adult fish at the start of 2020. State what happens to the population of the adult fish for large values of *n*. [2]

(iii) If the population of adult fish is to be maintained at 600 000, what should be the proposed number of adult fish to be harvested at the end of every year? [1]

2 The functions f and g are defined by

$$\mathrm{f} : x \mapsto \left|(2x+1)(2x-9)\right|, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto \ln(x+2), \qquad x > -2.$$

(i) Given that f^{-1} exist when domain is restricted to [*a*, *b*] where *a* < 0 and *b* > 0, find the least value of *a* and greatest value of *b*. [1]

Using the restricted domain found in part **(i)**,

(ii) find the domain of f^{-1} and the expression for $\mathrm{f}^{-1}(x)$, [5]

(iii) find the range of the composite function gf, leaving your answer in exact form. [2]

3 A curve *C* is defined by parametric equations

$$x = 4t - t^3 \qquad y = t^2 + 3t\ .$$

(i) Find the equation of the tangent at $t = -1$. [2]

(ii) Given that the normal at another point *P* on the curve is perpendicular to the tangent found in part **(i)**, find the value of *t* at point *P*. [2]

(iii) Sketch the graph *C* for $t \in [-1, 3]$, labeling the exact coordinates of the point where the gradient is undefined and the end points of *C*. [4]

[Turn Over]

4 The region R is bounded by the curve $y=\left(\mathrm{e}^{\frac{x}{2}}+1\right)^{2}$, the x-axis, the y-axis, and the line $x = a$, where $a > 0$.

(i) Find the area of the region R in terms of a. [3]

(ii) Hence find the exact value of $\pi\int_{0}^{2}\left[\mathrm{f}^{-1}(y)\right]^{2}\mathrm{d}y$, where $\mathrm{f}(x)=2\ln(x-1)$, and give a geometrical interpretation of the value found. [4]

5 The line l_1 passes through point A, whose position vector is $4\mathbf{i}+5\mathbf{j}-6\mathbf{k}$, and is parallel to the vector $-\mathbf{i}-2\mathbf{j}+3\mathbf{k}$. The plane p_1 has equation $x+2y-3z=4$.

(i) Show that l_1 is perpendicular to p_1. Hence find the coordinates of the foot of perpendicular from A to p_1. [3]

The line l_2 is given by $x+1=-y=z-1$.

(ii) Given that the plane p_2 is parallel to the line l_1 and contains the line l_2, show that the equation of p_2 is $x+4y+3z=2$. What is the relationship between the lines l_1 and l_2? [3]

(iii) The planes p_1 and p_2 intersect in a line l_3. Find the vector equation of the line l_3. [1]

(iv) The plane p_3 has equation $x-2y+7z-\beta-\alpha(2x+2y-z+1)=0$. Given that the three planes p_1, p_2 and p_3 have no point in common, what can be said about the values of α and β? [3]

Section B: Statistics [60 marks]

6 The table below shows the breakdown of the students in Tenaz Junior College.

	Males	Females
JC 1	480	560
JC 2	240	320

A sample of 100 students from different levels and gender is to be selected from the college to find out the amount of time they spent using the sports facilities.

(i) Roarbert plans to survey 25 students from each stratum. State the name of this sampling method. [1]

(ii) State a disadvantage in Roarbert's sampling method. [1]

(iii) Suggest another sampling method that would not have this disadvantage, and describe how it can be carried out. [3]

7 **(a)** Describe the difference in the hypothesis between a one-tailed test and a two-tailed test. [1]

(b) The mean weight of boys in a particular school is known to be m kg. A new weight gain programme was tried out on a random sample of 50 boys in this school. The weight of these 50 students, x, after the implementation of the new weight gain programme gave the following data:

$$\Sigma(x - 30) = 1279 \text{ and } \Sigma x^2 = 155233$$

A hypothesis test is carried out at the 5% significance level and it is found that the weight gain programme has been effective. Find the range of values of m. [5]

[Turn Over]

8 Three men, three women and a married couple are randomly seated at a round table with eight seats. Find the probability that

(i) the married couple is seated together, [3]

(ii) no two women are seated next to each other given that the married couple is seated together, [3]

giving each of your answers as a fraction in its lowest terms.

State, with a reason, whether or not the events 'no two women are seated next to each other' and 'the couple is seated together' are independent. [2]

9 A student wants to investigate the growth of mould on one side of a slice of expired bread measuring 8 cm by 8 cm. He placed the slice of bread in a sealed bag and measured the area of the bread covered by mould over 10 days. The following is the data recorded by the student.

Time, t days	0	2	3	4	6	8	9	10
Area, y cm^2	0.3	1.5	2.3	4.1	6.3	8.2	8.7	8.8

(i) Draw a scatter diagram to illustrate the data. [2]

(ii) Calculate the product moment correlation coefficient and use a suitable regression line to estimate the area of the bread covered by mould on the 7th day. Comment on the reliability of your answer. [4]

(iii) The student would like to use the regression line in part **(ii)** to predict the area of the bread covered by mould after 80 days. By calculating the predicted area of the mould, explain why a linear model may not be appropriate in the context of the question. [2]

10 The time taken for Miss Lau to wrap a large hamper is normally distributed with mean 18 minutes and standard deviation 4 minutes. The time taken for her to wrap a small hamper is normally distributed with mean 10 minutes and standard deviation σ minutes. The time taken by her to wrap different hampers may be assumed to be independent of one another.

(i) Given that the probability that she wraps a large hamper within 11 minutes is the same as the probability that she wraps a small hamper within 6.5 minutes, show that $\sigma = 2$. [2]

(ii) Find the probability that Miss Lau can wrap 2 randomly chosen small hampers faster than a randomly chosen large hamper. [4]

(iii) On a particular day, Miss Lau wrapped a total of 2 large hampers and *n* small hampers. Find the smallest possible value of *n* such that there is a probability of at least 0.7 that the mean time taken for her to wrap a hamper is less than 12 minutes. [4]

11 At PPQZ hospital, the number of babies delivered in a month is observed. State two conditions under which a Poisson distribution would be a suitable probability model. [2]

The average number of babies delivered in PPQZ hospital in a month is 5.

(i) Find the probability of more than 10 babies is delivered in a randomly chosen month. [2]

(ii) Given that the probability that at most *n* children are born in a month is less than 0.95, find the largest value of *n.* [2]

(iii) Find the probability that the average number of babies delivered in a month over 50 months is between 4 and 6. [2]

(iv) In another hospital, the average number of babies delivered in a month is 15. The number of babies delivered in a month from the two hospitals is independent. Find the probability that the total number of babies delivered in the two hospitals in a month is at most 11. [3]

[Turn Over]

12 **(a)** The random variable X is the number of successes in n independent trials of an experiment in which the probability of a success at a single trial is p. Denoting $\mathrm{P}(X = k)$ by p_k, show that $\dfrac{p_{k+1}}{p_k} = \dfrac{(n-k)p}{(k+1)(1-p)}$, $k = 0, 1, 2, \ldots, n-1$.

Hence find the most probable number of successes when n = 10 and $p = \dfrac{1}{3}$. [4]

(b) In a certain country, it is known that 30% of the adult population has some knowledge of a foreign language.

(i) Find the probability that, in a random sample of 8 adults, at most 2 have some knowledge of a foreign language. [1]

(ii) 400 adults are chosen at random. Use a suitable approximation to find the least value of n so that the probability that less than n adults having some knowledge of a foreign language is at least 0.9. [4]

For one particular foreign language, 99% of the adult population does not have some knowledge of it. Using a suitable approximation, find the probability that, in a random sample of 400 adults, more than 395 do not have some knowledge of the particular foreign language. [3]