XINMIN SECONDARY SCHOOL
新民中学
SEKOLAH MENENGAH XINMIN
Preliminary Examination 2024

CANDIDATE NAME	
CLASS	
INDEX NUMBER	

ADDITIONAL MATHEMATICS

4049/02

Paper 2

26 August 2024

Secondary 4 Express

2 hour 15 minutes

Setter: Ms Joanne Kong
Vetter: Ms Low Yan Jin
Moderator: Ms Pang Hui Chin

Candidates answer on the Question Paper

READ THESE INSTRUCTIONS FIRST

Write your name, index number and class in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total of the marks for this paper is **90**

Errors	Qn No.	Errors	Qn No.
Accuracy		Simplification	
Brackets		Units	
Geometry		***Marks Awarded***	
Presentation		***Marks Penalised***	

For Examiner's Use
90

Parent's/Guardian's Signature:

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Binomial Expansion

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n \ ,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!\,r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\Delta = \tfrac{1}{2} bc \sin A$$

1 A company purchased a colour copier machine at a cost of \$8500. The value of this machine decreases with time such that its value, \$$V$, after t months of usage is given by $V = 8500e^{-kt}$, where k is a constant.

(a) The value of the copier machine is expected to fall to \$6400 after 8 months of usage. Estimate the value, to the nearest dollar, of the machine after 2 years of usage. [4]

(b) Copier machines are to be replaced when its value reaches $\frac{1}{7}$ of its initial value.

The company's manager, Mrs Lee, claims that the machine will last for at least 5 years before a replacement is due. Showing all necessary working, explain whether you agree with Mrs Lee. [2]

2 **(a)** Differentiate $2x\sin\frac{x}{2}$. [2]

(b) Use the result in part **(a)** to evaluate $\int_0^{\pi} 3x\cos\frac{x}{2}\,dx$, leaving your answer as an exact value in the form $a\pi - b$, where a and b are constants. [4]

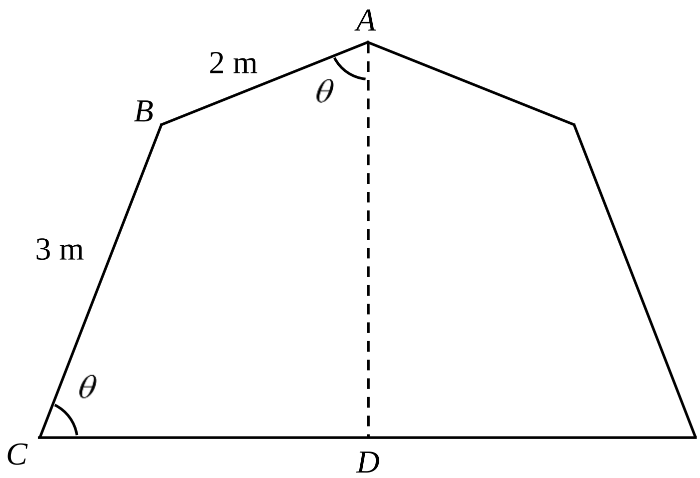
3 It is given that $f(x) = 2x^3 + px^2 + qx + 3$, where p and q are constants, has a factor of $2x - 1$ and leaves a remainder of -75 when divided by $x + 2$.

(a) Show that $p = -15$ and $q = 1$. [4]

(b) Solve the equation $f(x) = 0$. [4]

(c) Hence, solve the equation $2k\sqrt{k} + pk + q\sqrt{k} + 3 = 0$. [2]

4 The diagram shows a vertical cross section of a tent in which AB = 2 m, BC = 3 m and angle BAD = angle $BCD = \theta$. The tent is symmetrical about its vertical height AD and it is set up on horizontal ground.



(a) Show that $AD = 3\sin\theta + 2\cos\theta$. [2]

(b) Express AD in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [3]

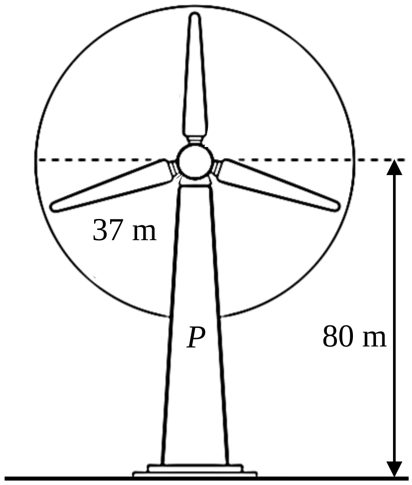
(c) Given that the vertical height of the tent is 3.45 m, calculate the value of θ. [2]

(d) Find the value of θ for which AD is a maximum. [2]

5 **(a)** Given that $\log_p A^2 = 10$ and $\log_p B = 2$, find the value of $\log_A pB$. [3]

(b) Solve $3^x = 6 - 5\left(3^{-x}\right)$. [4]

6 The diagram shows a wind turbine with propeller-like blades that have a length of 37 m each. Wind turns the blades that spin around a rotor in the centre to generate electricity. The height, h m, of the tip of each blade above the ground, t seconds after leaving a particular point P, can be modelled by $h = a - 37\cos bt$, where a and b are constants.



The centre of the wind turbine's rotor is 80 m from the ground and on average, the blades rotate in an anti-clockwise direction at a rate of 1 revolution every 8π seconds.

(a) Show that $a = 80$ and $b = \frac{1}{4}$. [2]

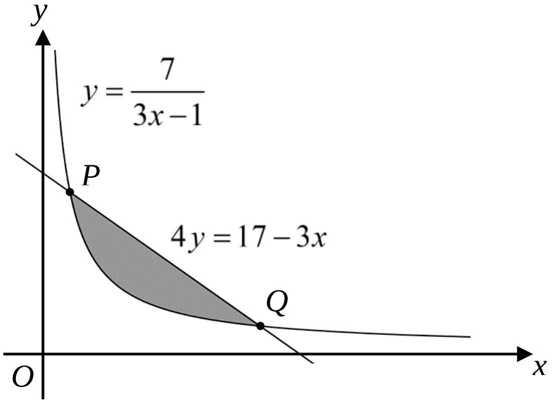
(b) Find the time taken, in seconds, for the blade to first reach a height of 89 m above ground after leaving P. [3]

7 The equation of a curve is $y = 5\ln x$. The tangent to the curve at $x = e^2$ intersects the x-axis at A.

(a) Show that the coordinates of A are $\left(-e^2,\ 0\right)$. [5]

(b) Find the area bounded by the tangent, the line $x = e^2$ and the x-axis. [2]

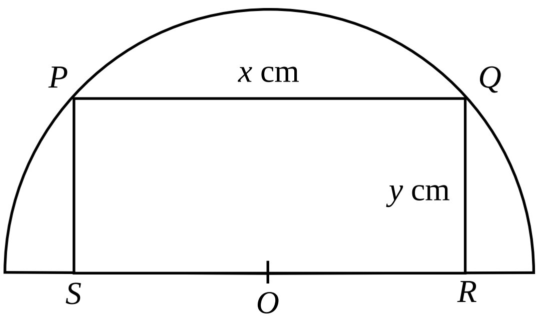
8



The diagram shows part of the curve $y = \frac{7}{3x-1}$ and the line $4y = 17 - 3x$, where the curve intersects the line at points P and Q.

Find, showing all necessary working, the area of the shaded region that can be expressed in the form $a - b\ln 7$, where a and b are constants. [6]

9 *PQRS* is a rectangle with *PQ* = x cm and *QR* = y cm. It is inscribed in a semicircle with centre *O* and radius 10 cm.



(a) Show that the area of the rectangle, A cm^2, is given by $A = \frac{x}{2}\sqrt{400 - x^2}$. [2]

(b) Given that x can vary, find the value of x for which the area of the rectangle is stationary. Leave your answer in the form $a\sqrt{b}$, where a and b are constants. [4]

(c) Explain why the value of x in **part (b)** gives the largest possible value of A and hence, find the maximum area of rectangle $PQRS$. [3]

10 AB is a chord of the circle C_1, where the coordinates of A and B are $(2,\ 5)$ and $(6,\ 3)$ respectively. The line $y = 5 - x$ passes through the centre of the circle.

(a) Find the coordinates of the centre of C_1. [4]

(b) Find the equation of the circle in the form $x^2 + y^2 + px + qy + r = 0$, where *p*, *q* and *r* are integers. [3]

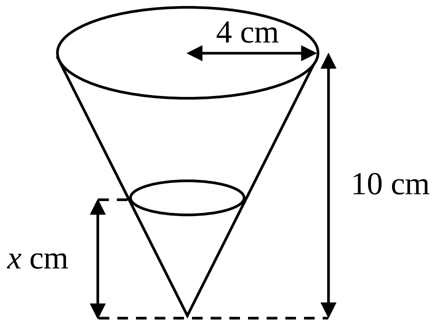
(c) Another circle C_2 with centre $(2,\ 3)$ passes through the centre of C_1.

Explain if the C_2 lies entirely within C_1. [2]

11 **(a)** Prove that $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = \tan x$. [4]

(b) Hence, solve the equation $\dfrac{\sin 2x - \cos 2x + 1}{\sin 2x + \cos 2x + 1} = 5 - 2\sec^2 x$ for $0° < x < 360°$. [4]

12 Water is dispensed at a constant rate into an empty paper cup in the form of an inverted cone of height 10 cm and radius 4 cm. After t seconds, the depth of the water in the conical cup is x cm.



(a) Show that the volume, V cm^3, of water in the cup is given by $\frac{4\pi x^3}{75}$. [2]

(b) The water dispenser is a cylindrical container with radius 12 cm.
Given that the depth of water in the cylinder dispenser decreases at a constant rate of 0.0035 cm/s, find the rate of increase in the volume of water dispensed into the conical cup, leaving your answer in terms of π. [2]

(c) Hence, find the rate of increase in the depth of water in the conical cup when the volume of water dispensed is $\frac{5\pi}{6}$ cm^3. [4]

END OF PAPER

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Answer Key

Qn No.		**Answers**
1	(a)	\$3628
	(b)	Disagree.
2	(a)	$x\cos\frac{x}{2}+2\sin\frac{x}{2}$
	(b)	$6\pi-12$
3	(b)	$x = 7.41$ or -0.405
	(c)	$k = 0.25$ or 54.8
4	(b)	$\sqrt{13}\cos(\theta-56.3°)$
	(c)	73.2°
	(d)	56.3°
5	(a)	$\frac{3}{5}$
	(b)	$x = 0$ or 1.46
6	(b)	$t = 7.27$ s
7	(b)	$10e^2$ or 73.9 units^2
8		$8-\frac{7}{3}\ln 7$
9	(b)	$10\sqrt{2}$
	(c)	Maximum area 100 cm^2
10	(a)	$(3,2)$
	(b)	$x^2+y^2-6x-4y+3=0$
	(c)	C_2 lies entirely within C_1
11	(b)	$x = 45°, 123.7°, 225°, 303.7°$
12	(b)	0.504π cm^3/s or $\frac{63\pi}{125}$ cm^3/s
	(c)	0.504 cm/s