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# **3.1 Introduction to Approximation**

Objectives: At the end of the lessons, students should be able to

- round off numbers to a required number of decimal places
- round off numbers to a required number of significant figures

Approximation is the **rounding** of numbers to a **required** degree of accuracy.

## Prerequisite

Let's recap on the number placing positions. Fill in the blanks with a number from the following figure.

# 1894.237

(i) Thousands place: \_\_\_\_\_

(ii) Hundreds place: \_\_\_\_\_

(iii) Tens place:

(iv) Ones place:

(v) Thousandths place:

(vi) Hundredths place: \_\_\_\_\_

(vii) Tenths place:

## Example 1 (Rounding off)

Based on the following number, correct to:

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16.49583
```

(a) the nearest whole number. Ans: 16

(b) two decimal places. Ans: 16.50

(c) the nearest integer Ans: 16

When rounding off numbers:

(1) Round down if the digit under consideration is 4 or less.

Round up if the digit under consideration is 5 or more.

(2) Check the value of approximation against the original value

## Practice 1

Round off the following correct to (i) two decimal places and (ii) the nearest whole number:

(a)	\$12.3125	(b)	57.6283
(c)	1.99976 cm	(d)	101.3333 kg
(e)	3.23495	(f)	\$ 4.5671

## **Example 2 (Significant figures)**

Let us consider the following case.

A grain of table salt weighs 0.0001208g. If the answer is rounded off to 3 decimal places, the

weight of the salt is now:

0.0001208 g = 0.000 g (3 d.p.)

This value we obtain is not useful for calculations! Instead of rounding off to 3 decimal places, we

#### round off according to significant figures.



In a rounded off **decimal**, all digits, other than zeros preceding the **first non-zero digit**, are significant figures.

E.g. 0.0503 has 3 significant figures '503'

0.05030 has 4 significant figures '5030'

In a rounded off **whole number**, the ending **zeros may or may not be significant**. If it is the result of rounding off to the nearest 10, 100, 1000, ..., then the last 1 zero, 2 zeros, 3 zeros, ..., respectively, are not significant. All the other digits are significant figures.

E.g. 23000 (to the nearest 10) has 4 significant figures

23000 (to the nearest 100) has 3 significant figures

23000 (to the nearest 1000) has 2 significant figures

## Practice 2

Based on the following number, correct to:

21.03460970

(a) 2 significant figures.

Ans: \_\_\_\_\_

(b) 3 significant figures.

Ans: \_\_\_\_\_

(c) 4 significant figures.

Ans: \_\_\_\_\_

(d) 5 significant figures.

Ans: \_\_\_\_\_

(e) 7 significant figures.

Ans: \_\_\_\_\_

# Example 3

Express the following correct to 2 significant figures.

(a) 0.04862	(b) 2.99
<b>Solution:</b> 0.049 (2SF)	Solution: 3.0 (2SF)
(c) 0.90945	(d) 54.0243
<b>Solution:</b> 0.91 (2SF)	Solution: 54 (2SF)
(e) 1.589	(f) 20.03
Solution: 1.6 (2SF)	Solution: 20 (2SF)

# Practice 3(a)

Write 1 354.154 correct to the number of significant figures indicated below.

(a)

- (b) (c) (d)
- 1 2 3 4 5
- (e)

# Practice 3(b)

Round off	
(a)	29 470 to 3 significant figures
(b)	98 836 to 2 significant figures
(c)	851.02 to 4 significant figures
(d)	750.645 to 4 significant figures
(e)	0.079816 to 3 significant figures
(f)	0.000 557 to 2 significant figures
(g)	6407.37 to 5 significant figures
(h)	3.0072 to 4 significant figures

# Practice 3(c)

Express each of the following correct to

(i) 2 decimal places, (ii) 2 significant figures.

(a)	3.825 1	(b)	0.013 527 6
(c)	0.207 9	(d)	5.068 4
(e)	12.384 7	(f)	197.143 92

## Practice 3(d)

State the number of significant figures in each of the following.

- (a) 0.00063
- (b) 7006.12
- (c) 27.3752
- (d) 392.6445
- (e) 1.0780

# Example 4

494.6

- (a) Calculate  $\overline{56.33 \times 98.12}$  showing all the figures on your calculator display.
- (b) Give your answer correct to 1 decimal place.

## Solution:

- (a) 0.08948635556
- (b) 0.1

## Practice 4

384.8

- (a) Calculate  $\overline{47.23 \times 91.43}$  showing all the figures on your calculator display.
- (b) Give your answer correct to 2 decimal place.

## Example 5

The mass of a pebble is 25 grams correct to 2 significant figures. Find the minimum mass of the pebble.

## Solution:

Thought process: To find the **minimum mass** of the pebble, we need to determine the range of values that round to **25 grams** when rounded to **2 significant figures**.

Possible values include 24.5, 24.6, 24.7, ..., 25.1, 25.2, 25.3, 25.4, and the smallest value is 24.5 grams.

## Practice 5

The length of a string is 340 cm correct to 3 significant figures. Find the maximum length of the string.

Complete WS 1 by \_\_\_\_\_

# 3.2 Approximation & Approximation errors in Real-world contexts

# Learning Experience 1

Paste your receipt collected below and answer the questions that follow.

- (a) Express the total amount of the items correct to 3 significant figures.
- (b) Express the cash given correct to
  - (i) 2 significant figures,
  - (ii) 3 significant figures,
  - (iii) 1 significant figure.

(c) Look at the below receipt issued by a restaurant in Singapore. Discuss with your partner and fill in the answers.



- (i) What was the amount charged on the customer?
- (ii) How was approximation used in this scenario?

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(iii) If you were the owner and did not want to lose out the few cents due to rounding off, what other mode of approximation can you do to solve this problem?

# 3.3 Estimation

Objectives: At the end of the lessons, students should be able to

- estimate quantities to an appropriate degree of accuracy
- estimate the results of computations
- understand the effects of rounding errors in intermediate steps of calculations

Estimation enables us to make a **prediction** on our calculations.

Symbol for estimation is  $\approx$ .

#### Example 6

By rounding each number to 1 significant figure, estimate the value of

 $\frac{8.86 \times 97.8}{0.043 \times 301}.$ 

Show your working.

#### Solution:

 $\frac{8.86 \times 97.8}{0.043 \times 301} \approx \frac{9 \times 100}{0.04 \times 300} = 75$ 

## Practice 6a

By rounding each number to 1 significant figure, estimate the value of

 $\frac{5.37 \times 89.8}{0.027 \times 503}.$ 

Show your working.

## Practice 6b

For a wedding reception, with 334 guests, the caterers charge \$59.95 per person. By approximating both the charge and the number of guests to 2 significant figures, estimate the cost of the catering for the reception. Show your working and give your answer to a reasonable degree of accuracy.

Complete WS 2 by \_\_\_\_\_

#### **HOMEWORK WORKSHEET 1**

## Ch 3.1 Rounding and Significant figures

Complete the following questions on foolscap.

#### **Compulsory Questions**

1. Textbook 1A (pg 79): Exercise 3A Qn 4

2. Textbook 1A (pg 79): Exercise 3A Qn 5

- 3. Textbook 1A (pg 79): Exercise 3A Qn 6
- 4. Textbook 1A (pg 79): Exercise 3A Qn 11

5. Express 204.016 to

(a) the nearest integer,

(b) 2 decimal places.

[1E | MYE | 2011 | Ang Mo Kio Sec Sch]

# $\frac{494.6}{56.33 \times 98.12}$ , showing all the figures on your calculator display.

6. (b) Give your answer correct to 1 decimal place.

['O' Level | Nov | 2009 |P1 Qn 19a]

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## **HOMEWORK WORKSHEET 2**

## Ch 3.2 Approximation & Approximation errors in Real-world contexts

Complete the following questions on foolscap.

#### **Compulsory Questions**

Question 1

The population of France in 2006 was 63 587 70. Give this answer correct to:

- (a) 3 significant figures.
- (b) 2 significant figures.

[New Maths Counts (2<sup>nd</sup> Edition)]

## Question 2

Express the following numbers correct to the degree of accuracy indicated within the brackets.

- (a) 18.749 (2 decimal places)
- (b) 59 510 (2 significant figures)
- (c) 0.030067 (3 significant figures)

[1E | MYE | 2011 | ACS (Barker)]

#### Question 3

By rounding each number to 2 significant figures estimate the value	$\sqrt{36.398 \times 24.9068}$		
By founding each number to 2 significant figures, estimate the value	49.5346		
of Show your workings.			

[1E | MYE | 2011 | Bowen Sec Sch]

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