9749 H2 Physics Topic 6

Motion in a Circle

DUNMAN HIGH SCHOOL

Year 5 (2024)

Guiding Questions

- How do we describe the motion of an object moving in a circular path?
- What causes an object to move in a circular path?

Content

- Kinematics of uniform circular motion
- Centripetal acceleration
- Centripetal force

Learning Outcomes

Candidates should be able to:

(a) express angular displacement in radians

(b) show an understanding of and use the concept of angular velocity to solve problems

(c) recall and use $v = r\omega$ to solve problems

(d) describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle

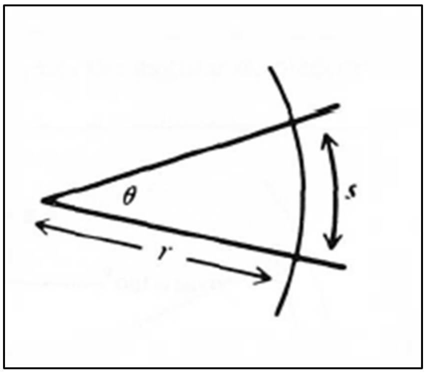
(e) recall and use centripetal acceleration $a = r\omega^2$, and $a = v^2/r$ to solve problems

(f) recall and use centripetal force $F = mr\omega^2$, and $F = mv^2/r$ to solve problems.

6.1 Circular Motion: An Introduction

(a) express angular displacement in radians.

So far, we have discussed the motion of an object in a **straight** line (linear motion) and a **parabolic** path (projectile motion). However, in real life, there are many cases of objects moving in an arc of a circle or a circular path.



For example, we can tie a ball to a string and swing it in a horizontal circle.

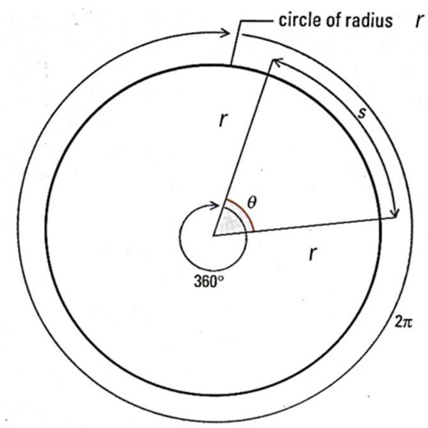
Suppose the radius of this circle is r and the ball travels a distance (arc length) of s, its angular displacement will be θ.

Angle θ in radians is the ratio of the distance s along a circular arc subtended by θ divided by the radius r.

$$\theta = \frac{s}{r}$$

If $s = r$, then θ =1 radian.

One **radian** is



Since $\pi \equiv \dfrac{\text{circumference of circle}}{\text{diameter of circle}}$, the circumference of a circle = $2\pi r$.

So $\theta = \dfrac{s}{r} = \dfrac{2\pi r}{r} = 2\pi \quad \Rightarrow \quad \theta$ for one revolution $= 2\pi$.

2π radian $\equiv 360^\circ$

Thus, 1 radian $\equiv \dfrac{360^\circ}{2\pi} = 57.3^\circ$

Conversion to degree can be done using: π (rad) $\equiv 180^\circ$

Translate degrees to radians: $\dfrac{\pi}{180} \times \theta(\text{degrees})$

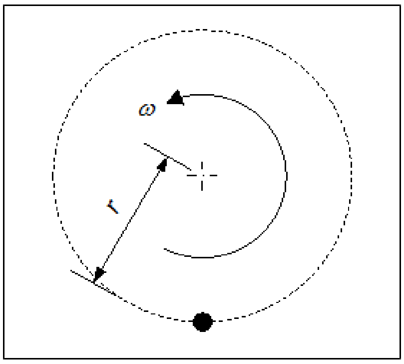
Translate radians to degrees: $\dfrac{180}{\pi} \times \theta(\text{radians})$

6.1.1 Angular velocity ω

(b) show an understanding of and use the concept of angular velocity to solve problems.

Angular velocity, ω, is the rate of change of angular displacement.

$$\omega = \frac{d\theta}{dt}$$



Its unit is rad s^{-1}.

When the angular velocity is constant,

$$\omega = \frac{\text{angle moved through in time } t}{\text{time taken}} = \frac{\theta}{t}$$

the body is then said to be undergoing **uniform circular motion**, i.e. the motion of an object moving in circular path at constant speed.

For our syllabus, we are **mainly** interested in objects moving in **uniform circular motion**.

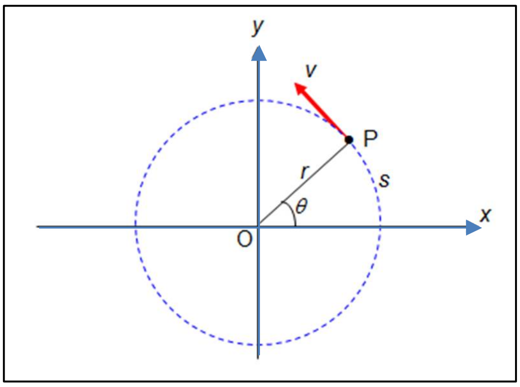
If a body undergoing uniform circular motion takes T seconds to complete 1 full revolution, i.e., its period is T seconds,

where f represents the frequency of the circular motion.

6.1.2 Relationship between angular velocity ω and linear speed v for uniform circular motion

(c) recall and use $v = r\omega$ to solve problems.

From $s = r\theta$, the linear speed of an object (e.g. P , as shown in the diagram) can be determined via calculus.



$$\frac{ds}{dt} = r\frac{d\theta}{dt} + \theta\frac{dr}{dt}$$

If r is a constant,

$$\frac{ds}{dt} = r\frac{d\theta}{dt}$$

Therefore,

This relationship is only valid when ω is measured in rad s^{-1}.

Note that the direction of the (linear) velocity is tangential to the circular path.

For an object rotating about an axis, every point on the object has the same angular velocity. The tangential speed of any point is proportional to its distance from the axis of rotation.

Question 6.1
Find the angular velocity of the second, minute and hour hands of a clock. If the second hand, minute hand and the hour hand is 5.0 cm, 5.0 cm and 4.0 cm long respectively, find the linear speed of the tip of each hand.

Using $\omega = \dfrac{2\pi}{T}$, for second hand, $\omega = \dfrac{2\pi}{\qquad\qquad} = 0.105 \text{ rad s}^{-1}$

for minute hand, $\omega = \dfrac{2\pi}{\qquad\qquad} = 1.75 \times 10^{-3} \text{ rad s}^{-1}$

for hour hand, $\omega = \dfrac{2\pi}{\qquad\qquad} = 1.45 \times 10^{-4} \text{ rad s}^{-1}$

Using $v = r\omega$, for second hand, $v = (\qquad\qquad)(0.105) = 5.25 \times 10^{-3} \text{ m s}^{-1}$

for minute hand, $v = (\qquad\qquad)(1.75 \times 10^{-3}) = 8.75 \times 10^{-5} \text{ m s}^{-1}$

for hour hand, $v = (\qquad\qquad)(1.45 \times 10^{-4}) = 5.80 \times 10^{-6} \text{ m s}^{-1}$

Question 6.2
A Blu-ray disc (BD) rotating inside a player must maintain a constant linear speed of 4.92 m s^{-1} in order for the information on it to be read correctly. The laser in the Blu-ray player starts at the inside of the disc and moves outwards.

(a) State and explain what must happen to the angular velocity of the disc as the BD is played.
(b) Calculate the radius at which the BD will have an angular velocity of 23.1 revolutions per second when the linear speed is 4.92 m s^{-1}.

Answers:

(a) The angular velocity of the disc must ……………………. when the laser is moving outwards to read the info. Since $v = r\omega$, when r increases, ω must ……………….. to keep v constant.

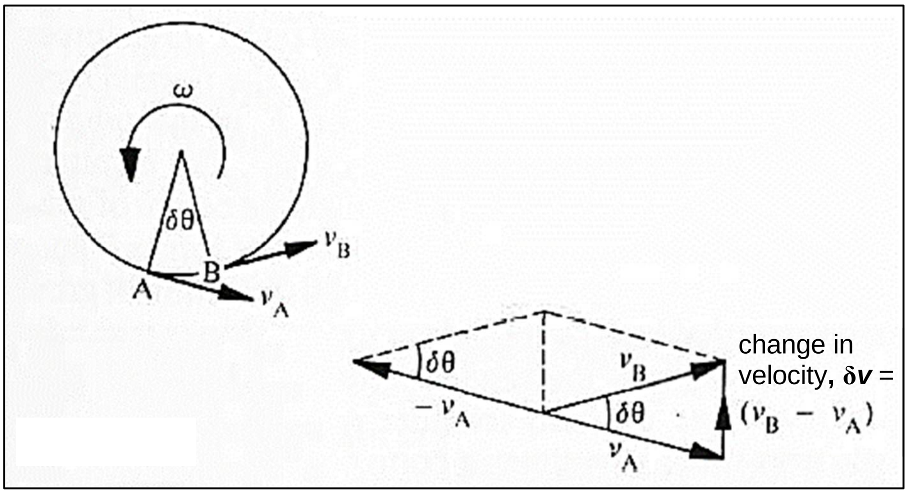
(b)

6.2 Analysis of uniform circular motion

6.2.1 Centripetal acceleration in uniform circular motion

(e) recall and use centripetal acceleration $a = r\omega^2$, and $a = v^2/r$ to solve problems.

A body is moving round a circle radius r with constant angular velocity ω. It moves from point A to point B in a brief time of δt. Its velocity changes from v_A to v_B in this time (change in direction but not magnitude).



In time δt, the body moves round the circle a distance arc AB of length $r\,\delta\theta$.

Its speed $v = r\dfrac{\delta\theta}{\delta t} = r\omega$.

The vector diagram shows that the change in velocity in time δt, $\delta v = v\,\delta\theta$ (if $\delta\theta$ is small)

The average acceleration of the body between A and B (rate of change of velocity) = $\dfrac{\delta v}{\delta t} = \dfrac{v\delta\theta}{\delta t}$

In the limit when δt approaches zero, $\lim\limits_{\delta t \to 0} \dfrac{\delta\theta}{\delta t} = \dfrac{d\theta}{dt} = \omega$,

The acceleration of the body at any point is

As $\delta\theta$ becomes infinitesimally small, δv is perpendicular to v_A. Since δv is in the same direction as the acceleration, the direction of acceleration of the body undergoing uniform circular motion is towards the centre of the circular path. This acceleration is called the **centripetal acceleration**.

Centripetal acceleration is one which is always perpendicular to the velocity and always acts towards the centre of the circular motion.

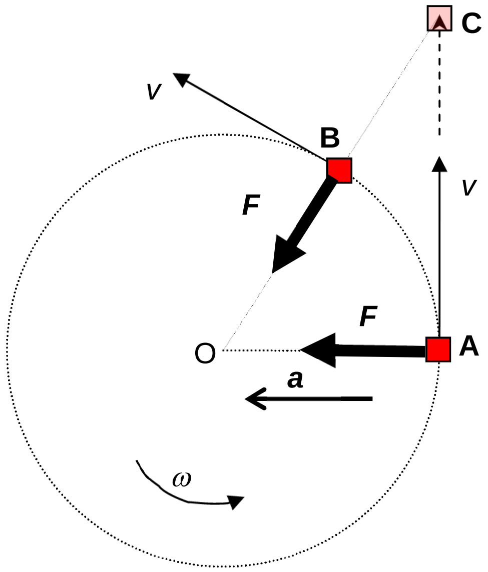
This centripetal acceleration can also be expressed as

6.2.2 Centripetal force in uniform circular motion

(d) describe qualitatively motion in a curved path due to a perpendicular force, and understand the centripetal acceleration in the case of uniform motion in a circle.

Recall: An object at rest will remain at rest and an object in motion will remain in motion at constant velocity in the absence of an external resultant force.

Consider an object moving in uniform circular motion at *constant angular velocity*, ω, from A to B.



If no external resultant force acts on the object, by Newton's 1st Law (N1L), the object will travel from A to C with a *constant velocity, v*.

However, since the object **travels along the curved path** from A to B, this implies that **an external resultant force *F* must have acted on the object to change its velocity**.

Instead of arriving at C, the object arrives at B. This means that the object must have been "pulled" towards the centre of the circle "by this external resultant force" ***F***.

Since the object is **moving at constant speed** in a circle, there is no component of acceleration in the direction of motion, implying there is **no component of the external resultant force acting along the tangential path** of the object.

[*Conversely, if the speed is changing (not constant), there will be a component of the external resultant force acting tangentially to the circle. This* component *causes a change in speed.*]

If there is no component of the external resultant force acting along the direction of motion, the **external resultant force must always be at right angles to the motion** and **directed along a radius towards the centre**. Since the object is turning steadily at constant rate (**constant ω**), the external resultant force must be **constant in magnitude** (**but always changing in direction**) towards the centre to produce this uniform circular motion.

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By Newton's 2nd Law (N2L), for an object with constant mass that is accelerating, the direction of its acceleration and the external resultant force acting on it are the same. Hence, the **direction of the acceleration** of an object undergoing *uniform* circular motion must be **acting towards the centre of the circular path** as well (in the same direction as its external resultant force).

The *resultant* acceleration and the external *resultant* force found in *uniform* circular motion is also known as *centripetal* acceleration ***a*** and *centripetal* force ***F*** respectively.

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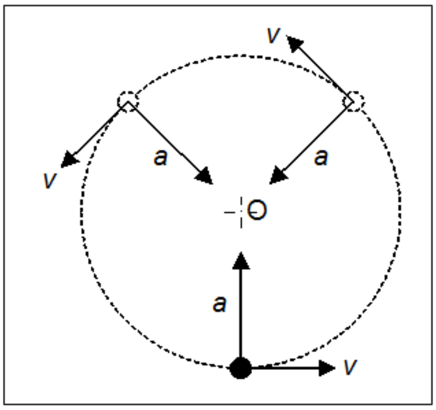
Similar to an external resultant force, the centripetal force is NOT an additional force acting on the object. It should not be drawn on any free-body diagrams. It can be a combination of forces like tension, friction, gravitational force, electric force etc.

6.3 Application of N2L

(f) recall and use centripetal force $F = mr\omega^2$, and $F = mv^2/r$ to solve problems.

Applying N2L to a body that has mass m, the centripetal force that must act to produce the uniform circular motion is

For uniform circular motion, the centripetal acceleration is constant in magnitude (but always changing in direction) and the associated centripetal force, which is required to produce this motion, is also constant in magnitude (but always changing in direction).



<u>Notes</u>:

As the centripetal force acts at right angles to the motion, it only changes the direction of the velocity, but not the speed of the object.

Centripetal acceleration is not constant as its direction is constantly changing.

Centripetal force is a resultant force towards the centre of the circle and therefore the object executing circular motion is not in equilibrium.

The fact that the centripetal force is normal to the velocity means that work is not being done by this force as the displacement of the object is always perpendicular to the force.

If the centripetal force producing the centripetal acceleration vanishes, the object does not continue to move in its circular path; instead, it will move along a tangential path.

6.3.1 Steps to solve problems involving circular motion

(a) Draw a free body diagram for the problem

(b) Identify the centre of the circular motion

(c) Choose a suitable coordinate system

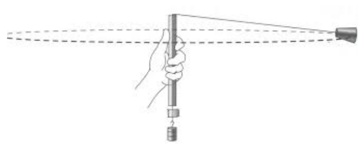
(d) Resolve the forces, if necessary

(e) Apply N2L to solve the problem

* Ensure that r is the radius of the circular path!

Question 6.3

When a bung of mass m is whirled with constant angular velocity ω in a horizontal circle at the end of a string of length l, the string itself will not be horizontal. The point at which the string is held will be above the plane of the circle.

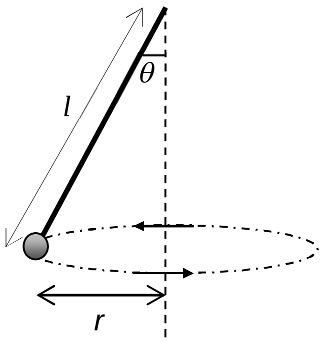


(a) Draw a diagram showing the forces acting on the bung.

(b) What force, in addition to the centripetal force, does the string provide?

(c) What happens to the angle which the string makes with the vertical if the angular velocity of the bung increases?

(a)



Free body diagram

Free body diagram
(After resolving the tension)

(b) It also provides a vertical component to balance the weight of the bung.

(c) Vertically, (1)

Horizontally, (2)

As ω increases, T …………………….. [if l is kept constant, from (2)]

From (1), as T increases, cos θ ……………………., thus θ ……………………..

Both θ and T ………………… so that the above equations are still true at the increased ω.

Note that cos $\theta = \underline{\hspace{4cm}}$.

6.4 Application I: Going around the bend

A vehicle going round a corner at a steady speed is accelerated, and a centripetal force must be acting. Consider the ways in which these forces are provided to enable cars, trains, aircraft and bicycles to turn.

6.4.1 Cars

Question 6.4
A car of mass 750 kg is taking a corner with a flat horizontal surface and a radius of 10 m. If the lateral friction forces cannot exceed 1/10 of the weight of the vehicle,

(a) what is the maximum speed at which it can take the bend?
(b) which piece of the above information is not needed? Explain your answer.

(a) Frictional force by the road on the tyres provides the centripetal force.

$$f = m\frac{v^2}{r}$$

For maximum speed,

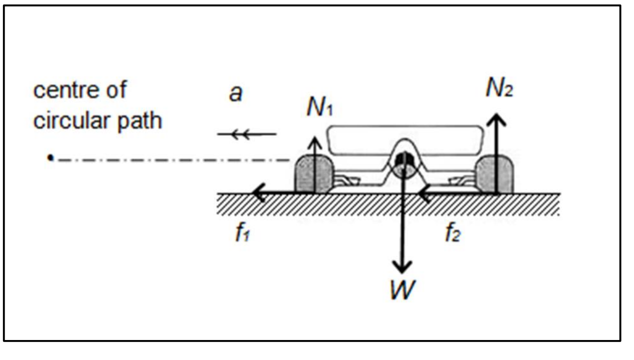
$$f_{\max} = m\frac{(v_{\max})^2}{r}$$

$$\Rightarrow \frac{mg}{10} = m\frac{(v_{\max})^2}{10} \qquad \text{--- (1)}$$

$$\Rightarrow v_{\max} = \qquad = \text{.........................} = \text{.........................}$$

(b)

Consider the forces acting on a racing car of mass *m* travelling round a circular bend along a flat horizontal track.



Neglecting air resistance, each tyre experiences its own normal reaction and frictional force, but the forces acting on front and rear tyres are combined so that there is just two normal reactions N_1 and N_2 and two frictional forces f_1 and f_2 as shown in the diagram.

Resolving forces horizontally:

Resolving forces vertically:

Because the tyres of the car can only provide a certain **……………………..** friction, a car travelling too fast may not get round the bend even though the wheels are pointing in the right direction. It may skid.

One way of reducing the possibility of slipping at a corner is to (slope or) bank the road so that the normal contact force of the road acting on the car provide a component directed to the centre of the circular path.

For a particular speed, this force will produce exactly the centripetal force required.

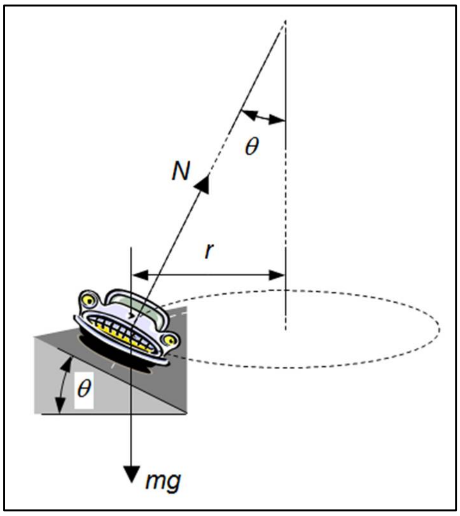
At other speeds some lateral frictional force needs to come into play.

Question 6.5
Show that, for a vehicle taking a bend at speed v, the banking of the curve to eliminate side slip (any tendency to skid) is given by

$$\tan\theta = \frac{v^2}{gr}$$

where θ is the angle between the road surface and the horizontal.

Here, the horizontal component of the normal contact force N between the car and the road provides the centripetal force required to keep the car moving round the bend.



6.4.2 Trains

Question 6.6

(a) A new high-speed train is designed to travel at 200 km h^{-1}. Calculate the banking angle needed on a track of radius 0.25 km if that speed is to be sustained without the train trying to push the rails out of place. (Hint: this means that the force on the train produced by the rails will be perpendicular to the ground.)
(b) What will passengers feel if the train takes this banked curve at speeds below 200 km h^{-1}?
(c) What will passengers feel if the train takes the curve at speeds above 200 km h^{-1}?

(a)

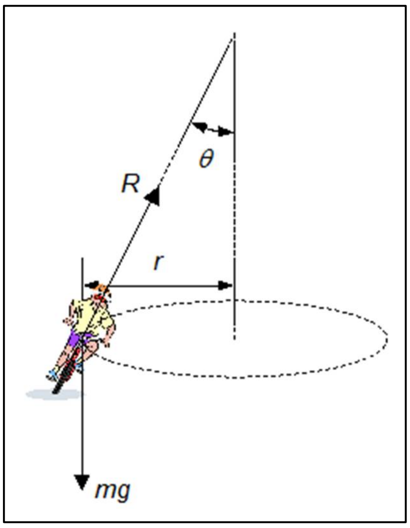
(In practice, the train will not travel round a bend at maximum speed and the track banking (normal push by the ground) is unlikely to provide all the necessary centripetal force. Hence the angle will probably be less than 52°.)

(b) Passengers feel a tendency to slide laterally …………………… the centre of the bend.

(c) Passengers feel a tendency to slide laterally …………………… from the centre of the bend.

6.4.3 Bicycles

A cyclist is moving with speed v around a bend of radius r on a horizontal road. The forces acting on the cyclist are the contact force R due to the road, and the weight mg of the cyclist.

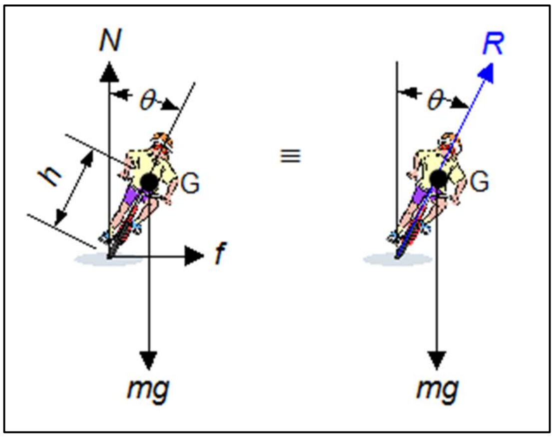


No vertical motion:

The cyclist is being accelerated towards the centre of circle:

Thus, $\tan\theta = \dfrac{v^2}{gr}$

Notes:
N is the normal contact force of the ground on the cycle, and f is the lateral frictional force of the ground on the cycle. The resultant of forces N and f is a force R acting through the CG of the rider and the machine.



The resultant of forces R and mg provides the centripetal force mv^2/r acting through the CG. This ensures that both cycle and cyclist accelerate towards the centre of curvature of the bend without any turning effect about G, and get round the bend safely.

Since the cyclist travels in a horizontal circular path,

..... (1)

The frictional force provides the centripetal force,

..... (2)

Zero net moment about G:

..... (3)

Sub (1) and (2) into (3), we get $\tan\theta = \dfrac{v^2}{gr}$

The resultant force on the bicycle plus cyclist must be a horizontal force through G, otherwise it will provide not only an acceleration but also a turning effect which will tilt them to a different angle. So there is only one **correct** angle of tilt for a given speed and bend radius. Too small (large) an angle, they will fall outwards (inwards).

Question 6.7
Why does the cyclist have to lean inwards, towards the centre of the circular motion?

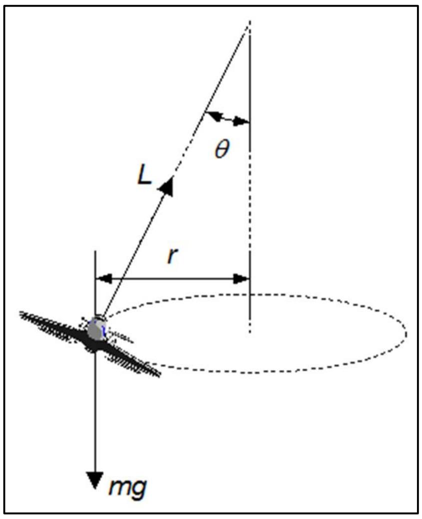
The ……………………………….. produces an anti-clockwise moment about the centre of gravity G.

The cyclist leans inwards so that the ……………………………….. produces a clockwise moment about G.

By leaning at the correct angle, the …………………………….. about G will be zero. This corresponds to the ……………………….. passing through G, to give a horizontal resultant force through G.

6.4.4 Aircrafts

An aircraft can only turn if it banks so that the lift force produced by its wings is no longer acting vertically.



Consider it moves with speed v and describes a circular path of radius r in a horizontal plane. The forces acting on the aircraft are the lift L of the aircraft and the weight mg of the aircraft.

Again, since the aircraft moves in a horizontal plane,

Horizontal component of the lift force provides the centripetal force,

Thus,

$$\tan\theta = \frac{v^2}{gr}$$

Question 6.8
Is the centripetal force required for circular motion directly or inversely proportional to radius r of circle?

Depends on the situation.

- If ………………………………………….. is constant (e.g. merry-go-round or turntable) then, by ………………………………..., F_c is …………………………… proportional to r.
- If we want to determine the magnitude of F_c (e.g. for a car rounding a bend) at a fixed ………………….…………., then ………………………………… should be used. A tighter turn will require a ………………………. centripetal force.

6.5 Application II: Defying gravity

6.5.1 Bucket of Water

A bucket of water can be swung in a vertical circular motion without spilling and a pilot loops the loop confident that he will stay in his seat in the aircraft. Think about the forces acting on a body and its kinetic energy as it moves in a vertical circle.

Question 6.9

Consider a body of mass *m* moving in a vertical circle of radius *r* on the end of a string.

(a) What force provides the centripetal force at L and R?

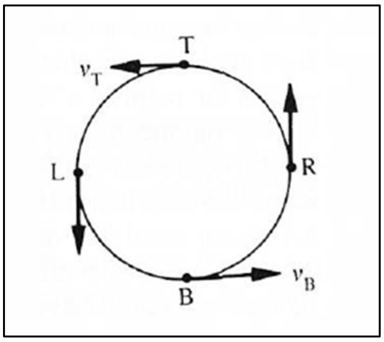
(b) What two forces may provide the centripetal force at T and B?

(c) Why does the speed change as the mass moves from B to T?

(d) Derive an expression for v_B^2 - v_T^2 in terms of *r* and *g*.

(e) Derive expressions for the tension in the string when the mass is at the top and when it is at the bottom of the circle and show that the difference between these tensions is 6 *mg*.

(f) What is the expression of the speed of the body at the top if the body moves in a circle but the tension in the string is zero?

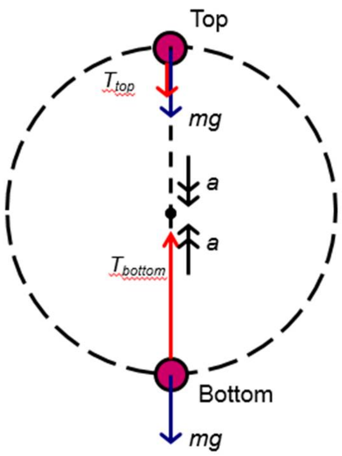


(a) ……………………………………………………………

(b) ……………………….……….. and ………………………………..

(c) The …………………………..……….. increases at the expense of ……………………………

(d) Increase in gravitational potential energy
= decrease in kinetic energy



(e) at the top,

----- (1)

at the bottom,

----- (2)

(2) – (1) :

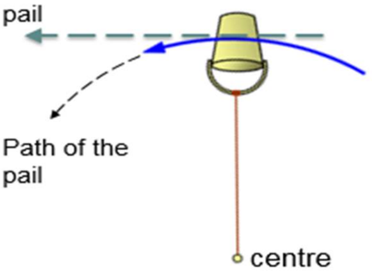
= 6*mg*

(f) Let its speed be v_c when at the top and tension in the string is zero.
Only the weight of the body provides the centripetal force:

Question 6.10
A pail of water can be whirled in a vertical circular path of radius r such that no water is spilled. If the acceleration of free fall near the surface of the Earth is g,

(a) why does the water remain in the pail, even when the pail is upside down?
(b) determine the minimum linear speed of the pail at the highest point such that the water remains in the pail.
(c) estimate the minimum angular velocity at which a pail of water must pass over your head at arm's length in a vertical circle if you are to remain dry.

(a) The water tends to follow a straight-line path. However, the pail is being pulled inwards, so the water in it gets pushed in, against its bottom. The pail exerts force on the water towards the centre.



The pail should be swung fast enough such that the required centripetal force is larger than the weight of water.

This force N and the weight of water W provide the centripetal force needed to keep the water in circular motion.

If the pail is swung very fast, a larger centripetal force is needed. If W is insufficient to provide this centripetal force, N needs to be present and act in the same direction as W (downwards), to help to provide the centripetal force.

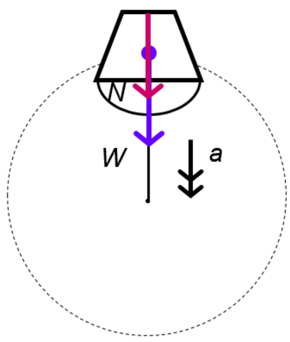
Non-zero N means the water needs to be in contact with the pail, and not spilling out.

Consider the water only as the system. By N2L,

$$(\downarrow): N + W = \frac{mv^2}{r} \rightarrow$$

If the water were to stay in the pail, the contact force with the pail must be present, i.e. $N > 0$.

(b) For the water to stay in contact with the pail, $N > 0$.



(c) For the water to stay in contact with the pail, the speed at the top of the circle:

$v > \sqrt{rg}$

If $r = 1.0$ m, $g = 9.81$ m s^{-2},

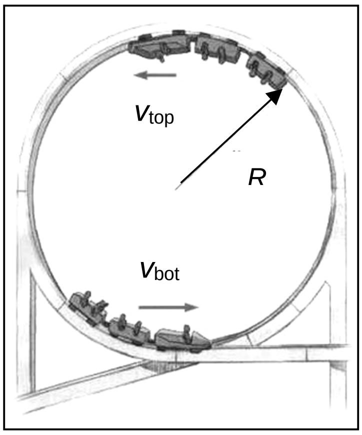
$v > \sqrt{(1.0)(9.81)} = 3.13$ m s^{-1}.

$$\Rightarrow \omega = \frac{v}{r} > \frac{3.13 \text{ m s}^{-1}}{1.0 \text{ m}} = \text{.......................} \text{ rad s}^{-1}$$

$$\Rightarrow f > \frac{(\qquad\qquad)}{(\qquad\qquad)} =$$

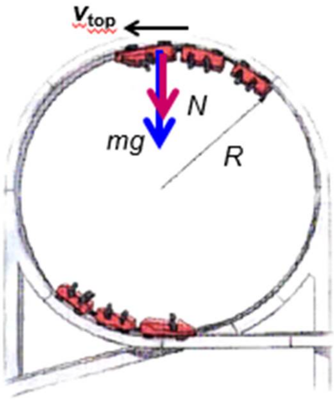
6.5.2 Roller Coaster

Question 6.11
The figure shows a roller coaster moving around an almost circular loop of radius R.



Assume that frictional forces may be neglected. Derive expression in terms of g and R for the
(a) minimum speed the car must have at the top for it to be just in contact with the loop,
(b) speed at which the car must enter at the bottom of the track so that the car just makes it over the top of the loop.

(a) At top of loop, by N2L:



If car is *just in contact*, $N = 0$:

$$mg = \frac{mv_{top}^2}{R} \rightarrow v_{top} = \sqrt{gR}$$

(b) For the car to ***just*** make it over the top of the loop,

$$v_{top} = \sqrt{gR}$$

By principle of conservation of energy, decrease in E_k = increase in E_p

$$\left(v_{bot}^2 - v_{top}^2\right) = 2g\left(2R\right)$$
$$v_{bot}^2 - gR = 2g(2R)$$
$$v_{bot}^2 = 5gR \Rightarrow v_{bot} = \sqrt{5gR}$$

6.6 Application III: Spin driers

In a spin drier, the clothes move around in a circle because the walls of the spinning cylinder provide the centripetal force. The holes in the cylinder are small enough to prevent clothes going through, but large and frequent enough not to impede the water. Very large forces are required to keep a mass spinning in a circle at high speed.

Question 6.12

(a) In which direction will the water travel when it is 'thrown out' of a spin drier?

(b) Calculate the maximum force a water molecule will need to keep it in the spinner, if the cylinder is turning 6000 times every minute and has a diameter of 0.5 m. Take the mass of a water molecule as 3×10^{-26} kg.

(c) An automatic washing-machine manufacturer advertises on TV that they have increased the rotational frequency of the cylinder from 5000 to 8000 revs per minute. What percentage increase in the 'squeezing force' does this represent?

(a) ……………………………………………………………..

(b) $\omega = 2\pi f = 2\pi \times \left(\frac{6000}{60}\right)$ rad s^{-1}

Using $F = m\omega^2 r$,

$F = \boxed{}$

(c) The squeezing force is proportional to ω^2, therefore, the squeezing force increases by a factor of $(8/5)^2 = 64/25$.

Percentage increase is $\left[\frac{64}{25} - 1\right] \times 100\% = \boxed{}$

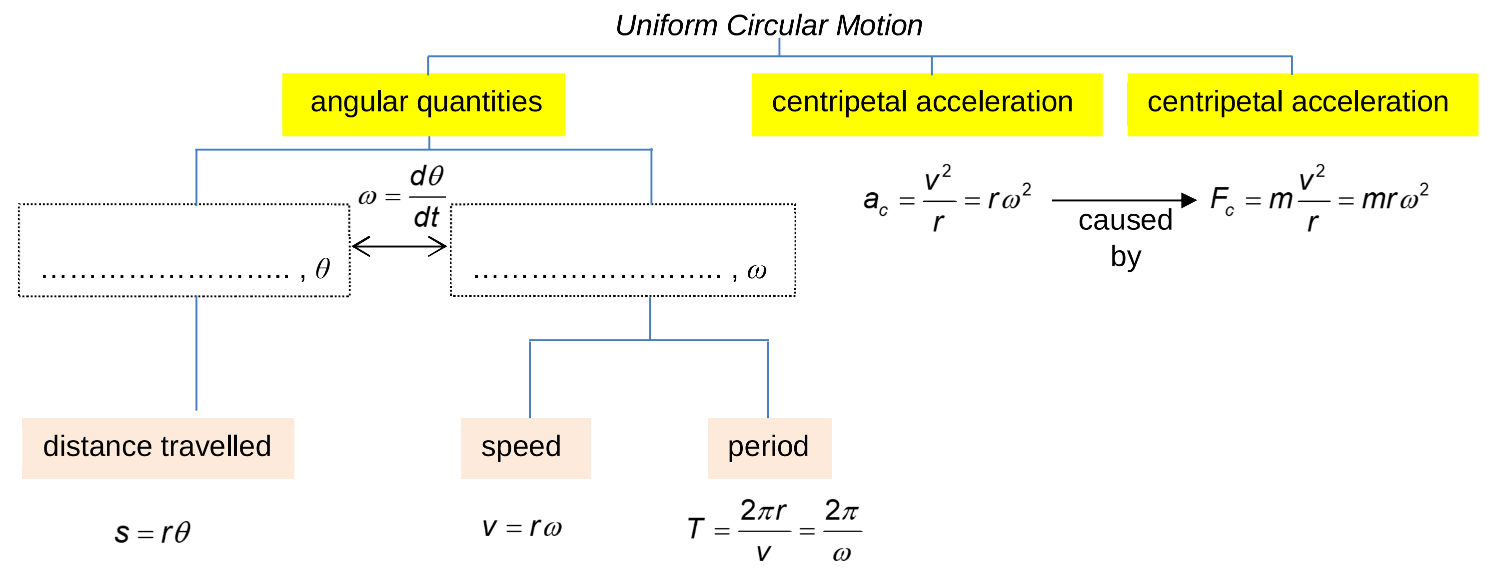
Definitions

Motion in a Circle	radian	The radian (or one radian) is the angle subtended at the centre of a circle by an arc length which is equal to the radius. Symbol of radian: rad
	angular displacement θ	Angle through which an object turns, usually measured in radians (rad).
	angular velocity ω	The rate of change of angular displacement of a radius joining the body to the centre of the circle. Unit: radian per second (rad s^{-1})
	centripetal acceleration (in the case of uniform motion in a circle)	Acceleration which is always perpendicular to the velocity of the object and points towards the centre of the circle. (In the case of uniform motion in a circle, it allows the object to continuously change its direction at a constant rate while moving in a circular path at a constant speed.)

~ END ~

2024 Year 5 Physics H2 - Tutorial Questions
Topic 6: <u>Motion in a Circle</u>

Summary



Kinematics of circular motion

<u>Self-Attempt Questions</u>

1. A spinning top is a toy which is typically set into spinning motion by unraveling a length of string initially wrapped around a circular body with a pointed tip. Such toys can reach speeds of 5000 RPM (revolution per minute).
   Express the rotational speed in rad s^{-1}.

   [524 rad s^{-1}]

2. A coin of mass 10 g, is placed on a horizontal turntable. The turntable makes 3 revolutions in π seconds.
   (a) Calculate the linear speed of the coin when it is at a distance of 5.0 cm from the centre of the turn table.
   (b) What is the acceleration of the coin?
   (c) What is the frictional force acting on the coin?

   [0.30 m s^{-1}; 1.8 m s^{-2}; 18 mN]

3. The moon takes 27.3 days to complete a revolution around the Earth and the radius of orbit is 3.83×10^8 m. Calculate
   (a) the angular velocity.
   (b) the linear speed.
   (c) the centripetal acceleration.

   [2.66×10^{-6} rad s^{-1}; 1020 m s^{-1}; 2.71×10^{-3} m s^{-2}]

4. Given that the radius of the Earth is approximately 6400 km, find the linear speed an object has if it is on the surface of the Earth, along the Equator. Assume that the Earth is spherical.

   [465 m s^{-1}]

Discussion Question

5. While driving on a meandering mountain road, you encounter a right turn with a steep drop over a cliff on your left. For the same linear speed, is it safer to keep towards the turn, or to stay on the outside of the road, nearer to the cliff edge?

Circular motion in a horizontal plane

Self-Attempt Questions

6. A 0.30 kg mass attached to a 1.5 m long string is whirled around in a horizontal circle of speed 6.0 m s^{-1}. By assuming that the string is nearly horizontal during its motion, determine
   (a) the centripetal acceleration of the mass.
   (b) the tension in the string.

[24 m s^{-2}; 7.2 N]

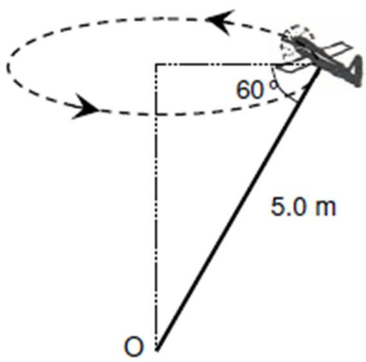
7. Determine the angular velocity of each of the following:
   (a) the Earth in its orbit around the Sun [2.0 × 10^{-7} rad s^{-1}]
   (b) the Earth about its axis [7.3 × 10^{-5} rad s^{-1}]

8. A spaceman in training is rotated in a seat at the end of a horizontal rotating arm of length 5.00 m. If he can withstand accelerations of up to 9*g*, what is the maximum number of revolutions permissible? The acceleration of free fall may be taken as 10 m s^{-2}.

[0.675 rev s^{-1}]

Discussion Questions

9. A model aeroplane has a mass 0.40 kg and has a control wire of length 5.0 m attached to it when it flies in a horizontal circle. Its wings are horizontal, creating vertical upward lift on the aeroplane. The taut wire is then inclined 60° to the horizontal and fixed to a point O.



   (a) If each revolution takes 3.5 s, show that its centripetal acceleration is 8.06 m s^{-2}.
   (b) Calculate the tension *T* in the control wire.
   (c) Calculate the upward lift, *L* on the model aeroplane due to the air.

[6.45 N; 9.51 N]

10. A racing car of mass 1000 kg, moves around a banked track with no lateral frictional force at a constant speed of 108 km h^{-1}. If the horizontal radius of the track is 100 m, calculate
   (a) the angle of inclination of the track to the horizontal.
   (b) the normal contact force at the wheels.

[42.5°; 13.3 kN]

11. In the Bohr's model of an atom, an electron moves in a circular motion about a proton in a hydrogen atom about the same centre. Calculate the distance between the electron and the proton given the following information:

Electrostatic force of attraction	=	8.24×10^{-8} N
Linear speed of electron	=	2.19×10^{6} m s^{-1}
Mass of electron	=	9.11×10^{-31} kg

[0.0530 nm]

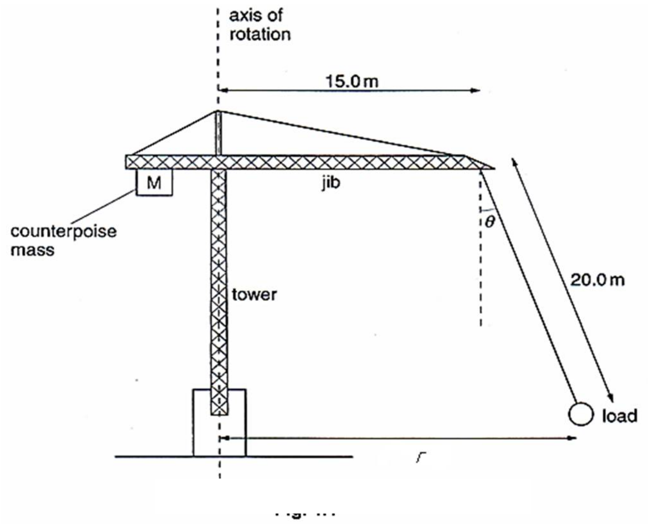
12. Top gun pilots worry about taking a turn too tightly. As a pilot's body undergoes centripetal acceleration, with the head towards the centre of curvature, the blood pressure in the brain decreases and diminishes brain functions. When centripetal acceleration is 2*g* or 3*g*, the pilots feels heavy. At 4*g*, the pilot risks becoming unconscious, a condition known as *g*-LOC or "*g*-induced loss of consciousness."

Determine the smallest horizontal circle a F16 fighter plane can navigate to avoid *g*-LOC if it travels at a linear speed of 2500 km h^{-1}.

[12.3 km]

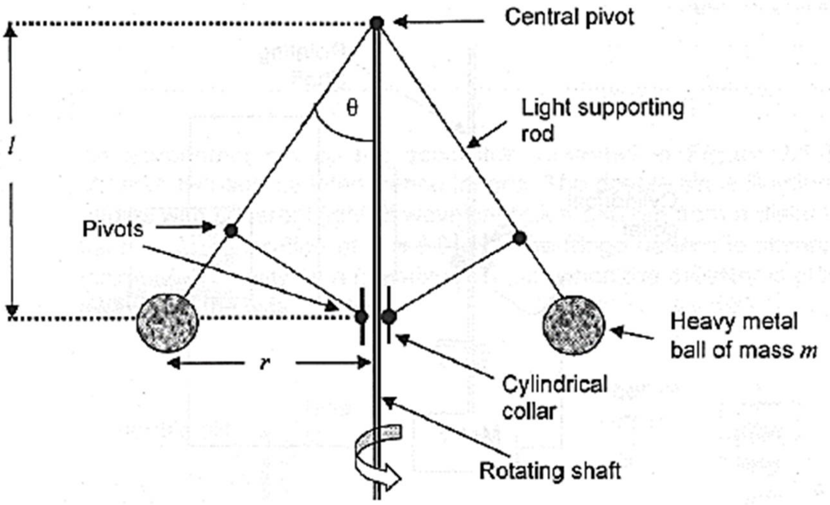
13. [AJC/09/P2/1]
The jib on a tower crane is rotating with constant angular velocity about a vertical axis through the centre of the tower as shown in the diagram. A fixed load is suspended by a light cable from the end of the jib.

Under these conditions, the load swings away from the axis of rotation so that the cable makes an angle θ with the vertical and the load travels in a horizontal circle of radius *r*. Forces that the air exerts on the load may be neglected.



(a) State how the vertical equilibrium of the load is maintained.
(b) In order that the crane does not topple, a counterpoise mass *M* is attached to the jib as shown in the diagram. Explain why the position of *M* does not need to be altered as the jib begins to rotate.
(c) State and explain the work done on the load as the jib is rotating.

14. The figure below shows the basic design of a Watt's Governor, a device previously used to control the speed of the steam engines.



HCI/07/P3/2

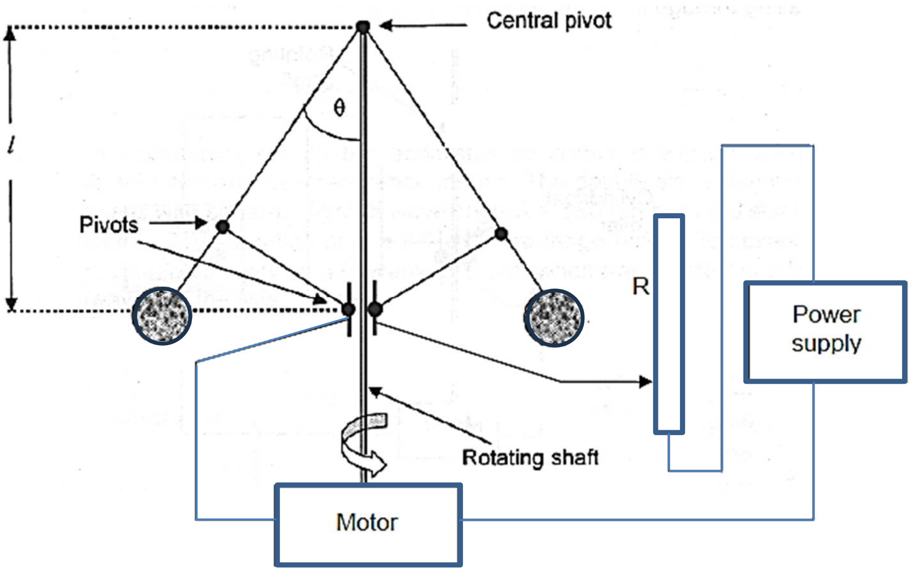
(a) Assuming that the light supporting rods are of negligible mass, show that

$$\tan\theta = \frac{r\omega^2}{g}$$

where θ and r are as shown in the figure and ω is the angular velocity of the rotating shaft.

(b) Hence express the distance of the collar from the central pivot, l, in terms of g and ω.

(c) The collar is placed in electrical contact with an external circuit as shown in the following figure by means of a slider in contact with a variable resistor R. You may assume that the speed of the motor (and hence the rotating shaft) is proportional to the current passing through it.



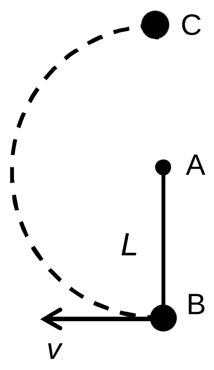
With reference to your answer in (b), explain how this arrangement would help to regulate the speed of the motor during a sudden power surge from the power supply.

(d) Discuss the feasibility of fitting such a device to an electric car as "cruise control" (a device which would allow a car to maintain the same speed over different sorts of terrain without any intervention by the driver).

Circular motion in a vertical plane

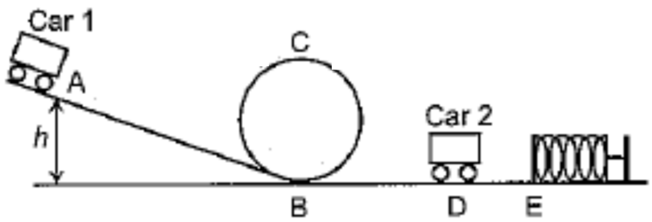
Discussion Questions

15. A particle is suspended from point A by an inextensible massless string of length L. It is projected from B with a speed v, perpendicular to AB, which is just sufficient for it to reach point C.
   (a) Show that, if the string is just to be taut when the particle reaches C, its speed there is $v = \sqrt{gL}$.
   (b) Find the speed v, in terms of g and L, with which the particle should be projected from B. [$v = \sqrt{5gL}$]



16. A 0.0200 kg mass is attached to a motor via a spring of spring constant 4.00 N m^{-1}. The motor is the centre of rotation in a vertical plane for the mass. The distance between the motor and the mass was found to be 0.150 m when the mass is at the lowest point of the motion, during which the mass traveled with a linear speed of 0.942 m s^{-1}.
   Determine the natural length of the un-extended spring. [7.14 cm]

17. The diagram shows a toy runway. Upon release from a point A, a car of mass 0.100 kg will run down the slope and move round the loop, passing through points B and C. It will then collide head-on with a stationary toy car of mass 0.080 kg at D. Upon collision, the two cars will stick together and continue to travel towards E until it is stopped by a buffer of spring constant 120 N m^{-1}. The radius of the loop BC is 0.25 m.

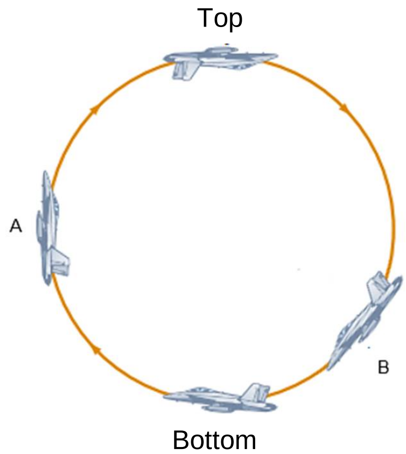


   (a) What is the minimum speed above with which the car must pass point C at the top of the loop if it is to remain in contact with the runway?
   (b) What is the minimum value of h which allows the speed calculated in part (a) to be achieved?
   (c) Car 1 moves around the loop and travels at constant speed towards car 2 at D. Calculate the speed at which both cars approach the spring buffer at E.
   (d) Hence, calculate the maximum compression of the spring buffer when the cars collide into it.

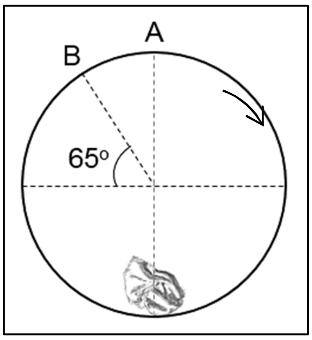
[1.57 m s^{-1}; 0.625 m; 1.95 m s^{-1}; 0.076 m]

18. At an air show, an aircraft travelling at a constant speed of 170 m s^{-1} performs the loop-the-loop manoeuvre, as shown in the diagram.
(a) Indicate on a sketch the forces acting on the pilot at the positions indicated.
(b) Calculate the minimum radius of this circle if the centripetal acceleration of the aircraft is not to exceed six times the acceleration of free fall.
(c) If the pilot has a mass of 85 kg, what is his apparent weight at the instant when the aircraft is at
(i) its highest point,
(ii) its lowest point?

[491 m; 4.2×10^3 N; 5.8×10^3 N]



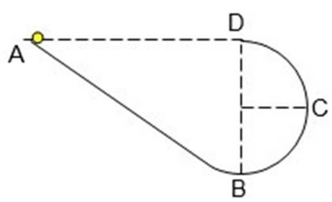
19. In a home laundry dryer, the cylindrical tub of a washer rotates steadily about a horizontal axis. The smooth-walled washer rotates at a steady rate of 0.84 revolutions in one second and the radius of the tub is 0.36 m. Assume that g = 10 m s^{-2}.
(a) By considering the forces acting on a piece of cloth, suggest what happens to this piece of cloth as it reaches the highest point, A, in a spin cycle.



A manufacturer proposed a washer design which he claims is an improvement to the washer design mentioned above. In this "improved" design, the rate of rotation of the tub is chosen such that a small piece of cloth loses contact with the tub when the cloth is at an angle of 65° with the horizontal, as shown in the diagram.
(b) Calculate the rate of rotation required in this new design. [0.80 rev s^{-1}]
(c) Suggest one reason why this washer design is preferred over the previous washer design.

Challenging Questions

20. A small sphere is released from rest and allowed to slide down along a smooth track ABCD. BCD is circular with D levelled with A.
At which part of the track will the small sphere break contact?



21. A 0.30 kg mass attached to a 1.5 m long string is whirled around in a horizontal circle of speed 6.0 m s^{-1}. By NOT assuming that the string is nearly horizontal during its motion, determine
(a) the centripetal acceleration of the mass.
(b) the tension in the string.
[Hint: $\sin^2\theta + \cos^2\theta = 1$]

[26 m s^{-2}; 8.2 N]

~ THE END ~