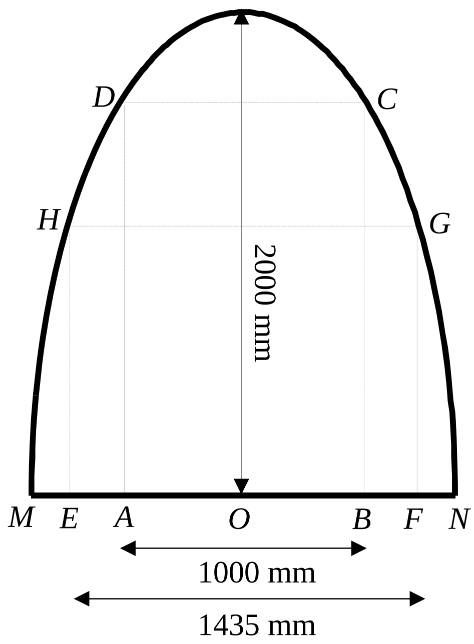
1. **(a)** Use the substitution $u = 2x - 1$ to find $\int (x-1)\sqrt{2x-1}\ \mathrm{d}x$. [3]

(b) Find $\int \sin 3x \sin x\ \mathrm{d}x$. [2]

2. Given that $\mathrm{f}(x) = \ln\left(\dfrac{1-x}{1+\cos x}\right)$, find $\mathrm{f}(0)$, $\mathrm{f}'(0)$, $\mathrm{f}''(0)$ and $\mathrm{f}'''(0)$. Hence, write down the first four non-zero terms in the Maclaurin's series for $\mathrm{f}(x)$. [6]

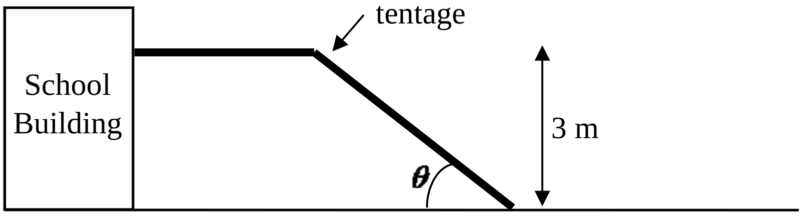
Use your result to deduce the first four non-zero terms in the Maclaurin's series for $\ln\left(\dfrac{1+\cos x}{1+x}\right)$. [1]

3. A tunnel is built to facilitate the transportation of goods by trains between Country X and Country Y. Due to differences in the rail systems between the two countries, two types of tracks are used – the international track with track gauge of 1435 mm and the narrow track with track gauge of 1000 mm (in rail transport, track gauge is the spacing on a railway track). It is known that the cross-section of the tunnel is a half ellipse with centre O and width MN (see diagram). The maximum height of the tunnel is 2000 mm. To standardize the volumes of the goods to be transported, the areas $ABCD$ and $EFGH$ are made equal. Find the width of the tunnel MN, giving your answer to the nearest mm. [4]



[Turn over

4. A group of student councillors bought an 8 m long piece of tarpaulin canvas to build a tentage for a school event. The canvas would extend diagonally at an angle of θ from the ground to a height of 3 m, where it will then stretch horizontally to the school building (see diagram for the cross-sectional view).



(i) Given that the total cross-sectional area covered by the canvas is A m^2, show that $A = 24 - 9\operatorname{cosec}\theta + \frac{9}{2}\cot\theta$. [2]

(ii) Find, by differentiation, the largest possible value of A. [5]

5. The function f is defined by $\mathrm{f} : x \mapsto \ln(2-x)^2 - 2,\ x \in \mathbb{R},\ x \neq 2.$

(i) By considering the graph of $y = \mathrm{f}(x)$, give a reason why f^{-1} does not exist. [2]

The function h is defined by $\mathrm{h} : x \mapsto \mathrm{f}(x),\ x \geq a,\ a \in \mathbb{R}$.

(ii) Find the largest possible domain of h such that h^{-1} exists. [1]

(iii) Define h^{-1} in a similar form. [3]

(iv) Find the set of values of x which satisfies the equation $\mathrm{hh}^{-1}(x) = \mathrm{h}^{-1}\mathrm{h}(x)$. [2]

6. A curve C has parametric equations $x = \mathrm{e}^t + \sin t$, $y = \mathrm{e}^t - \cos t$.

(i) Describe the shape of C as $t \to -\infty$. [2]

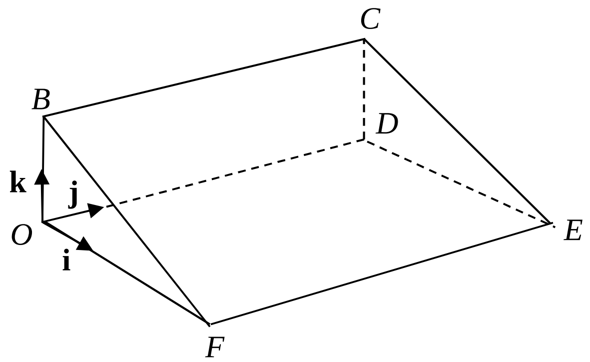
(ii) Find the Cartesian equation of the normal to C at the point $P\left(\mathrm{e}^\theta + \sin\theta, \mathrm{e}^\theta - \cos\theta\right)$, where $\theta > 0$, giving your answer in the form $y = mx + c$. [3]

The normal to C at P meets the y-axis at the point D, and the curve C meets the positive x-axis at the point E that has integral coordinates.

(iii) Find the coordinates of D and E. [3]

(iv) Describe the locus of the mid-point of DE as θ varies. [2]

7.



The diagram shows a vehicle ramp $OBCDEF$ with horizontal rectangular base $ODEF$ and vertical rectangular face $OBCD$. Taking the point O as the origin, the perpendicular unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to the edges OF, OD and OB respectively. The lengths of OF, OD and OB are $2h$ units, 3 units and h units respectively.

(i) Show that $\overrightarrow{OC} = 3\mathbf{j} + h\mathbf{k}$. [1]

(ii) The point P divides the segment BC in the ratio $2:1$. Find $\overrightarrow{OP}$ in terms of h. [1]

(iii) A vector parallel to the normal of the plane $BCEF$ is given as $a\mathbf{i} + b\mathbf{k}$. By the use of a scalar product, find the value of $\frac{a}{b}$. Hence find the Cartesian equation of the plane $BCEF$ in terms of h. [4]

(iv) Take $h = 3$. Find the shortest distance from the point $Q(1, 2, 2)$ to the plane OPF. [4]

8. Mac has a $400\,000\text{ m}^2$ farm and on his farm, an area of $60\,000\text{ m}^2$ is covered in weeds in June, and in September, the area increases to $69\,500\text{ m}^2$. The growth of weeds is such that the area covered in weeds increases at a monthly rate directly proportional to its area. At the same time, Mac does weeding at a constant rate of $4\,000\text{ m}^2$ per month. Let the area of the farm covered in weeds at time t (in months) be $A\text{ m}^2$.

(i) By considering a differential equation, show that $A = \alpha e^{kt} + \lambda$, where α, k and λ are constants to be determined. [5]

(ii) The region covered in weeds is in the shape of a circle. Find the monthly rate at which the radius of the region changes when the radius is 200 m. [2]

(iii) Mac understands that having some weeds on the farm can be beneficial. Find the monthly rate at which Mac needs to do weeding if $\frac{dA}{dt} = 0$ in September. [2]

(iv) Comment on the significance of $\frac{dA}{dt} = 0$ in the context of this question. [1]

9. A researcher conducted a study on the radioisotope, Iodine-131 (I-131) which has a half-life of 8 days (i.e., the amount of I-131 is halved every 8 days). He first introduced $1000\,\text{mg}$ of I-131 in an empty Petri dish on Day 1 and tracked the amount of I-131 in the dish.

(i) State the amount of I-131 in the dish at the end of 16 days. [1]

After every 16 days, i.e., on Day 17, Day 33, Day 49 etc., the researcher added $1000\,\text{mg}$ of I-131 to the dish.

(ii) Find the amount of I-131, to the nearest mg, in the dish immediately after $1000\,\text{mg}$ of I-131 was added on Day 49. [3]

(iii) Show that the amount of I-131 in the dish will never exceed $1334\,\text{mg}$. [2]

The researcher discovered that he accidentally used a different radioisotope, Iodine-125 (I-125) on Day 1, which has a half-life of 60 days instead. He checked that he had indeed used the correct I-131 on other occasions.

(iv) Find the total amount of radioisotopes I-125 and I-131 in the dish on Day 121, giving your answer correct to the nearest mg. [4]

10. **(a)** The equation $z^3 - az^2 + 2az - 4\text{i} = 0$, where a is a constant, has a root i.

(i) Briefly explain why i^* may not necessarily be a root of the equation. [1]

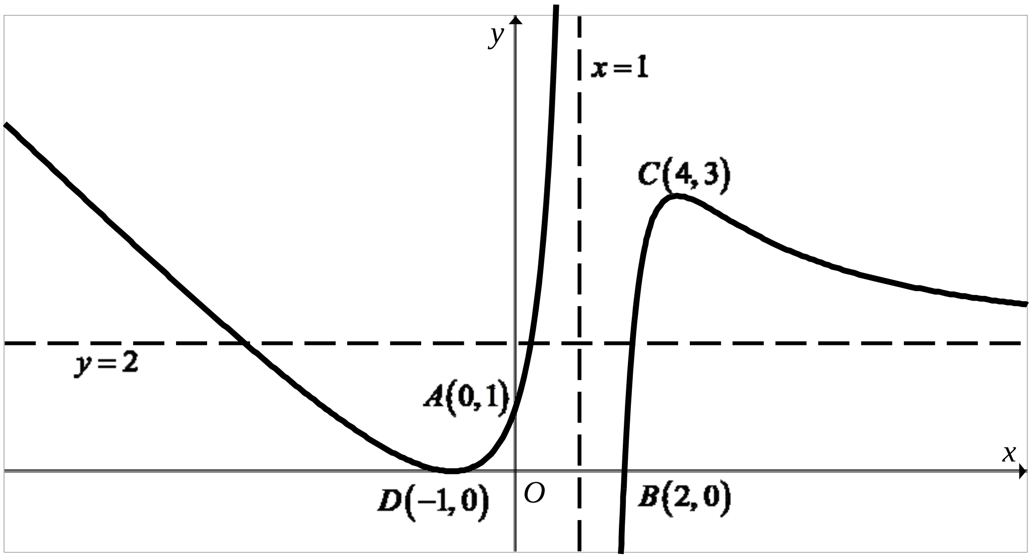
(ii) Show that $a = 2 + \text{i}$. [2]

(iii) Hence, find the remaining roots of the equation in exact form. [5]

(b) The complex number z satisfies the equations $|z^* - 1 + \text{i}| = 2$ and $\arg(z - 2\text{i}) = \frac{\pi}{4}$.

By considering $z = x + \text{i}y$, find z. [5]

11. **(a)**



The graph of $y=\mathrm{f}(x)$ is shown in the diagram above. It has asymptotes $x=1$ and $y=2$. The points A, B, C and D have coordinates $(0,1)$, $(2,0)$, $(4,3)$ and $(-1,0)$ respectively, with C and D being stationary points.

On separate diagrams, sketch the graphs of

(i) $y=\mathrm{f}(2x-1)$, [3]

(ii) $y=\mathrm{f}'(x)$, [3]

(iii) $y=-\sqrt{\mathrm{f}(x)}$. [3]

In each case, state the coordinates of A, B, C and D whenever applicable, and the equations of any asymptotes.

(b) A curve G with equation $y=\dfrac{2x+a}{x^2-b}$, where a and b are constants, has a stationary point at $\left(-4,-\dfrac{1}{4}\right)$ and a vertical asymptote $x=-2$.

(i) Find the values of a and b. [2]

(ii) Find the range of values of x for which G is increasing and is concave downwards. [2]

(iii) By sketching a suitable line on the same diagram as G, find the number of distinct real roots of the equation $5x^3+2x^2-14x+7=0$. [3]