JURONG PIONEER JUNIOR COLLEGE
9749 H2 PHYSICS

WORK, ENERGY AND POWER

Content

- **Work**
- **Energy conversion and conservation**
- **Efficiency**
- **Potential energy and kinetic energy**
- **Power**

Learning Outcomes

Candidates should be able to:

(a) define and use work done by a force as the product of the force and displacement in the direction of the force

(b) calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = p\Delta V$

(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation

(d) show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems

(e) derive, from the equations for uniformly accelerated motion in a straight line, the equation $E_k = ½ mv^2$

(f) recall and use the equation $E_k = ½ mv^2$

(g) distinguish between gravitational potential energy, electric potential energy and elastic potential energy

(h) deduce that the elastic potential energy in a deformed material is related to the area under the force-extension graph

(i) show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems

(j) derive, from the definition of work done by a force, the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface

(k) recall and use the equation $E_p = mgh$ for gravitational potential energy changes near the Earth's surface

(l) define power as work done per unit time and derive power as the product of a force and velocity in the direction of the force

Introduction

This topic involves the core ideas of Conservation Laws as well as Systems and Interactions. The concept of energy is one of the most important concepts in Science. Energy is present in various forms. The transformation of energy from one form to another and its conservation are essential in the study of physics. The concept of work links energy and force. Work is a means of transferring energy through application of a force. The concepts of work and energy can be applied to the dynamics of a mechanical system without resorting to Newton's laws. Beyond mechanics, this approach can be applied in a wide range of phenomena in electromagnetism, thermal and nuclear physics. In addition, the energy approach often provides a simpler analysis than direct application of Newton's laws because (1) energy is a scalar whereas force is a vector, and (2) only the initial and final states of a situation need to be considered without involving the intermediate processes.

1 Work

(a) define and use work done by a force as the product of the force and displacement in the direction of the force

1.1 Definition of Work

The work done by a force on an object is defined as the **product of the force and the displacement** of the object **in the direction of the force**.

1.2 Work done by a constant force

If the force acting on the object is constant in magnitude and direction throughout the motion of the object, then the work done W by a force F in moving an object through a displacement s is

$$W = Fs\cos\theta$$

where W is the work done on the object by the constant force F (unit: J)
F is the magnitude of the constant force acting on the object (unit: N)
s is the displacement of the object (unit: m)
θ is the angle between F and s

Fig.1 to Fig. 4 show different scenarios which the equation above can be applied.

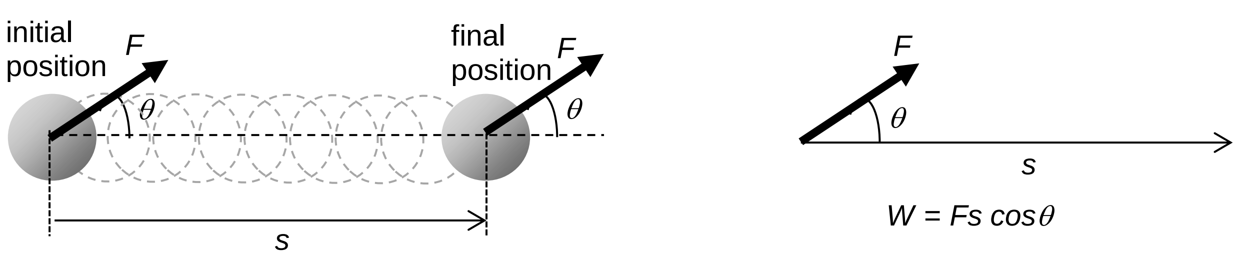


Fig. 1: Constant force at an angle θ to the displacement

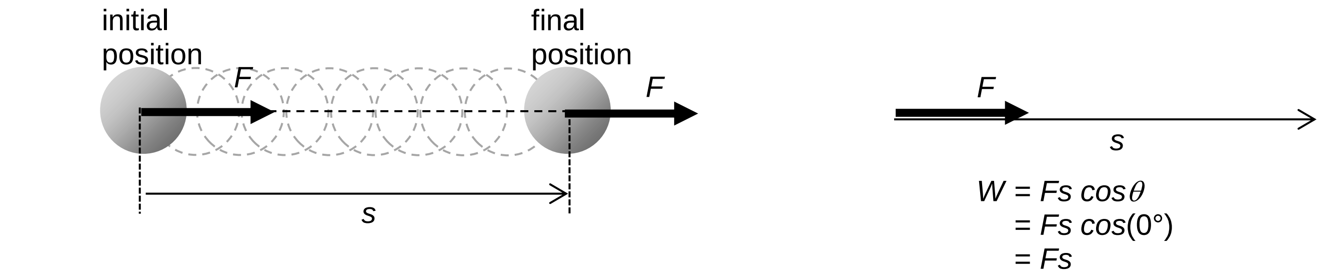


Fig. 2: Constant force in the same direction as the displacement

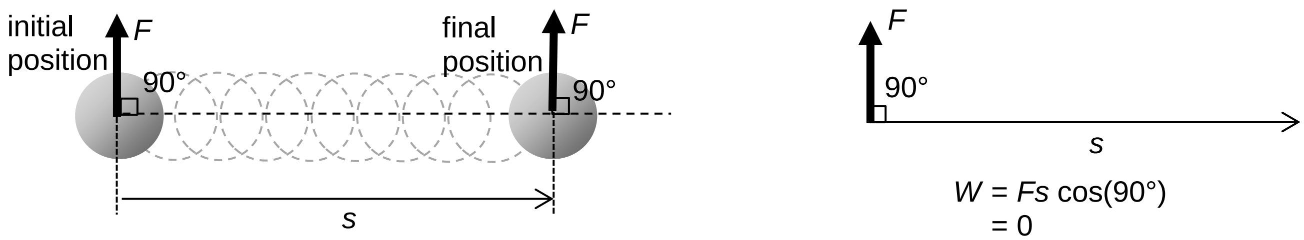


Fig. 3: Constant force 90° to the displacement

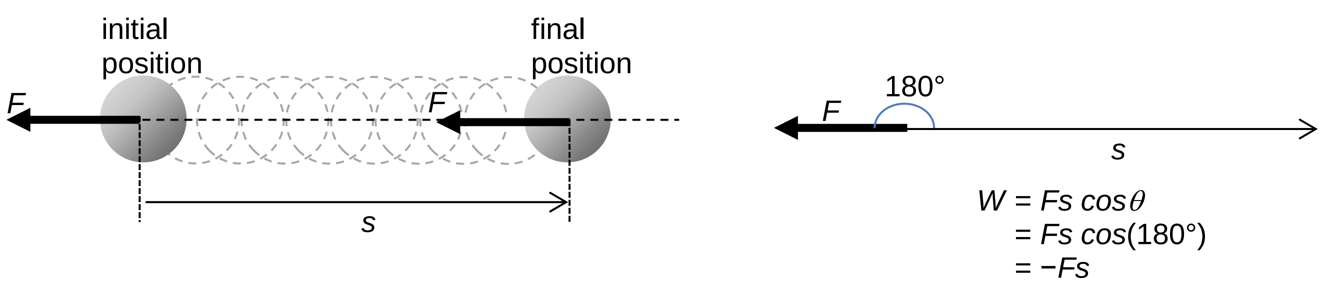


Fig. 4: Constant force opposite to the displacement

NOTE:

- The SI unit of work done is the joule (J).

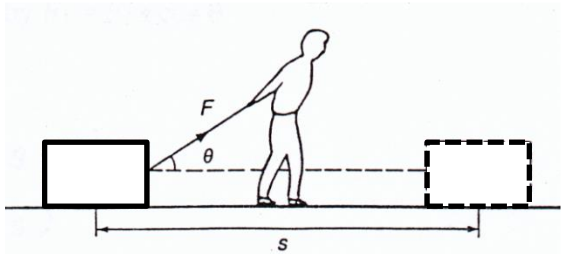
One joule (1 J) is defined as the work done by a force of one newton (1 N) when its point of application moves through a displacement of one metre (1 m) in the direction of the force.

$$\text{Work done} = (1\text{ N})\,(1\text{ m})\,(\cos 0°) = 1\text{ J}$$

- Work done is a **scalar** quantity.

Example 1

The figure below shows a man pulling a box of mass 20 kg with a constant force of 50 N at an angle θ of 35° to the ground. The box moves through a horizontal distance of 10 m and the frictional force between the box and the ground is 11 N.



Determine the work done by the following forces on the box:

(a) The force applied by the man.

(b) The frictional force on the box.

(c) The normal contact force on the box by the ground.

(d) The gravitational pull by Earth.

(e) The resultant force on the box.

<u>Solution</u>

(a) Work done by the force applied by the man $= (50)\ (10)\ (\cos 35°)$
$= 410$ J

(b) Work done by the frictional force $= (11)\ (10)\ (\cos 180°)$
$= -110$ J

(c) Work done by normal contact force $= (N)\ (10)\ (\cos 90°)$
$= 0$ J

(d) Work done by gravitational pull $= (W)\ (10)\ (\cos 90°)$
$= 0$ J

(e) Work done by resultant force $= (F_R)\ (s)\ (\cos \theta)$
$= (F\cos\theta - f)\ (10)\ (\cos 0°)$
$= (50 \cos 35° - 11)\ (10)$
$= 300$ J

Alternatively,
Part **(a)** to **(d)** constitute all the forces acting on the box. Hence the sum of all the work done by each of the force is also the work done by the resultant force on the box.

Work done by resultant force = (a) + (b) + (c) + (d)
$= 410 + (-110) + 0 + 0$
$= 300$ J

NOTE:

- If a force *F* is in the **same direction** as the object's displacement, or has a component in the same direction as the object's displacement, the work done by force *F* on the object is **positive**.

- If a force *F* is in the **opposite** direction to the object's displacement, or has a component in the opposite direction to the object's displacement, the work done by force *F* on the object is **negative.**

- Work is a process of transferring energy through the application of a force. If the work done on an object is **positive**, it means energy is **transferred to** the object. If the work done is **negative**, it means energy is **transferred out** from the object.

  In Example 1, the 50 N force exerted by the man on the box results in the man transferring energy to the box. This could be in the form of kinetic energy if the man is pulling a box which is initially at rest, until the box reaches a certain final velocity. On the other hand, the 11 N frictional force exerted on the box results in energy being removed from the box. This is in the form of heat loss to the surroundings. Hence the box cannot reach a higher speed than it could have if the frictional force was absent.

- **Work done on an object <u>by</u> a <u>force</u>**

  A statement like 'Determine the work done on the box' is not clear as there are many forces acting on the box.

  A statement like 'Determine the work done on the box by the man when pulling the box' should be interpreted as 'Determine the work done on the box <u>by</u> the <u>force exerted by the man</u> when pulling the box'.

  A statement like 'Determine the work done on the box by Earth should be interpreted as 'Determine the work done on the box <u>by</u> the <u>gravitational force</u> that is acting on the box'.

- **Work done <u>against</u> a <u>force</u>**

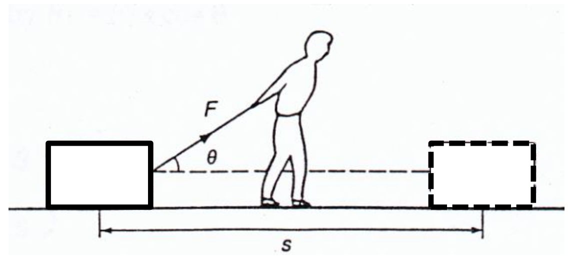
  A statement like 'work done <u>against</u> force *F*' should be interpreted as:

Work done against force *F* = − Work done by force *F* on object

  In Example 1, the work done by the frictional force on the box = −110 J. Therefore, the work done against the frictional force = 110 J.

Example 2

Relook at Example 1.



Determine

(a) the work done by the man.

(b) the work done on the box by the ground.

(c) the work done against friction.

<u>Solution</u>

(a) Work done by the man = Work done by the 50 N pull
= (50) (10) (cos 35°)
= 410 J

(b) Work done on the box by ground = Work done by frictional force
= (11) (10) (cos 180°)
= −110 J

(c) Work done against friction = − Work done by friction
= 110 J

1.3 Work done by a variable force

- If the force F acting on a body is not constant, the work done by a force can be given by the **<u>area under the *F-x* graph</u>** where **<u>*x* is the displacement in the direction of the force</u>**. Refer to Fig. 5.

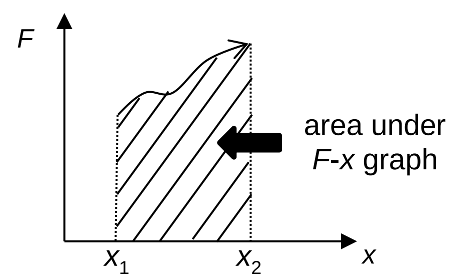


Fig. 5: Work done is given by area under *F-x* graph

- In general, the work done over a displacement x_1 *to* x_2 is calculated by applying:

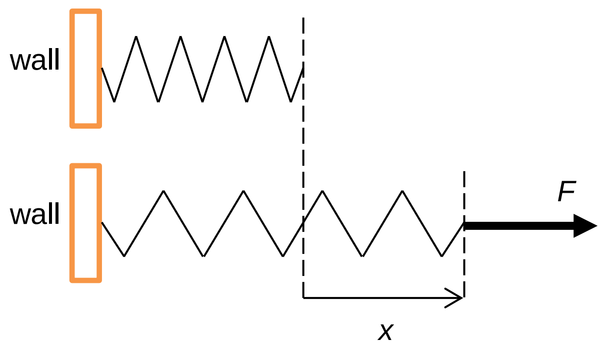
$$W = \int_{x_1}^{x_2} F\,dx$$

- We will examine two cases of work done by variable force, namely, work done to stretch a spring and work done by an expanding gas.

1.4 Work done to stretch a spring

(h) deduce that the elastic potential energy in a deformed material is related to the area under the force-extension graph

- Fig. 6 below shows an elastic spring with one end attached to the wall and the other end stretched to a horizontal displacement *x* by a variable force *F*. The spring obeys Hooke's Law and has a spring constant *k*.

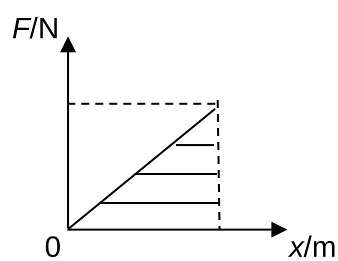


when no force exerts on the spring, the extension is zero

when a variable force *F* with a magnitude of $F = kx$ exerts on the spring, the extension is *x*

Fig. 6: Extension of a spring

- Work done by the variable force is determined by plotting a *F*-*x* graph, where *F* has a magnitude of $F = kx$.



Work done, *W* in stretching spring = **Area under *F*-*x* graph (shaded)**

$$= \frac{1}{2}Fx$$

$$= \frac{1}{2}(kx)x$$

$$= \frac{1}{2}kx^2$$

- The work done by the variable force is transferred to the spring and stored as **elastic potential energy.**

Elastic potential energy stored in the spring = $\frac{1}{2}kx^2$

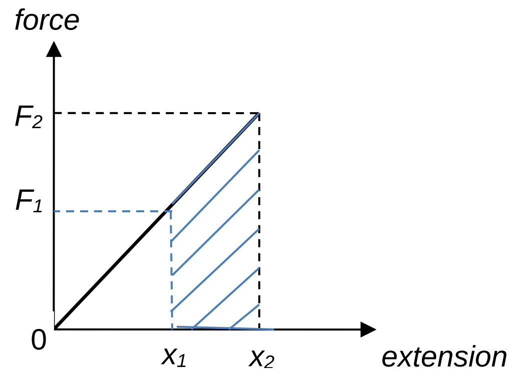
Example 3

Calculate the work done in stretching a spring of force constant 25 N m^{-1} from an extension of 1.0 cm to an extension of 2.0 cm.

Solution

Assume that Hooke's Law is obeyed.

Work done by an external agent in stretching the spring can be found from the area under the force-extension graph.



Work done by external agent $W = \int_{x_1}^{x_2} F\,dx$

$= \frac{1}{2}F_2x_2 - \frac{1}{2}F_1x_1$

$= \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$

$= \frac{1}{2}(25)(0.020)^2 - \frac{1}{2}(25)(0.010)^2$

= 0.00375 J

1.5 Work done by gas which is expanding against an external pressure

(b) calculate the work done in a number of situations including the work done by a gas which is expanding against a constant external pressure: $W = p\Delta V$

- Fig. 7 shows a volume of gas contained in a cylinder fitted with a movable piston of cross-sectional area *A*.

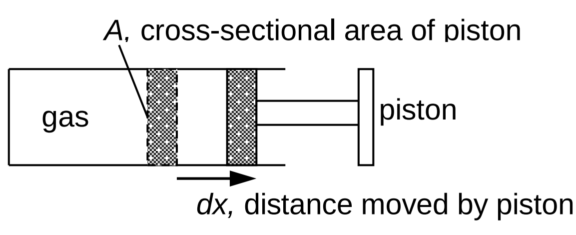


Fig. 7: Work done by gas against external pressure

- The piston is frictionless and the gas is expanding slowly, at a constant speed against an external pressure, *p*.
- The volume of the gas increases from V_i to V_f as the piston is pushed out by a distance of *dx*.
- The force exerted by the gas on the piston is $F = pA$.

- The work done by the gas,

$$W = \int_{x_1}^{x_2} F\,dx$$
$$= \int_{x_1}^{x_2} pA\,dx$$
$$= \int_{x_1}^{x_2} p\,(A\,dx)$$
$$= \int_{V_1}^{V_2} p\,dV$$

= **area under a *p*-*V* graph**

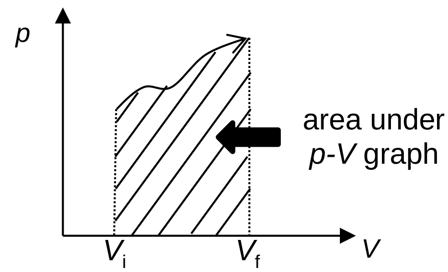


Fig. 8: Work done by a gas expanding from V_i to V_f

- If the gas is expanding against a constant external pressure p, then the work done by the gas is

$$W = p\,(V_f - V_i)$$
$$W = p\,\Delta V$$

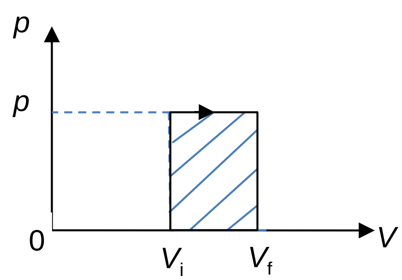


Fig. 9: Work done against constant external pressure

Example 4

A gas is kept in a cylinder with a movable and frictionless piston. The volume of the gas changes slowly from 2.0×10^{-3} m^3 to 1.0×10^{-3} m^3 against a constant atmospheric pressure of 1.0×10^5 Pa.

Determine the work done

(a) by the gas against external pressure,

(b) on the gas by external pressure.

Solution

(a) Work done by the gas against external pressure

$= p\Delta V$

$= (1.0\times10^5)(1.0\times10^{-3} - 2.0\times10^{-3})$

$= -1.0\times10^2$ J (The negative sign indicates that the gas is being compressed.)

(b) Work done on the gas by external pressure

= − work done by the gas against external pressure

$= 1.0\times10^2$ J

2 Energy Conversion and Conservation

(c) give examples of energy in different forms, its conversion and conservation, and apply the principle of energy conservation

(g) distinguish between gravitational potential energy, electric potential energy and elastic potential energy

2.1 Different forms of energy

- Energy is defined as the **capacity to do work**.
- Energy is a scalar quantity. The SI unit of energy is the joule (J).

The following are some examples of different forms of energy:

energy	source
mechanical kinetic energy potential energy types of potential energy: **gravitational** potential energy **electric** potential energy **elastic** potential energy	total mechanical energy = Sum of kinetic energies and potential energies energy possessed by all objects in motion. e.g. moving car energy possessed by a system by virtue of its relative position or its state of deformation energy possessed by an object by virtue of its own mass and its position in a gravitational field. e.g. object on top of a building energy possessed by an object by virtue of its own charge and its position in an electric field energy possessed by an object by virtue of its state of deformation. e.g. compressed or stretched springs, a bent diving board
electrical	due to flow of charge or current. e.g. current in a closed circuit
chemical	energy possessed by a fuel by virtue of its chemical composition. e.g. oils, wood, food, chemicals in electric cells
nuclear	energy in nucleus of atoms due to nuclear composition. e.g. energy released in atomic bombs, produced by nuclear reactors
radiant (light)	energy in the form of electromagnetic waves. e.g. visible light, radio waves, ultraviolet, x-rays, microwaves
internal energy	energy possessed by particles of matter in the form of kinetic energy due to motion of the particles in the matter and potential energy which depends on the separation between these particles

Renewable and Non-renewable energy

Renewable sources of energy are those that can be **replaced or replenished** each day by the Earth's natural processes. e.g. wind, geothermal, solar and tidal energy.

Wind Park off Copenhagen, Denmark.

Wind generates 20% of Denmark's electricity.

Rise and fall of a buoy generates 20 kW.

World largest Solar Park in Longyangxia Dam, China.

With 4 million solar panels and an output capacity of 850 megawatts, Longyangxia Dam Solar Park is the largest solar farm in the world.

Non-renewable sources of energy are those that are **finite or exhaustible** because it takes several million years to replace them, e.g. fossil fuels like coal, oil and natural gas, nuclear energy from the fission of uranium nuclei.

2.2 Examples of energy conversion

examples	energy conversions
bouncing ball	• As the ball falls, its gravitational potential energy is converted into kinetic energy. • When the ball hits the ground, the ball is deformed during the collision. Its kinetic energy is converted into elastic potential energy. Some kinetic energy may be lost as thermal energy or sound energy. • The elastic potential energy is converted back into kinetic energy as the ball regains its original shape. • The kinetic energy is converted into gravitational potential energy as the ball bounces upwards, until it reaches its highest position.
diver jumping off a diving board	• The diver uses his gravitational potential energy to do work in bending the diving board. • The work done by man is transferred to the board and stored as elastic potential energy in the board, which is then converted into kinetic energy of the diver as the diver is pushed upwards and off the diving board. • At the same time, some of the elastic potential energy in the board is lost as heat and sound due to dissipative forces in the diving board.
hammering a nail into a wooden block	• A person uses the chemical energy in his muscles to do work against the gravitational force acting on the hammer by the Earth in order to lift the hammer. • The work done is converted into the gravitational potential energy of the hammer in its raised position. • As the hammer falls, its gravitational potential energy is converted into kinetic energy. • When the hammer hits the nail, its kinetic energy is used to do work in driving the nail into the wooden block, producing sound energy in the air and thermal energy in the block, nail and hammer.
hydroelectric plant	• In a hydroelectric plant, water falls from a height on to a turbine causing it to turn. • The turbine turns a coil placed in a magnetic field, thereby generating an electric current. • Therefore, potential energy of the water is converted into kinetic energy of the turbine, which is converted into electrical energy.

2.3 Energy conservation

(f) *recall and use the equation* $E_k = \frac{1}{2} mv^2$

(k) *recall and use the equation* $E_p = mgh$ *for gravitational potential energy changes near the Earth's surface*

- ***Principle of conservation of energy*** *states that:*

> Energy can **neither be created nor destroyed** in any process. It can only be **transformed (converted)** from one form to another or **transferred** from one body to another but the total amount in any isolated system must remain constant.

- We can apply this principle to perform detailed calculations of the total mechanical energy of an isolated system.

- For an isolated system, there is no transfer of energy between the system and its surroundings (e.g. no heat transfer).

- When an isolated system undergoes a change (e.g. an object is displaced, or an object's velocity is changed), the system's total energy remains constant.

- For an isolated system undergoing a change, these are a few common ways to calculate the system's changes in energy:

$$(\text{potential energy} + \text{kinetic energy})_{\text{initial}} = (\text{potential energy} + \text{kinetic energy})_{\text{final}}$$

$$(E_p + E_k)_{\text{initial}} = (E_p + E_k)_{\text{final}}$$

$$(\text{change in potential energy}) + (\text{change in kinetic energy}) = 0$$

$$\Delta E_p + \Delta E_k = 0$$

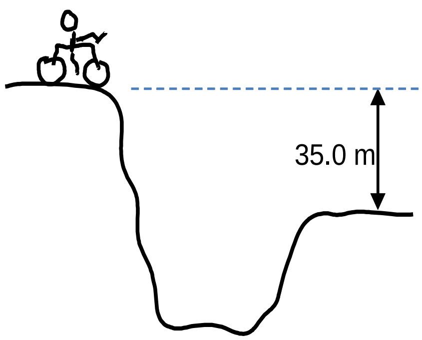
loss in E_p = gain in E_k

gain in E_p = loss in E_k

Example 5

The figure below shows a motorcycle rider leaping between cliffs, across a canyon. At the instant when the motorcycle leaves the higher cliff, it has a horizontal velocity of 38.0 m s^{-1}. The difference in height between the two cliffs is 35.0 m.

Assuming air resistance is negligible, determine the speed with which the motorcycle reaches the lower cliff.



Solution

By the Principle of Conservation of Energy,

$(E_p + E_k)_{\text{initial}} = (E_p + E_k)_{\text{final}}$

$$\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$$

$$v_f^2 = v_i^2 + 2g(h_i - h_f)$$

$$v_f = \sqrt{38.0^2 + 2(9.81)(35.0)}$$

$$v_f = 46.2 \text{ m s}^{-1}$$

OR

By the Principle of Conservation of Energy,

$\Delta E_p + \Delta E_k = 0$

$$-mgh + \left(\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2\right) = 0$$

$$v_f^2 = v_i^2 + 2gh$$

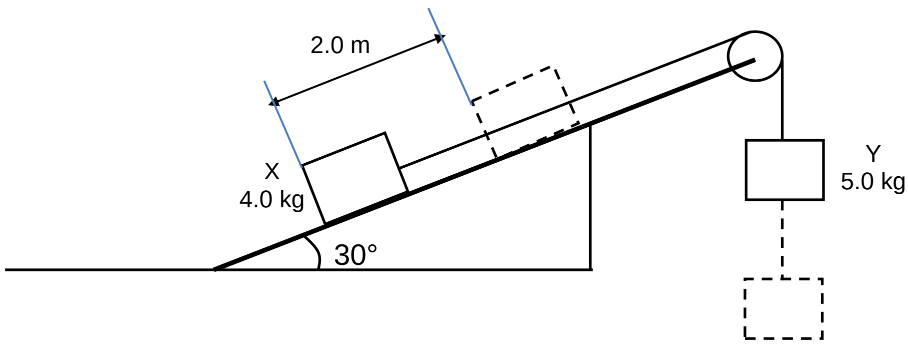
$$v_f = 46.2 \text{ m s}^{-1}$$

Example 6

The diagram shows two bodies X and Y connected by a light cord passing over a light, free-running pulley. X starts from rest and moves on a smooth plane inclined at 30° to the horizontal.

(a) Calculate the total final kinetic energy of the system after X has travelled 2.0 m along the plane.

(b) If the plane is rough, and exerts an average frictional force of 5.0 N, calculate the total final kinetic energy of the system after X has travelled 2.0 m along the plane.



Solution

(a) Applying the Principle of conservation of energy,

$$\text{loss in } E_\text{p} = \text{gain in } E_\text{k}$$
$$(mgh)_Y - (mgh)_X = E_k - 0$$
$$(5.0)(9.81)(2.0) - (4.0)(9.81)(2.0\sin 30°) = E_k$$
$$E_k = 59 \text{ J}$$

(b) Applying the Principle of conservation of energy,

$$\text{loss in } E_\text{p} = \text{gain in } E_\text{k} + \text{work done against friction}$$
$$(mgh)_Y - (mgh)_X = (E_k - 0) + (5.0)(2.0)$$
$$(5.0)(9.81)(2.0) - (4.0)(9.81)(2.0\sin 30°) = E_k + 10$$
$$E_k = 49 \text{ J}$$

3 Efficiency

(d) show an appreciation for the implications of energy losses in practical devices and use the concept of efficiency to solve problems

- Practical devices are used to convert one form of energy into a form which is useful for mankind. For example, a diesel engine converts chemical energy stored in diesel fuel into mechanical energy for lifting objects.

- When a practical device works, besides converting energy input into useful energy output, some unwanted energy in the form of heat and sound are also produced. These unwanted energies are lost to the surrounding, and should be kept as small as possible.

energy input = useful energy output + wasted energy output

- Efficiency of a practical device is a measure of the useful energy output to the total energy input.

$$\eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

Example 7

A worker pushes a cart of iron ore from a mine to a collection point. He expends 0.10 MJ of energy in the process while the total work done by him is 18 kJ.

Calculate his efficiency.

<u>Solution</u>

$$\text{efficiency, } \eta = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%$$

$$= \frac{18 \times 10^3}{0.10 \times 10^6} \times 100\%$$

$$= 18\%$$

Example 8

A car has a mass of 800 kg and the efficiency of its engine is rated at 18 %.

Determine the amount of fuel needed to accelerate the car from rest to 60 km h^{-1}. (Assume that the energy equivalent of 1 litre of fuel is 1.3 × 10^8 J.)

The engine converts chemical potential energy in the fuel (input energy) to kinetic energy of the car (useful output energy).

$$\eta = \frac{\text{useful energy output}}{\text{energy input}} \times 100\%$$

$$18\% = \frac{\text{car's gain in } E_K}{E_{\text{input}}} \times 100\%$$

$$18\% = \frac{(E_K)_{final} - (E_K)_{initial}}{E_{\text{input}}} \times 100\%$$

$$18\% = \frac{\frac{1}{2}(800)(\frac{60000}{3600})^2 - 0}{E_{\text{input}}} \times 100\%$$

$$E_{\text{input}} = 6.17 \times 10^5 \text{ J}$$

$$\text{Amount of fuel needed} = \frac{6.17 \times 10^5}{1.3 \times 10^8} = 4.7 \times 10^{-3} \text{ litre}$$

4 Derivation of Kinetic Energy and Potential Energy

(e) *derive, from the equations for uniformly accelerated motion in a straight line, the equation* $E_k = ½ mv^2$

(j) *derive, from the definition of work done by a force, the equation* $E_p = mgh$ *for gravitational potential energy changes near the Earth's surface*

4.1 Derivation of $E_k = ½mv^2$

- Kinetic energy is a scalar quantity that represents the energy associated with the body due to its motion.

- Consider an object of mass *m* on a horizontal frictionless surface. A constant force *F* acts on the object, causing it to accelerate from an initial velocity of *u* to a final velocity *v*, covering a displacement of *s*.

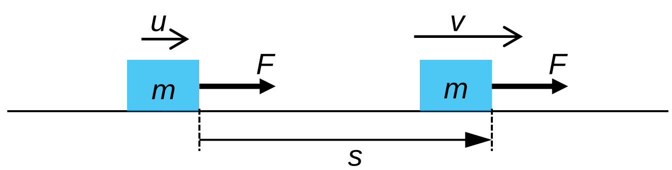


Fig. 10: Object accelerating under constant force

- **Work done by force *F*** on the object is **transferred to** the object as its **kinetic energy** (law of conservation of energy). An equation for kinetic energy would need to link work done to the velocity of the object. This can be done by applying the kinematics equations.

- Work done by force F on the object,

$$W = Fs \quad \text{- - - (1)}$$

- Since the surface is frictionless, F is the net force acting on the mass. Hence F in equation (1) can be replaced by $F = ma$.

$$W = Fs$$
$$W = (ma)s \quad \text{- - - (2)}$$

- Since the net force is constant, **acceleration *a* is constant,** and the kinematics equation of motion applies.

$$v^2 = u^2 + 2as$$
$$a = \frac{1}{2s}(v^2 - u^2) \quad \text{- - - (3)}$$

- Substitute (3) into (2), work done by force F is:

$$W = (ma)s$$
$$W = \mathrm{m}\left[\frac{1}{2s}(v^2 - u^2)\right]s$$
$$W = \frac{1}{2}m(v^2 - u^2)$$
$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

- **Work-Energy Theorem** states that:

The ***net work done*** by external forces acting on an object (or a system) is equal to the object's (or system's) ***change in kinetic energy***.

work done by net force, $W = (E_\mathrm{k})_{\mathrm{final}} - (E_\mathrm{k})_{\mathrm{initial}}$

$W = \Delta E_\mathrm{k}$

- If the object accelerates from rest to a final velocity of v, all the work done is transferred into the object's kinetic energy E_k:

$$E_\mathrm{k} = \frac{1}{2}mv^2$$

Example 9

A car of mass 800 kg moving at 30 m s^{-1} along a horizontal road is brought to rest by a constant frictional force of 5000 N.

(a) Determine the work done by friction.
(b) Hence determine the displacement of the car.

Solution

(a) Net force on the car is the frictional force.
Hence using work-energy theorem,

$$\begin{aligned}\text{work done by friction} &= \text{work done by net force}\\ &= (E_k)_{\text{final}} - (E_k)_{\text{initial}}\\ &= \frac{1}{2}(800)(0)^2 - \frac{1}{2}(800)(30)^2\\ &= -3.6\times10^5 \text{ J}\end{aligned}$$

(b) Work done by friction = $Fs \cos \theta$

$$-3.6\times10^5 = (5000)\, s \cos(180°)$$
$$s = 72 \text{ m}$$

Example 10

A box of mass 10 kg is accelerated from rest to a speed of 5.0 m s^{-1} along a smooth horizontal road by a horizontal pulling force of 200 N.

Calculate the distance travelled by the box in reaching its final speed.

Solution

Taking rightwards as positive,
W.D. by pulling force = change in kinetic energy

$$W = Fs = \tfrac{1}{2}mv^2 - \tfrac{1}{2}mu^2$$
$$(200)s = \tfrac{1}{2}(10)(5.0)^2 - 0$$
$$s = 0.625 \text{ m} = 0.63 \text{ m}$$

Alternative method

Using Equations of Motion:

$$v^2 = u^2 + 2as$$
$$(5.0)^2 = 0 + 2\left(\frac{200}{10}\right)s$$
$$s = 0.625 \text{ m} = 0.63 \text{ m}$$

4.2 Derivation of $E_p = mgh$

- Near the Earth's surface, the gravitational field is **uniform** and the acceleration of free fall *g* is taken to be constant.
- Consider an object of mass *m* lifted by a constant force *F*, vertically upwards at **constant speed** from ground level to a height *h*.
- At constant speed, the applied force must **balance** the weight of object, $F = mg$
- Since the object's velocity is constant, its kinetic energy is also **constant**.

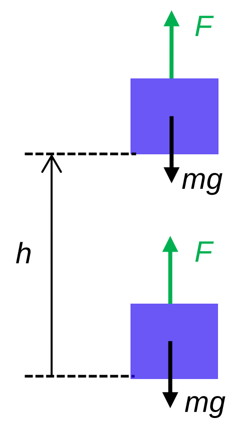


Fig. 11: object raising at constant velocity

- Hence work done by the applied force *F* on the object is transferred to the object to increase its gravitational potential energy (law of conservation of energy).
- Work done by *F* is

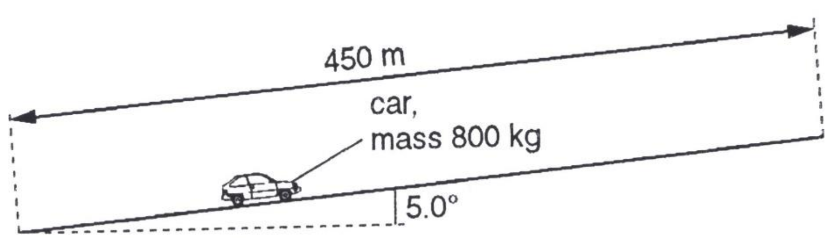
$$W = Fs$$
$$W = (mg)h$$
$$\boxed{E_p = mgh}$$

Example 11

A car travels upwards along a straight road inclined at an angle of 5.0° to the horizontal, as shown in the figure below.

The length of the road is 450 m and the car has a mass 800 kg.

What is the gain in gravitational potential energy of the car when it reaches the top of the slope?



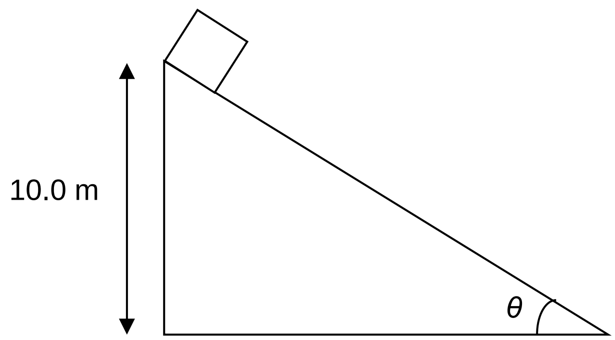
<u>Solution</u>
Increase in height, $h = (450)(\sin 5.0^o) = 39.2$ m

Gain in G.P.E = $mgh = (800)(9.81)(39.2) = 3.08 \times 10^5$ J $= 3.1 \times 10^5$ J

Example 12

A block weighing 800 N moves down a frictionless slope inclined at 20º to the horizontal of height 10.0 m as shown below.

Determine the speed of the block at the bottom of the slope, assuming that the block moves off from the top at an initial speed of 5.0 m s^{-1}.



Solution

Total initial energy of the block (at the top) = total final energy of the block (at the bottom)

$$KE_i + GPE_i = KE_f + GPE_f$$

$$\tfrac{1}{2}mv_i^2 + mgh_i = \tfrac{1}{2}mv_f^2 + mgh_f$$

$$mgh_i - mgh_f = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

(loss in GPE = gain in KE)

$$gh_i - gh_f = \tfrac{1}{2}v_f^2 - \tfrac{1}{2}v_i^2$$

$$(9.81)(10) - (9.81)(0) = \tfrac{1}{2}v_f^2 - \tfrac{1}{2}(5.0)^2$$

$$v_f = 14.9 \text{ m s}^{-1} = 15 \text{ m s}^{-1}$$

4.3 Force and Potential Energy in a uniform field

(i) show an understanding of and use the relationship between force and potential energy in a uniform field to solve problems

- When a body is placed in a uniform field, the body will experience a constant force F acting on it, regardless of which point the body is placed at.

- Examples of a body in a uniform field include:

  - <u>A ball of mass m falling freely near Earth's surface.</u>
    In the region near Earth's surface, Earth's gravitational field is constant (the acceleration of free fall, g, is a constant value of 9.81 m s^{-2}). At any point in this region, the ball always experiences a constant gravitational force of $F_g = mg$.

    As the ball moves from rest under the effect of the gravitational force F_g, the ball will lose gravitational potential energy. That is, work done by the gravitational force causes the ball to lose gravitational potential energy.

  - <u>A particle of charge q placed between 2 oppositely-charged parallel metal plates</u>
    The region between the 2 metal plates has a constant electric field strength of E. At any point in this region, the charge will experience a constant electric force of $F_E = qE$.

    As the charge moves from rest under the effect of the electric force F_E, the charge will lose electric potential energy. That is, work done by the electric force causes the charge to lose electric potential energy.

- Consider a body placed in a uniform field. A constant field force F acts on the body and moves the body along a distance Δx in the direction of the force. The work done by this force F is

$$W = F\Delta x$$

- In a closed system, the principle of conservation of energy dictates that this work done must be compensated by a decrease in potential energy (i.e. ΔU is negative).

$$W = F\Delta x = -\Delta U$$

$$F = -\frac{\Delta U}{\Delta x}$$

In general (including non-uniform field), $\boxed{F = -\frac{dU}{dx}}$

where F is the force acting on the body placed at that point in the field

$\frac{dU}{dx}$ is the **potential energy gradient**.

- Potential energy gradient $\frac{dU}{dx}$ is defined as the change in the potential energy of the body with variation of the distance x from the source of the force field (units: J m^{-1}).

- The negative sign shows that the field force acts in the direction of decreasing potential energy.

- In words, the equation $F = -\frac{dU}{dx}$ means the field force F is equal in magnitude to the potential energy gradient and it acts in the direction of decreasing potential energy.

- Graphically, if a graph of the body's potential energy is plotted against its position in the field, the **negative** of the gradient of the graph gives the value of the force at that position.

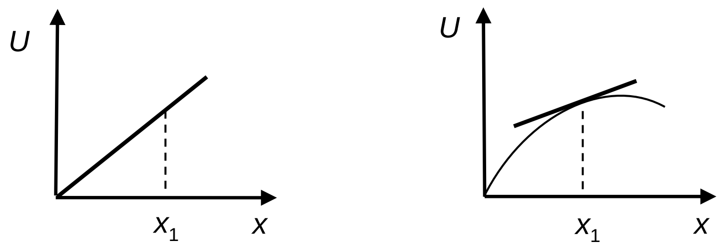


Fig. 12: U-x graphs in Uniform and Non-uniform fields

Example 13

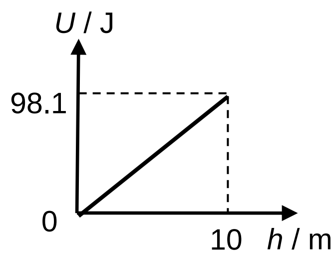
An object of mass 1 kg is raised from Earth's surface to a height of 10 m.

(a) Plot a graph to show how the object's gravitational potential energy varies with its height from Earth's surface.

(b) Use the graph to determine Earth's gravitational pull on the object.

Solution

(a)



(b)
$$F = -\frac{dU}{dx}$$
$$= -\frac{98.1-0}{10-0}$$
$$= -9.81\text{N}$$

The negative sign shows that the gravitational force is downwards.

5 Power

(l) define power as work done per unit time and derive power as the product of a force and velocity in the direction of the force

5.1 Definition of Power

- Power is defined as the **rate at which work is done.**

- Power is a **scalar** quantity.

- The S.I. unit for power is the watt, W.

One watt is defined as the power when one joule of work is done per second, or one joule of energy is converted per second.

5.2 Average Power

- Average power <P> is the total work done W divided by the total time interval t, OR the change in energy ΔE divided by the total time interval t.

Mathematically, $$P = \frac{W}{t}$$ or $$P = \frac{\Delta E}{t}$$

where P is power (unit: W or J s^{-1})
W is the work done (unit: J)
ΔE is the energy converted (unit: J)
t is the time taken (unit: s)

5.3 Instantaneous Power

- Consider a constant force F being applied on an object such that F cancels other forces on the object, resulting in a net force of zero on the object. Under the influence of this force F, the object moves at a constant velocity of v and cover a displacement s over a time interval ***t.*** Refer to Fig. 13.

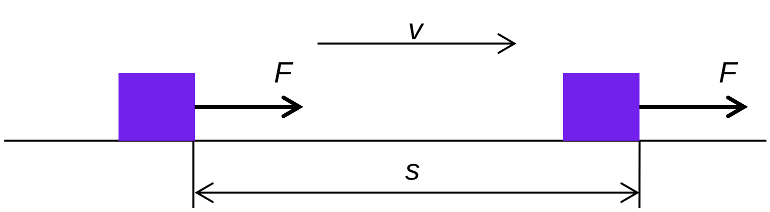


Fig. 13: Work done by a constant force moving object at constant velocity

- Work done by the force F, $W = Fs$

- Power delivered by the force F,

$$P = \frac{W}{t}$$

$$P = \frac{Fs}{t}$$

$$P = F\left(\frac{s}{t}\right)$$

$$\boxed{P = Fv}$$

NOTE:

- Force F is **<u>not the net force</u>** of the object. It is the **applied force** that does work to propel the object forward. If the object is moving at constant velocity, force F will have the same magnitude as the opposing force (e.g. friction, drag). For such a case, the net force on the object is zero.

- The equation is true for both constant and changing force F. When a changing force F acts on the object, the instantaneous power at one instant is the product of the applied force F at that instant and the object's velocity at the same instant.

  Instantaneous Power, $\boxed{P_{instantaneous} = F_{instantaneous}\ v_{instantaneous}}$

Example 14

A cyclist travelling along a horizontal road experiences a constant drag of 20 N. The combine mass of the cyclist and the bicycle is 100 kg and he delivers a constant power.

(a) If his maximum velocity is 5.0 m s^{-1}, what is the power output of the cyclist?
(b) If his velocity at one instant is 4.0 m s^{-1}, what is his acceleration at this instant?

<u>Solution</u>

(a) At max velocity, cyclist cannot increase his velocity anymore, implying that his acceleration and hence the resultant force is zero.

Define the direction of travel of the cyclist as +ve.

$$F = ma$$
$$F_{\text{drive}} + (-F_{\text{drag}}) = 0$$
$$F_{\text{drive}} = F_{\text{drag}} = 20 \text{ N}$$

Power of cyclist, $P = F_{\text{drive}}\, v$
$= (20)\,(5.0)$
$= 100$ W

(b) Power of cyclist, $P = F_{\text{drive}}\, v$
$100 = F_{\text{drive}}\,(4.0)$
$F_{\text{drive}} = 25$ N

$$F = ma$$
$$F_{\text{drive}} + (-F_{\text{drag}}) = ma$$
$$25 + (-20) = (100)\,a$$
$$a = 0.050 \text{ m s}^{-2}$$

5.4 Electrical Energy Consumption - kilowatt hour (kWh)

- When calculating the cost of electricity consumption, the amount of electrical energy used is denoted in kilowatt-hours (kWh) and not in J (joule).

- 1 kWh is the amount of energy supplied by a power source of 1 kW over a time interval of 1 h.

  i.e. 1 kWh = (1000 W) (3600 s)
  $= 3.6 \times 10^6$ J
  = 3.6 MJ

Example 15

A rice cooker uses 680 W when plugged into a 240 V mains.

It takes 25 min to cook the rice. If the electrical energy costs 23.02 cents per kilowatt-hour, how much does it cost to cook the rice?

Electricity consumed $= 0.680 \times \frac{25}{60}$ kWh

Cost of electricity $= \$\left(0.2302 \times 0.680 \times \frac{25}{60}\right) = \$\ 0.065$

References

1 Robert Hutchings, *Bath Advanced Science Physics (2^{nd} Edition)*, Nelson Thornes

2 Chris Mee, Mike Crundell, *Advanced Level Practical Work for Physics*, Hodder & Stoughton

3 Chris Mee, Mike Crundell, Brian Arnold, Wendy Brown, *AS/A2 Physics*, Hodder & Stoughton

4 Douglas C. Giancoli, *Physics - Principles with Applications (6^{th} edition)*, Prentice Hall

Appendix

Non-renewable/ Exhaustive sources of Energy

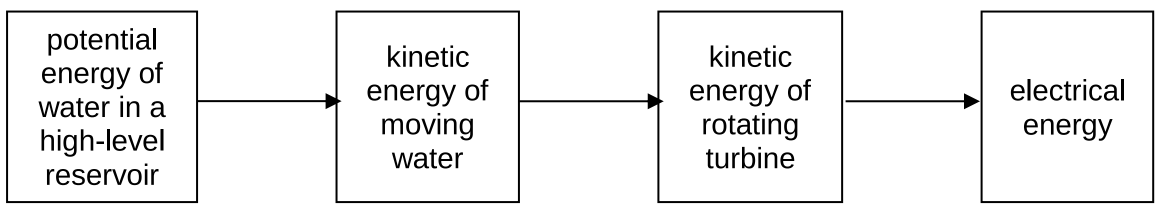
1. Oil and gas
2. Coal
3. Nuclear power

Renewable sources

Energy sources that are continually being replenished naturally.

1. <u>Hydroelectric power</u>

   Lots of rainfall in the mountains is the essential requirement for hydroelectricity.



2. <u>Solar power</u>

   Solar cells are thin wafers made from silicon and they convert solar energy directly into electrical energy. Electrons in the silicon gain energy from light photons to create a voltage.

3. <u>Wind power</u>

   Windmills sited in windy coastal areas convert the kinetic energy of the wind into electrical energy.

4. <u>Tidal power</u>

   The tide rises and falls twice each day. By trapping each high tide and allowing it to flow out through generators, tidal energy can be converted to electrical energy.

5. <u>Geothermal power</u>

   The Earth's interior is very hot and the heat flow can increase the temperature of underground rock basins to 200°C or more. By pumping water down to the hot rocks, steam can be drawn up to generate electricity.

Understanding power efficiency at power stations in Singapore

Brief Description:

The power generation sector accounts for about half of the total fuel consumption in Singapore. This energy is used to generate electricity, which is then used to power Singapore homes and industries. Given the large amounts of fuel consumed by the power generation sector, improving energy efficiency in power generation would have a significant impact on the overall **energy efficiency** in Singapore.

Power Generation Efficiency:

Since the liberalisation of the electricity market, the power generation companies have moved towards the increased use of natural gas combined-cycle turbine technology. From 2000 to 2006, the electricity generated by natural gas has increased from 19% to 78%. As a result, Singapore's overall power generation efficiency has improved from 38% in 2000 to 44% in 2006.

	2000	2001	2002	2003	2004	2005	2006	2020
electricity generated by natural gas	19%	29%	44%	60%	69%	74%	78%	95%

(Source: Energy Market Authority. www.ema.gov.sg)

Regenerative Braking (Hybrid vehicles, Mass Transit Rail, F1 cars)

Vehicles driven by electric motors use the motor as a generator when using regenerative braking: it is operated as a generator during braking and its output is supplied to an electrical load; the transfer of energy to the load provides the braking effect.

Regenerative braking is used on hybrid/electric cars to recoup some of the energy lost during stopping. This energy is saved in a storage battery and used later to power the motor whenever the car is in electric mode.

Applied to MRT trains: A conventional electric train braking system uses dynamic braking, where the remaining kinetic energy of a moving train is dissipated as waste, mainly in the form of heat. In SMRT trains, this excess energy is not wasted.

Using Regenerative Braking, the electric motors reverse the current, which slows down the train. At the same time, it generates electricity to be returned to the power distribution system.

This generated electricity is used to power other trains within the network. Any excess electricity is used to offset power demands of other loads such as air conditioners and lighting in stations.

This saves about 15% of the energy costs for running the trains.

http://www.smrt.com.sg/about_us/documents/environment/Regenerative_Braking_in_SMRT_Trains.pdf

~~ THE END ~~