RVHS 2021 JC2 H2 MATHS COMMON TEST

Section A: Pure Mathematics [60 marks]

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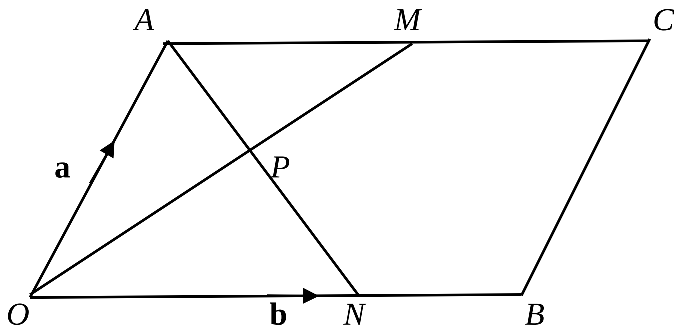

As shown in the above parallelogram *OACB*, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. It is given that *M* is the midpoint of *AC* and $ON : NB = 2:1$.The point *P* is the intersection of the line *OM* and AN.

(i) By letting $\overrightarrow{OP} = \lambda\overrightarrow{OM}$ and $\overrightarrow{AP} = \mu\overrightarrow{AN}$, find the value of λ and μ and hence the position vector of *P* in terms of **a** and **b**. [3]

(ii) Given that the point *Q* lies on *AN* produced and is such that $AN : NQ = 2:1$, find the position vector of *Q* and hence determine if *Q*, *B* and *C* are collinear. [3]

2 Solve the following simultaneous equations for complex numbers *z* and *w*.

$$2\mathrm{i}z^* - w = 1$$
$$z + w^* = 4$$
[6]

3 Let $z = 1 + \sqrt{3}\mathrm{i}$ $\arg(w) = \frac{\pi}{4}$ and $|zw| = 2\sqrt{2}$.

(i) Find *w* in the form $x + \mathrm{i}y$ where $x, y \in \mathbb{R}$. [2]

(ii) Find the least positive integer value of *n* such that $(zw)^n$ is purely imaginary. [3]

Sequence & Series not tested in 2023 H2 MA CT

4 It is given that $\mathrm{f}(r) = 1 - \frac{3}{(r+2)!}$, where *r* is a non-negative integer.

Show that $\mathrm{f}(r) - \mathrm{f}(r-1) = \frac{a(r+1)}{(r+2)!}$ for some constant *a* to be determined. [1]

(i) Using the above result, find $\sum_{r=1}^{n} \frac{r+1}{(r+2)!}$ in terms of n . [3]

(ii) By using the result in part (i), deduce the value for $\sum_{r=3}^{n} \frac{r-1}{r!}$, simplifying your answer. [2]

5 The functions f and g are defined by

$$\mathrm{f} : x \to 1 - \frac{1}{x^2 - 1} \quad \text{for } x \in \mathbb{R},\ x < -1,$$
$$g : x \to x^2 + 4x + 5 \quad \text{for } x \in \mathbb{R},\ x \le 1.$$

(i) Find $\mathrm{f}^{-1}(x)$ and state the domain of f^{-1}. [3]

(ii) State whether the composite functions fg and gf exist, justifying your answer. Hence find the range of the composite function(s) that exist(s). [4]

Integration not tested in 2023 H2 MA CT

6 A curve C has parametric equations

$$x = \mathrm{e}^t,\ y = \sin t$$

where $0 \le t \le \pi$.

(i) Find $\int \mathrm{e}^t \sin t \,\mathrm{d}t$. [4]

(ii) Sketch the graph of C. [1]

(iii) Find the area of the region bounded by C, the x-axis, the lines $x = 1$ and $x = \mathrm{e}^a$, where a is a positive real constant, in terms of a. [2]

7 The plane π passes through the points with coordinates $(-1, 0, 1)$, $(2, 1, -1)$ and $(1, -3, 2)$. The line l passes through the point $P(16, 20, 16)$ and is parallel to the vector $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(i) Find the cartesian equation of π. [3]

(ii) Find the coordinates of the point of intersection of l and π. [2]

(iii) Find the acute angle between l and π. [2]

(iv) Determine a vector equation of the line of reflection of l in π. [4]

8 The R_0 value of a virus refers to the average number of new infections resulting from each infected person.

(a) A new virus is discovered. Extensive testing reveals that by the n^{th} day, a total of 268 people were infected. In the following day, 686 people were newly infected. In the day following that, 2401 people were newly infected. Virologists propose to use a geometric progression to model the number of infections due to the virus. They propose the model:

$$u_n = aR_0^{\,n-1}$$

where u_n is the number of new infections on the n^{th} day after the virus emerged, and a is the number of people infected initially.

(i) Using this model, estimate the value of R_0. [2]

(ii) Contact tracers identify 16 people who were likely originally infected by the virus. Estimate how long the virus had been spreading before it was identified. [3]

(iii) The government introduces a series of measures that reduces the value of R_0 to less than 1. Explain the trend in the number of new infections in the long run. [1]

(b) Administrators of a hospital in a region suffering from an outbreak observe a pattern in their data. As new patients are admitted overnight, the number of patients each morning is 20% higher than the night before. During the course of the day they are able to discharge 10 patients. The hospital has a total of 500 beds available.
The hospital has 200 patients on the morning of the first day of the observation.
Given that v_n denotes the number of patients in hospital on the morning of the n^{th} day, show that $v_n = 140 \times 1.2^{n-1} + k$, where k is a constant to be determined.
Hence, find the least value of n when the hospital first runs out of capacity. [6]

Section B: Statistics [40 marks]

9 An ice-cream café sells a mega sundae in which the customer will order 5 scoops of ice-cream. The staff will arrange the 5 scoops on a plate in either a circular or row manner. The café offers exactly 10 distinct flavors of ice-cream daily.

(a) Find the number of ways the mega sundae can be arranged in a circular manner if 5 distinct flavors of ice-cream are to be chosen. [1]

(b) A customer decides to order the vanilla, chocolate and banana flavour among the flavours for all 5 scoops of ice-cream but does not specify the number of scoops for each flavor, i.e. some flavor of ice-cream can have 2 or more scoops.
Find the number of ways the staff can arrange the mega sundae in a row manner.[2]

10 Mr Tan, who is the school Admin Manager, believes that JC students in his school take 22 minutes on average to buy and consume their lunch in the school canteen.

In the data collection process, Mr Tan uses the school's computer programme to assign every JC students in his school a number based on the alphabetical order of their statutory names and select 50 students using a random number generator. The selected students do a survey on the duration they spent for lunch on a particular day in the school canteen.

The random variable X, which denotes the time spent for lunch in the school canteen, in minutes gives the following results: $\sum x = 1117$, $\sum x^2 = 25061$.

(i) Justify whether the group of 50 JC students chosen by Mr Tan is a random sample. [1]

(ii) Calculate the unbiased estimates for the population mean and variance for the variable X. [2]

(iii) Carry out a test at the 5% level of significance to determine if Mr Tan's belief is valid. [3]

11 A game is played in which a tetrahedral die is thrown. The sides of the die are numbered 1, 2, 3, 4. Whatever number on the face that the die land on, that many number of fair coins are tossed. A player's final score X, is the number of coins showing heads.

(i) Determine the probability distribution table of X. [3]

(ii) Find $\mathrm{E}(X)$ and $\mathrm{Var}(X)$. [3]

12 (a) A particular disease is known to infect approximately 1 in every 10 000 people. A test for this disease is 99.9% accurate, meaning the outcome of the test matches the true result 99.9% of the time. Find the probability that a person has the disease given that the person is tested positive. [2]

(b) Researchers looking into whether wearing masks helps reduce the spread of a particular disease. A representative sample of 50 people is taken. In that sample, 7 people who wore masks got sick, and 6 people who did not wear masks got sick. A total of 31 people in the sample wore masks.

The events M and C are defined as follows:

M: The event that a randomly chosen person wore a mask.
C: The event that a randomly chosen person caught the virus.

(i) Determine if M and C are independent. [2]

(ii) After looking at the result of the sample collected, the researcher comments that: "Wearing a mask is pointless, since more people who got sick wore masks than didn't." By comparing suitable probabilities, justify if you agree with the comment. [2]

13 On average, one in 40 scientific calculators manufactured by a company is faulty. An educational store manager decides to order n scientific calculators from the company. The event that a randomly chosen scientific calculator is faulty is independent of that of another randomly chosen calculator. The number of these scientific calculators which are faulty is denoted by the random variable X.

(i) State, in the context of the question, an assumption needed for X to be well modelled by a binomial distribution. [1]

Assume that the number of faulty scientific calculators in the manager's order follows a binomial distribution.

(ii) For the case when $n = 30$, find the mean and variance for the distribution of X and hence determine the probability that X differs from its mean by an amount less than 2 times its standard deviation. [3]

(iii) Find the largest number of scientific calculators that the manager needs to order if the probability that the number of faulty calculators being more than 2 is at most 0.05. [3]

14 In this question, you should state the parameters of any distribution that you use.

A local fish market is famous for two types of fish it sells. The masses in kg of each type of fish may be assumed to follow independent normal distributions, with means and standard deviations as shown in the following table.

Fish	Mean / kg	Standard Deviation / kg
Garoupa	0.50	0.025
Snapper	0.45	0.020

(a) On a particular day, Mrs Tan brought \$50 to the fish market to purchase some fish. The price of Snapper was \$18.50 per kg.

(i) Find the probability that 2 Garoupas and 3 Snappers weigh more than 2.25 kg. [4]

(ii) Suppose Mrs Tan decided to purchase only Snapper. Determine the maximum number k of Snappers she could buy so that she was at least 95% certain that she had enough money to pay for the purchase. [4]

(b) On another day, Mrs Tan decided to purchase only Garoupas from the market. She brought sufficient money to buy 6 Garoupas. Find the probability that among the 6 Garoupas purchased, more than 3 of them were of masses greater than 0.48 kg each. [3]

(c) Suppose Mrs Tan bought 6 Garoupas, find the probability that the lightest Garoupa has mass greater then 0.48 kg. [1]

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