Title	Junior College 'A' Levels H1/H2 – Binomial Distribution
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Conditions for a random variable to be modelled by a Binomial Distribution includes the following:

The experiment must consist of Bernoulli trials (Where there are two possible outcomes in the experiment, which we can call as "outcome" and "complement outcome".)

All trials in the experiment have to be independent (Where the probability of obtaining "outcome" of each trial isn't affected by a previous trial or will affect a future trial within the experiment).

All trials within the experiment have to be identically distributed. (Such that each Bernoulli trial has constant probability of obtaining "outcome" and "complement outcome".)

Example of common outcomes and complement outcomes as follows:

Outcome	**Complement Outcome**
Yes	No
No	Yes
Success	Failure
Failure	Success
Picking a red ball	Not picking a red ball

If the experiment in question satisfies the above requirements, it is said the follow a Binomial Distribution with parameters n and p,where,
n refers to the total number of trials.
p refers to the probability of obtaining the "outcome" of each trial.

When it is written in standard Binomial Notation, it looks like the following, where X refers to the random variable.

$$X \sim B(n, p)$$

The formula for Binomial Probability distribution of a specific number of trials to be calculated for is given below:

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

The formula for Binomial Probability distribution from 0 up till x number of trials can be calculated as follows:

$$P(X \leq x)$$
$$= \binom{n}{0} p^0 (1-p)^{n-0} + \binom{n}{1} p^1 (1-p)^{n-1} + \binom{n}{2} p^2 (1-p)^{n-2} + \cdots + \binom{n}{x} p^x (1-p)^{n-x}$$

Since A Levels permit the use of Texas Instrument Graphing Calculators in exam condition, I would also have to demonstrate the two rather commonly used functionality in TI-84 Plus CE, namely BinomialPDF and BinomialCDF that is equivalent the above two respectively.

BinomialPDF can be used when you are tasked to find $P(X = x)$ given parameters n and p.

Example 1. Given the random variable $X \sim B\left(3, \frac{1}{6}\right)$, find $P(X = 2)$.

This case requires the use of BinomialPDF functionality, which can be accessed by pressing the following buttons on the TI-84 PLUS CE in the following order:

Press [2ND] then [VARS] in exact order as mentioned and press the down arrow key repeatedly until you see your calculator cursor reaching an option called “binompdf”.

Press [ENTER] key on the calculator and you should see something similar to the below example on the screen

trials:
p:
x value:
Paste

Key in value of n into the number of trials, and press [ENTER]
Key in value of p into the field “p” and press [ENTER]
Key in number of trials being computed for into the field “x value” and press [ENTER]
Once the cursor is on the “Paste”, you should have the following,

trials: 3
p:1/6
x value: 2
Paste

Press [ENTER] after checking if the values are correct and you should see the following on your graphing calculator screen

binompdf(3, 1/6, 2)

Press [ENTER] and the answer should appear as follows (If the calculator isn't in fraction mode):

binompdf(3, 1/6, 2)
.0694444444

Example 2. Given the random variable $X \sim B\left(3, \frac{1}{6}\right)$, find the probability that $P(X \leq 2)$.

This case requires the use of BinomialCDF functionality, which can be accessed by pressing the following buttons on the TI-84 PLUS CE in the following order.

Press [2ND] key, then Press [VARS] in exact order as mentioned and press the down arrow key repeatedly until you see your calculator cursor reaching an option called "binomcdf".

Press [ENTER] key on the calculator and you should see something similar to the below example on the screen

trials:
p:
x value:
Paste

Key in value of n into the number of trials, and press [ENTER]
Key in value of p into the field "p" and press [ENTER]
Key in number of trials being computed for into the field "x value" and press [ENTER]
Once the cursor is on the "Paste", you should have the following,

trials: 3
p:1/6
x value: 2
Paste

Press [ENTER] after checking if the values are correct and you should see the following on your graphing calculator screen

binomcdf(3,1/6,2)

Press [ENTER] and the answer should appear as follows (If calculator isn't in fraction mode)

binomcdf(3,1/6,2)
.9953703704

Using graphing calculator manipulation to solve problems involving binomial distribution:

(Questions mostly taken from Power Math H2 Second Edition Volume 2 by PK Lim and slightly changed)

Q1. In XYZ Junior College, 65% of the student population are male.
12 students are randomly selected from this school. Find the probability that

(i) Exactly 3 of them are male.
(ii) At most 3 of them are male.
(iii) Not less than 3 of them are male.
(iv) More than 5 of them are male.
(v) Between 4 to 8 of them inclusively are male.

Written working	Graphing Calculator Actions Performed
(i) X: Number of male students selected, out of 12 students $X \sim B(12, 0.65)$ $P(X = 3) = 0.00476$	Go to "binompdf" option and press enter Key in trials as 12, Key in p as 0.65 Key in x value as 3 Answer obtained = 0.00476

(ii) $P(X \leq 3) = 0.00561$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 3 Answer obtained = 0.00561 Final Answer: 0.00561
(iii) $P(X \geq 3) = 1 - P(X \leq 2) = 0.999$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 2 Answer obtained = 8.479084920E-4 Press 1 – 8.479084920E-4 to get 0.9991520915 Final Answer: 0.999
(iv) $P(X > 5) = 1 - P(X \leq 5) = 0.915$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 5 Answer obtained = 0.0846320652 Press 1 – 0.0846320652 to get 0.9153679348 Final Answer: 0.915
(v) $P(4 \leq X \leq 8) = P(X \leq 8) - P(X \leq 3)$ $= 0.648$	Go to "binomcdf" option and press enter. Key in trials as 12 Key in p as 0.65 Key in x value as 8 Answer obtained = 0.6533473038 Go to "binomcdf" options and press enter.

	Key in trials as 12 Key in p as 0.65 Key in x value as 3 Answer obtained = 0.0056097523 Press 0.6533473038- 0.0056097523=0.6477375515 Final answer: 0.648

Q2. A survey shows that only 60% of all drivers in a town uses their seatbelts.

(i) 5 drivers in the town are randomly selected. Let X be the number of drivers who use their seat belts, out of 5 drivers from this town. Find the exact standard deviation of X.

(ii) If a sample of 500 drivers are taken, what is the expected number of drivers who use their seat belts?

(i)

X: Number of drivers who uses their seat belts, out of 5 drivers selected from this town.

$X \sim B(5, 0.6)$

Variance of Binomial Distribution $\sigma^2 = np(1-p)$

Standard Deviation of Binomial Distribution $\sigma = \sqrt{np(1-p)} = \sqrt{5(0.6)(1-0.6)} = \sqrt{1.2}$

(ii)

Y: Number of drivers who uses their seat belts, out of 500 drivers selected from this town

$Y \sim B(500, 0.6)$

Expected number of drivers who uses their seat belts $E(Y) = np = 500(0.6) = 300$

Q3. A random variable $X \sim B(n, p)$ has mean of 8 and variance of 6.

(i) Find the value of both n and p.

(ii) Find the probability that X lies within 1 standard deviation of the mean.

(i)

Equation 1: $np = 8$

Equation 2: $np(1-p) = 6$

Sub Equation 1 into Equation 2.

$8(1-p)=6$

$8-8p=6$

$-8p=6-8=-2$

Equation 3: $p=\frac{2}{8}=0.25$

Substitute Equation 3 into Equation 1.

$n(0.25)=8$

$$n=\frac{8}{0.25}=32$$

(ii)

Let X represent the binomial random variable above.

$X \sim B(32, 0.25)$

$\sigma^2=np(1-p)=32(0.25)(1-0.25)$

$\sigma^2=6$

$\sigma=\sqrt{6}$

$\sqrt{6}=2.449489743$

$\mu=np=8$

Range of values as follows: $\mu-\sqrt{6}<X<\mu+\sqrt{6}$

$P(8-\sqrt{6}<X<8+\sqrt{6})$

$P(5.5505<X<10.4495)$

Rewritten to fit a discrete probability distribution where X has to be **strictly** within 1 standard deviation from the mean, it looks like the following.

$P(6 \leq X \leq 10)=P(X \leq 10)-P(X \leq 5)=0.8464053667-0.1530030994$

$=0.693$

Q4. In a particular IT show, the probability that a customer bought the newest "Tiun" brand laptop is given by p. Sixty-five customers were randomly chosen.

Given that $p < 0.5$ and the variance of the number of "Tiun" laptop bought is 2.7, find the most probable of number of "Tiun" laptop bought by customers out of 65 randomly chosen customers.

Variance = $\sigma^2 = 2.7 = 65p(1-p) = 65p - 65p^2$
$-65p^2 + 65p = 2.7$
$p = 0.04342$ or $p = 0.95658$ (Rejected)

Since it is stated in question that $p < 0.5$ we have to reject one solution above as shown.

X: Number of customers who bought "Tiun" brand laptop out of 65 randomly chosen customers
$X \sim B(65, 0.04342)$

Graphing Calculator instructions for finding most probable number (AKA the mode) of the probability distribution above.

Press the [Y=] button
Press [2ND] and then press [VARS]
Scroll down repeatedly until you see "binompdf" and press [ENTER]

Key in "65" into the trials field. Key in 0.0434 as value of p. Key in letter X by pressing [Alpha] then [STO]. Press Enter twice and you should see the following on your graphing calculator

```
Plot1  Plot2  Plot3
\Y1= binompdf(65, 0.0434, X)
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

Once you reach the above stage, press [2ND] and then press [Graph] and the following should appear on the graphing calculator screen below.

X	Y1
0	.05583
1	.16473
2	.23927
3	.22807
4	.16064
5	.08886
6	.04033

In this case we already know that the most probable value of number of customers who bought the "Tiun" laptop is 2 as it has the highest probability (0.23927), but in other cases, you may need to scroll down all the way from 0 to n to locate the most probable value.

Q5. The random variable R denotes the number of red cars observed in a survey of n cars.

(i) Write down, in context, two assumptions needed to model R by a binomial distribution
It may be assumed that R has the distribution $B(20, p)$

(ii) Given that $p = 0.15$, find $P(4 \leq R < 8)$.

Written working and answers	Graphing calculator actions performed
(i) The event of observing red cars every time within the survey has to be independent and the probability of observing any one red car is constant for all observation within the survey.	
(ii) X: Number of red cars observed out of 20 cars $X \sim B(20, 0.15)$ $P(4 \leq R < 8) = P(X \leq 7) - P(X \leq 3)$ $= 0.351$	Go to binomcdf option and press enter Key in trials as 20 Key in p as 0.15 Key in X value as: 7 Press [ENTER] button twice Answer: 0.9940788545

	Go to binomcdf option and press [ENTER] Key in trials as 20 Key in p as 0.15 Key in X value as 3 Press [Enter] button twice Answer: 0.6477251743 0.9940788545−0.6477251743=0.346 Final answer = 0.346

Q6. Given that $X \sim B(15, 0.4)$,
Find the largest integer r, such that $P(X > r) > 0.1$

$P(X > r) > 0.1$ $P(X \leq r) < 0.9$ $r = 7$	Press [Y=] button Press [2ND] button and then press [VARS] button Scroll down until your cursor is on "binomcdf" and press [ENTER] After pressing [ENTER], Key in trials: 15 p: 0.4 x value: X Press [ENTER] twice and you should see the following on your graphing calculator screen Plot 1 Plot 2 Plot 3 \Y1= binomcdf(15, 0.4, X) \Y2= \Y3= \Y4= \Y5= \Y6= \Y7=

	Press [2ND] and then Press [Graph] and the following should appear on the screen. X1 | Y1 0 | 4.7E-4 1 | .0517 2 | .02711 3 | .0905 4 | .21728 5 | .40322 6 | .60981 Keep scrolling down until you find the largest integer that satisfy the condition stated and derived from the question In this case, after scrolling, the largest integer $r = 7$ has the highest probability (0.7869 that satisfy the question of $P(X \leq r) < 0.9$ Final answer, largest integer $r = 7$

Q7. Given that $X \sim B(15, p)$ and that $p > 0.3$, find the value of p such that $P(X = 4) = 0.12$.

$X \sim B(15, p)$ $P(X = 4) = 0.12$ $p = 0.406$	Press [Y=] button Press [2ND] followed by [VARS] Scroll down to binompdf and press [ENTER] Set the values as follow below trials: 15 p: X X value: 4 And press [ENTER] twice

	You should see the following on the screen by this time now. \Y1=binompdf(15, X, 4) \Y2= \Y3= \Y4= \Y5= \Y6= \Y7= Press [MATH] button and select "MATH" and press [ENTER] Scroll down to Numeric Solver and press [ENTER] You should see the following on the screen by this time now. E1: E2: In the E1 box field, press [ALPHA], followed by [TRACE] and a pop-up should appear, select Y1 and press [ENTER] In the E2 box field, key in the value of $P(X = 4)$ which is 0.12

	You should see the following on your screen: E1: Y1 E2: 0.12 OK Press [GRAPH] and you should see the following on your screen: Y1=0.12 X= bound = {-1E99, 1E99} Once at this stage, always set X = 0.5 and press [GRAPH] button. And the following should appear. Y1=0.12 X= 0.5 bound = {-1E99, 1E99} E1-E2 = 0 SOLVE

	Set bounds as {0.3, 1}, since question mentioned $p > 0.3$ and probability cannot exceed 1 in value. And the following should appear. Y1=0.12 X= 0.5 bound = {0.3, 1} E1-E2 = 0 SOLVE Press [ALPHA], followed by [ENTER} key Y1=0.12 X= 0.40645273391598 bound = {0.3, 1} E1-E2 = 0 SOLVE Answer: $p = 0.406$

Q8. Given that the random variable $X \sim B(40, p)$ and find the value of p such that $P(X \leq 2) = 0.3$

$X \sim B(40, p)$ $P(X \leq 2) = 0.3$ $p = 0.0885746347538$	Press [Y=] button Press [2ND] followed by [VARS] Scroll down to binomcdf and press [ENTER] Set the values as follows

	The following should appear on your screen \Y1=binomcdf(40, X, 2) \Y2= \Y3= \Y4= \Y5= \Y6= \Y7= Press [MATH] button and select "MATH" and press [ENTER] Scroll down to Numeric Solver and press [ENTER] You should have the following on your graphing calculator screen now E1: E2: In the E1 box field, press [ALPHA], followed by [TRACE] and a pop-up should appear, select Y1 and press [ENTER] In the E2 box field, key in the probability value as "0.3" and press [ENTER] and set the bound to "{0,1}" and you should have the following on the display screen.

	Y1=0.3 X=0 Bound {0, 1} Always set X=0.5 and press [ALPHA] key followed by [ENTER] key and you should see the following on the screen now. Y1=0.3 X=0.0885746347538 Bound {0,1} Answer: $p = 0.0886$