AISS 2024 Sec 4NA Prelim Math Marking Scheme

Qn		Working	Mark Awarded	Sub-total	Remarks
1		9.962571767 = 9.963 (to 4 s.f.)	B1	1	
2		39%, $0.25^{\frac{2}{3}}$, $\sqrt{0.16}$, 0.412, $\frac{21}{50}$	B2	2	B1 if order is correct if one of the terms is covered.
3		$x < -7.5$ $x = -8$	M1 A1	2	
4	**(a)**	5 units → 1.4 kg 1 unit →0.28 kg 7 units → 1.96 kg	B1	1	
	(b)	2 : 3 = 8 : 12 4 : 9 = 12 : 27 8 : 12 : 27	B1	1	No marks if ratio is not simplified.
5		$y = \frac{k}{x^2}$ When $x = 3$, $y = 15$. $15 = \frac{k}{3^2}$ $k = 15(3^2)$ $k = 135$ $y = \frac{135}{x^2}$ When $x = 5$, $y = \frac{135}{5^2}$ $y = 5.4$	M1 A1	2	M1 for finding k.
6	**(a)**	$24x^6$	B1	1	
	(b)	$9y^4$	B1	1	

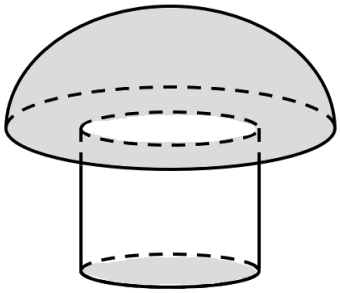
Qn		Working	Mark Awarded	Sub-total	Remarks
7		$2^{-a} \times \frac{1}{8} = 32$ $2^{-a} \times 2^{-3} = 32$ $2^{-a-3} = 2^5$ $-a - 3 = 5$ $-3 - 5 = a$ $a = -8$	M1 A1	2	M1 for 2^{-3} and 2^5.
8		$60\% \times \frac{7}{10} = \frac{21}{50}$ 21 units → 21 marbles 1 unit → 1 marble 50 units → 50 marbles Or $60\% \times \frac{7}{10} = 0.42$ $\frac{21}{0.42} = 50$	M1 A1 [M1] [A1]	2	M1 for finding the fraction / percentage of marbles that are yellow.
9		Method 1: Substitution $y = x - 5$ --------- (1) $3x + 4y = 29$ ---------- (2) Sub (1) into (2) $3x + 4(x - 5) = 29$ $3x + 4x - 20 = 29$ $7x = 49$ $x = 7$ Sub $x = 7$ into (1) $y = 7 - 5$ $y = 2$ $P(7, 2)$ <u>Method 2: Elimination</u> $y = x - 5$ $x - y = 5$ --------- (1) $3x + 4y = 29$ ---------- (2) (1) × 3: $3x - 3y = 15$ ---------- (3) (2) – (3): $(3x + 4y) - (3x - 3y) = 29 - 15$	 M1 A1 A1 [M1]		M1 for correct substitution / elimination.

Qn		Working	Mark Awarded	Sub-total	Remarks
		$3x + 4y – 3x + 3y = 14$ $7y = 14$ $y = 2$ Sub $y = 2$ into (1) $x – 2 = 5$ $x = 7$ $P(7, 2)$	 [A1] [A1]	 3	
10		$5(x – 4) = 6 – (2 – x)$ $5x – 20 = 6 – 2 + x$ $4x = 24$ $x = 6$	 M1 A1	 2	M1 for expanding correctly
11		$\frac{4y^2-1}{6y^2-13y+5}$ $= \frac{(2y+1)(2y-1)}{(2y-1)(3y-5)}$ $= \frac{2y+1}{3y-5}$	 M1 M1 A1	 3	M1 for factorizing numerator M1 for factorizing denominator
12	**(a)**	19×0.5 $= 9.5$ km	 B1	 1	
	(b)	<table><tr><th>Map</th><th>Actual</th></tr><tr><td>1 cm</td><td>0.5 km</td></tr><tr><td>1^2 cm^2 $= 1$ cm^2</td><td>0.5^2 km^2 $= 0.25$ km^2</td></tr><tr><td>4 cm^2</td><td>1 km^2</td></tr><tr><td>140 cm^2</td><td>35 km^2</td></tr></table> *Answer*: 140 cm^2	 M1 A1	 2	M1 for ratio of area (1 : 0.25)
13		Area of triangle $= \frac{1}{2}ab\sin C$ $= \frac{1}{2}\times 10\times 10\times \sin 60°$ $= 43.30127019$ cm^2 Area of the 3 sectors	 M1	 	M1 for finding area of triangle.

Qn		Working	Mark Awarded	Sub-total	Remarks
		= area of semicircle $= \frac{\pi r^2}{2}$ $= \frac{\pi \times 5^2}{2}$ = 39.26990817 cm^2 Area of shaded region = 43.30127019 – 39.26990817 = 4.031362019 = 4.03 cm^2 (to 3 s.f.)	M1 A1	3	M1 for finding area of 3 sectors
14		$A = P\left(1+\frac{r}{100}\right)^n$ $5000+800 = 5000\left(1+\frac{r}{100}\right)^3$ $5800 = 5000\left(1+\frac{r}{100}\right)^3$ $\frac{5800}{5000} = \left(1+\frac{r}{100}\right)^3$ $\sqrt[3]{\frac{5800}{5000}} = 1+\frac{r}{100}$ $\sqrt[3]{\frac{5800}{5000}} - 1 = \frac{r}{100}$ $100\left[\sqrt[3]{\frac{5800}{5000}} - 1\right] = r$ $r = 100\left[\sqrt[3]{\frac{5800}{5000}} - 1\right]$ r = 5.07175745 r = 5.07 (to 3 s.f.)	M1 M1 A1	3	M1 for formula of compound interest, with correct components substituted. M1 for cube root.
15	**(a)**	$x^2 - 18x + 19$ $= (x-9)^2 - 9^2 + 19$ $= (x-9)^2 - 62$ $n = -62$	B1	1	
	(b)	$(x-9)^2 - 62 = 0$ $(x-9)^2 = 62$	M1		M1 for ± and square root.

Qn		Working	Mark Awarded	Sub-total	Remarks
		$x-9=\pm\sqrt{62}$ $x=9\pm\sqrt{62}$ x = 16.87400787 or x = 1.125992126 x = 16.9 (3 s.f.) x = 1.13 (3 s.f.)	A1	2	
16	**(a)**	$a=\frac{13bc+1}{2b+c}$ $a=\frac{13(1.5)(-1.2)+1}{2(1.5)+(-1.2)}$ $a=-12\frac{4}{9}$	B1	1	Accept $-\frac{112}{9}$ too.
	(b)	$a=\frac{13bc+1}{2b+c}$ $a(2b+c)=13bc+1$ $2ab + ac = 13bc + 1$ $2ab - 13bc = 1 - ac$ $b\,(2a - 13c) = 1 - ac$ $b=\frac{1-ac}{2a-13c}$	M1 A1	2	M1 for correct expansion
17	**(a)**	$\frac{2}{x+7}-\frac{1}{x}=5$ $\frac{2x}{x(x+7)}-\frac{1(x+7)}{x(x+7)}=5$ $\frac{2x-1(x+7)}{x(x+7)}=5$ $2x-1(x+7)=5x(x+7)$ $2x-x-7=5x^2+35x$ $0=5x^2+34x+7$ $5x^2+34x+7=0$ $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ $x=\frac{-34\pm\sqrt{34^2-4(5)(7)}}{2(5)}$ $x=\frac{-34\pm\sqrt{1016}}{10}$ x = –0.2125245099 or x = –6.58747549 x = –0.21 (to 2 d.p..) x = –6.59 (to 2 d.p)	M1 M1 M1 A1	4	M1 for combining into a single fraction correctly. M1 for getting the equation. M1 for quadratic formula A1 for both answers, correct to 2 d.p.

Qn		Working	Mark Awarded	Sub-total	Remarks
18	**(a)**	$1 + 3n$	B1	1	
	(b)	46	B1	1	
	(c)	$1 + 3n = 547$ $3n = 546$ $n = 182$, which is an integer. Hence, 547 is a term of the sequence. Yes, Chye Joo is correct.	 B1	 1	B1 for finding n and stating that it is an integer / whole number.
	(d)	Analyse the relationship between the two sequences. First sequence: 4, 7, 10, 13, 16 + 1, + 4, + 9, + 16, + 25 Second sequence: 5, 11, 19, 29, 41 Note that $1 = 1^2$, $4 = 2^2$, $9 = 3^2$, $16 = 4^2$ and $25 = 5^2$. Hence, using (b), $1 + 3n + n^2$.	 B1	 1	
19	**(a)**	angle $PSU = 180^o - 65^o$ (interior angles, $PQ//SR$) $= 115^o$ reflex angle $PSU = 360^o - 115^o$ (angles at a point) $= 245^o$	M1 A1	 2	Deduct marks for missing angle properties.
	(b)	angle $PTU = 121^o$ (alternate angles, $PQ//SR$)	B1	1	
	(c)	angle $PTS = 180^o - 65^o - 65^o$ (base angle of an isosceles triangle) $= 50^o$ angle STU = angle PTU – angle PTS $= 121^o - 50^o$ $= 71^o$	M1 A1	 2	
20	**(a)**	Volume of hemisphere = 170 cm^3 $\frac{2}{3}\pi r^3 = 170$ $r^3 = 170 \div \frac{2}{3}\pi$ $r = \sqrt[3]{170 \div \frac{2}{3}\pi}$	 M1	 	M1 for formula involving volume of hemisphere.

Qn		Working	Mark Awarded	Sub-total	Remarks
		r = 4.329756132 r = 4.33 cm (to 3 s.f.)	A1	2	
	(b)	Volume of cylinder = 190 – 170 = 20 cm^3 $\pi r^2 h = 20$ $\pi \times 1.5^2 \times h = 20$ $h = 20 \div \pi \div 1.5^2$ h = 2.829421211 h = 2.83 cm (to 3 s.f.)	M1 A1	2	M1 for $\pi r^2 h = 20$
	(c)	 Surface area of hemisphere and base of cylinder = $2\pi r^2 + \pi r^2$ = $3\pi r^2$ = $3\pi(4.329756132)^2$ = 176.6843159 cm^2 Surface area of curved surface of cylinder = $2\pi rh$ = $2\pi \times 1.5 \times 2.829421211$ = 26.6666666667 cm^2 Total surface area = 176.6843159 + 26.6666666667 = 203 cm^2 (to 3 s.f.)	M1 M1 A1	3	
21	**(a)**	By Pythagoras' Theorem, $20^2 = AB^2 + 13^2$ $AB^2 = 20^2 - 13^2$ $AB = \pm\sqrt{20^2 - 13^2}$ $AB = \sqrt{20^2 - 13^2}$ (reject negative, length > 0) AB = 15.19868415 AB = 15.2 m (to 3 s.f.)	M1 A1	2	Minus one mark if "By Pythagoras' Theorem" is missing.
	(b)	$BC^2 = 29^2 = 841$ $BE^2 + CE^2 = 20^2 + 21^2 = 841$	 M1		M1 for finding both BC^2 and $BE^2 + CE^2$

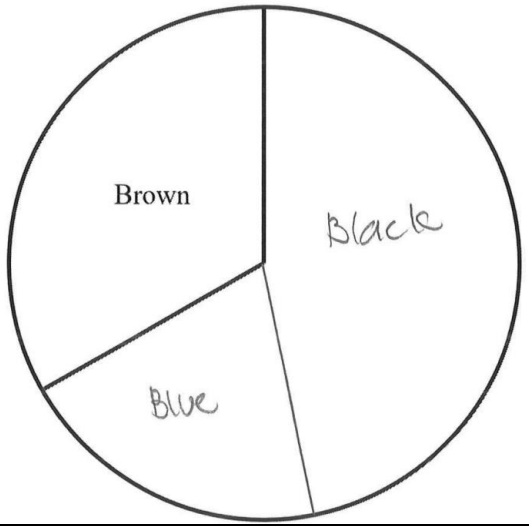
<table>
<tr><th colspan="2">Qn</th><th>Working</th><th>Mark Awarded</th><th>Sub-total</th><th>Remarks</th></tr>
<tr><td></td><td></td><td>Hence, $BC^2 = BE^2 + CE^2$
and by the Converse of Pythagoras' Theorem,
triangle BCE is a right-angled triangle (proven).</td><td>A1</td><td>2</td><td>A1 for stating Converse of Pythagoras' Theorem.</td></tr>
<tr><td></td><td>(c)</td><td>TOA CAH SOH
angle CBE = $\sin^{-1}\left(\frac{21}{29}\right)$
= 46.39718103
= 46.4° (to 1 d.p.)</td><td>M1

A1</td><td>

2</td><td></td></tr>
<tr><td></td><td>(d)</td><td>$\cos 50° = \frac{CD}{21}$
$CD = 21\cos 50°$
= 13.4985398
= 13.5 m (to 3 s.f.)</td><td>M1

A1</td><td>

2</td><td></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>22</td><td>(a)</td><td>
<table>
<tr><th></th><th>Brown</th><th>Black</th><th>Blue</th><th>Total</th></tr>
<tr><td>Male</td><td>12</td><td>17</td><td>8</td><td>37</td></tr>
<tr><td>Female</td><td>13</td><td>18</td><td>7</td><td>38</td></tr>
<tr><td>Total</td><td>25</td><td>35</td><td>15</td><td>75</td></tr>
</table>
</td><td>B2</td><td>2</td><td>B1 for any 2 correct answers.

B2 for all 4 correct answers.</td></tr>
<tr><td></td><td>(b)</td><td>75 students → 360°
1 student → 4.8°
35 students → 168°</td><td>B1</td><td>1</td><td></td></tr>
<tr><td></td><td>(c)</td><td>Black eyes → 168°
Blue eyes → 72°
 
</td><td>B1</td><td>1</td><td></td></tr>
</table>