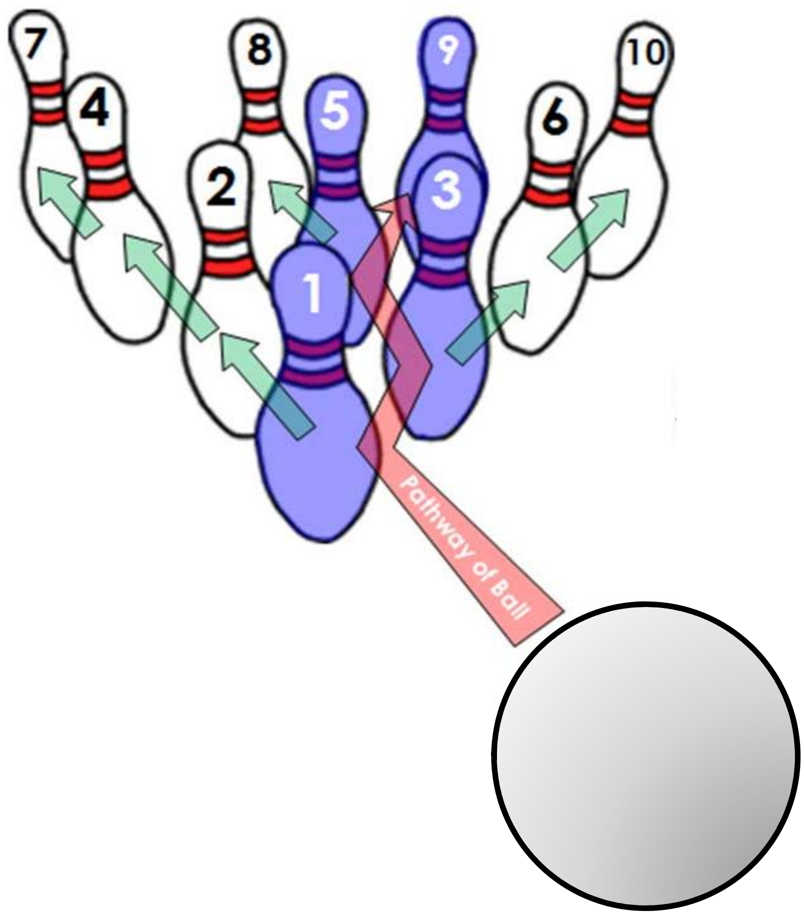
Chapter 3

Dynamics



"Although you can get a strike by hitting the head pin square-on, it is not a very high percentage shot. Hitting the pins in "the pocket", which is the gap between the head pin and the next pin on the right (the 3-pin) for right handed bowlers, and the next pin to the left (the 2-pin) for lefties, will strike with the most consistency."

(Neil Lazarski, "How to Raise Your Bowling Average")

H2 Physics Syllabus 9749

Topic 3: Dynamics

Content

- Newton's laws of motion
- Linear momentum and its conservation

Learning Outcomes

Students should be able to:

(a) state and apply each of Newton's laws of motion.

(b) show an understanding that mass is the property of a body which resists change in motion (inertia).

(c) describe and use the concept of weight as the force experienced by a mass in a gravitational field.

(d) define and use linear momentum as the product of mass and velocity.

(e) define and use impulse as the product of force and time of impact.

(f) relate resultant force to the rate of change of momentum.

(g) recall and solve problems using the relationship $F = ma$, appreciating that resultant force and acceleration are always in the same direction.

(h) state the principle of conservation of momentum.

(i) apply the principle of conservation of momentum to solve simple problems including inelastic and (perfectly) elastic interactions between two bodies in one dimension. (Knowledge of the concept of coefficient of restitution is not required.)

(j) show an understanding that, for a (perfectly) elastic collision between two bodies, the relative speed of approach is equal to the relative speed of separation.

(k) show an understanding that, whilst the momentum of a closed system is always conserved in interactions between bodies, some change in kinetic energy usually takes place.

Topic 4: Forces (Part)

Learning Outcomes

Students should be able to:

(b) describe the forces on a mass, charge and current-carrying conductor in gravitational, electric and magnetic fields, as appropriate.

(c) show a qualitative understanding of normal contact forces, frictional forces and viscous forces including air resistance. (No treatment of the coefficients of friction and viscosity is required.)

(d) show an understanding that the weight of a body may be taken as acting at a single point known as its centre of gravity.

Contents

Videos of Lecture Examples can be found at
https://youtube.com/playlist?list=PL_b5cjrUKDlYDkaMJ2sKNvZNiASvpiSfT

3.1 Introduction

Dynamics is the branch of classical mechanics concerned with the study of forces and their effects on motion. Isaac Newton defined the fundamental physical laws which govern dynamics in physics, especially his second law of motion.

3.1.1 Types of Forces

Broadly speaking, forces may be categorized into two kinds: **contact** forces and **non-contact** forces.

3.1.1.1 Contact Forces

Forces acting between two bodies that are in physical contact.

Contact forces	Description
Friction	Force that resists the **relative** motion or the tendency for **relative** motion between two surfaces that are in contact.
Normal contact force	Net electrostatic repulsion between two surfaces that are in contact.
Contact force	Vector sum of friction and normal contact force, meaning the total contact force.
Spring force	Force exerted by one end of a spring when the spring is extended or compressed from its natural length. The spring forces acts throughout a spring, in both directions.
Tension	Force exerted by one end of a string/rope which is being pulled taut. The tension is constant throughout the string/rope and acts in both directions.
Upthrust	**Net** upward force exerted by the fluid on a body fully or partially submerged in the fluid.
Viscous drag force	Force that resists a body moving **relative** to a fluid.
Air resistance	Drag force exerted by air on a body moving in air. Air resistance is a specific form of drag.

3.1.1.2 Non-contact Forces

Forces acting between two bodies which are NOT in physical contact (separated by a distance). This characteristic of the force is described as **action-at-a-distance**.

Non-contact forces	Description	Physical property of a body on which the force acts on.
Gravitational force	Attractive force exerted between masses.	Mass
Electric force	Attractive/repulsive force exerted between electrically charged bodies.	Electric charge
Magnetic force	Force exerted between currents	Moving charges

3.1.2 Weight

The weight of a body is due to the body being in a gravitational field. In most cases, the object of interest is close to the Earth and its weight refers to the gravitational force exerted on its body by the Earth. However, an object may have a different weight elsewhere, for instance, when it is in the gravitational field of the Moon.

Notes:

- Weight is a **force** (hence a **vector**).
- Direction: For everyday-life problems, the weight of the object is taken to act **vertically downwards** at the centre of gravity of the object.
- Magnitude: $W = mg$

  W: weight of an object (SI unit: N)
  m: mass of the body (SI unit: kg)
  g: acceleration of free fall/ gravitational field strength at the point (SI unit: m s^{-2})

- Weight is a non-contact force. It does not only act on a person sitting in a chair, but also on a flying bird, regardless of whether we take into account air resistance.

Distinction between mass and weight

Mass	Weight
A measure of the inertia of a body	A measure of the gravitational pull on a body
Scalar quantity	Vector quantity
SI unit: kg	SI unit: N
Same regardless of location	Varies with gravitational field strength at each location
Same regardless of the method of measurement.	Some methods of measurement do not measure the true weight but the apparent weight (see example 7)

⚠ In everyday language, the terms 'mass' and 'weight' are used interchangeably, e.g. I "weigh" 70 kg. In physics, and especially during examinations, such casual usage is not tolerated.

3.1.2.1 Centre of Gravity

An object of a significant size has its mass distributed (not necessarily uniformly) throughout its volume.

> The **centre of gravity** of an object is the point at which the weight of the object *appears to* act.

Notes:
- In free-body diagrams, the weight should be drawn from the centre of gravity of an object.
- The centre of gravity of an object is not necessarily on the object. It may lie outside the object.
- In a uniform gravitational field, the centre of gravity always coincides with the centre of mass of the object.

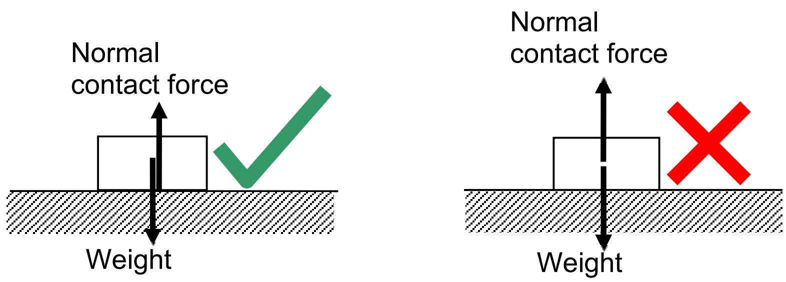
3.1.3 Contact Forces

When two surfaces come into contact, they both experience contact forces arising from electrostatic forces between atoms of the surfaces. For ease of analysis, this contact force is often resolved into two components: the component perpendicular to the surfaces is called **normal contact force**, and the component parallel to the surfaces is given the name **friction**.

3.1.3.1 Normal Contact Force

A block can rest on a table because its weight is balanced by an upward force the table exerts on it. This force arises from the electrostatic repulsion between the molecules of the table and the molecules of the block. It is called the normal contact force since it acts perpendicularly (normal) to the surfaces.

In free-body diagrams (see Section 3.2.5.1 for details), contact forces must be drawn from a contact point (e.g. somewhere along the base of the block). A common mistake is contact forces that are drawn from the centre of gravity (see diagram below).



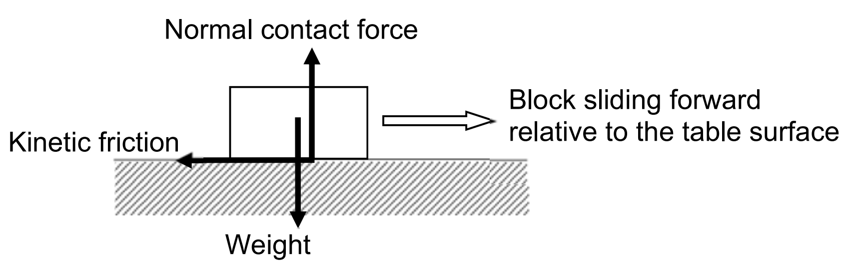
- By definition, the normal contact force is always directed **perpendicular to the surfaces**.
- The magnitude depends on how hard the two surfaces are pressed into each other.

3.1.3.2 Friction

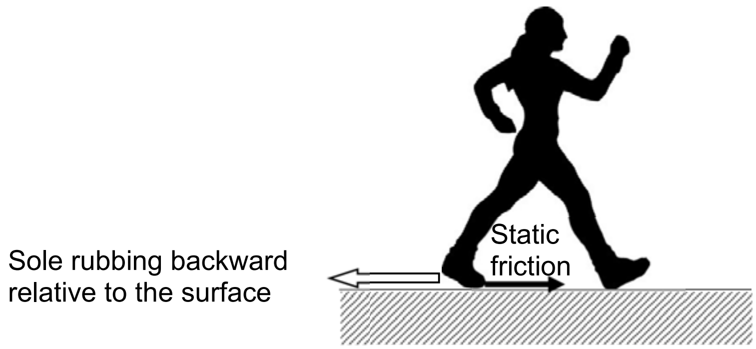
Friction is the force that resists relative motion between two sliding surfaces (called kinetic friction), or the tendency for relative motion between two stationary surfaces (called static friction).

<u>Direction</u>
Friction is always directed <u>in the direction opposite to the direction of **relative motion**.</u>

For example, a block sliding along a rough table is gradually decelerated to rest. It experiences a (kinetic) friction directed backward because the block's surface is sliding forward relative to the table's surface.



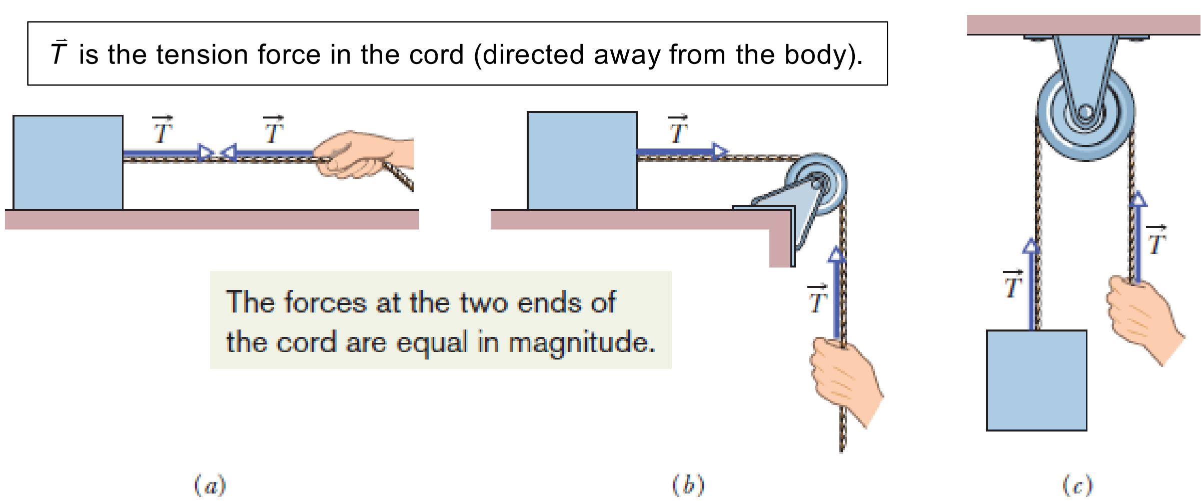
On the other hand, a person is able to walk forward only if he experiences a (static) friction that is directed forward. The sole of his shoe tends to rub backward relative to the floor's surface. When an exam question refers to friction, you have to infer from the question whether the relevant force is a *retarding* frictional force or a *propelling* frictional force.



3.1.4 Tension

In physics, tension refers to the magnitude of the pulling force exerted by a string, rope, cable or chain on the object tied to its end when it is pulled **taut**.

- *At the end of a string*, the direction of the tension force exerted on the object is **along the string**, directed **away from the object**.
- *Within the string*, the magnitude of the tension is constant, and the tension acts in both directions equally. The magnitude of the tension in the string is equal to the magnitude exerted at the ends.
- The magnitude of the tension force depends on how much the string is stretched. Once the string is **slack**, the tension falls to zero.
- In many situations, we idealise the string, rope, cable, etc. to be **massless** and **inextensible** to simplify analysis.
- Strings or ropes are often used together with pulleys. Again, in many situations, we idealise a pulley to be **massless** and **frictionless** (at the axle) to simplify analysis.
- The tension is equal to the magnitude of the forces applied by the ends of the string, even if the bodies and the cord are accelerating and even if the cord runs around a massless, frictionless pulley (figs. b and c).



3.2 Newton's Laws of Motion

3.2.1 Newton's 1st Law

<u>Statement of Newton's 1st law of motion</u>

A body stays at rest or continues to move with a constant speed in a straight line unless acted upon by a ***net*** *external force*.

Notes

- Changes in a body's state of motion refer to changes in its velocity. It can be a change in either its **speed** or **direction** (or both).
- An object's resistance to change its state of motion is called **inertia**. As such, Newton's 1st law is also referred to as the Law of Inertia.
- An object's inertia is dependent on its mass: The larger its mass, the larger its inertia.

3.2.2 Linear Momentum

Everyday experience tells us that two objects moving at the same velocity can result in very different consequences. Consider a 3-gram ping pong ball and a 3-ton lorry, both travelling at 5 m s^{-1}. Which would you rather collide with?

In physics, we define a "quantity of motion" called **momentum**, ***p***.

<u>Definition of Linear Momentum</u>

The linear momentum of a body is the product of its mass and its velocity.
Mathematically, linear momentum, $\vec{p} = m\vec{v}$

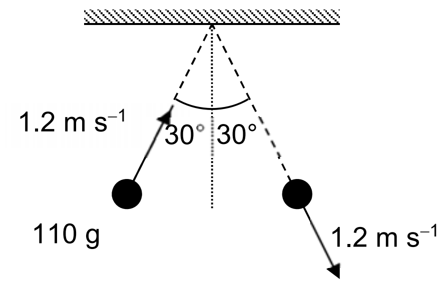
Notes

- Momentum is a **vector quantity**. It has the same direction as the object's velocity.
- Momentum has the SI unit: **kg m s^{-1}**. Sometimes the alternative unit **N s** is used.

In H2 Physics, when we refer to momentum, we mean linear momentum. It is not to be confused with angular momentum, which is not in the syllabus.

Example 1a: Change in Momentum

A 110 g billiard ball rebounds off a wall, with velocities as shown in the (top view) diagram. The ball moves on a horizontal plane. Evaluate the change in momentum of the ball.



3.2.3 Newton's 2nd Law

<u>Statement of Newton's 2nd law of motion</u>

The ***<u>rate of change of linear momentum</u>*** of a body is ***directly proportional*** to the ***resultant force***[1] acting on it and is in the direction of the resultant force.

Strictly speaking, Newton's 2nd law only states that $\vec{F}_{net} \propto \frac{d\vec{p}}{dt}$.

Hence, $F_{net} = k\frac{dp}{dt}$, where k is some constant of proportionality. When SI units are used for all quantities in the equation, the constant k has a value of 1.

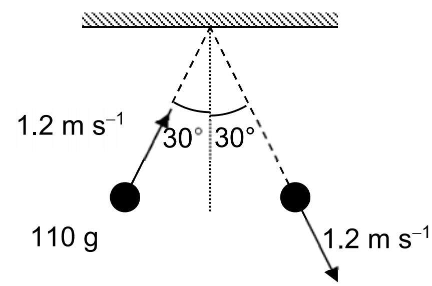
Mathematically, net force, $\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt}$

Average net force, $\langle F_{net} \rangle = \frac{\Delta\vec{p}}{\Delta t} = \frac{\Delta(m\vec{v})}{\Delta t}$

The SI unit for force is the newton (N).

<u>Example 1b: Rate of Change in Momentum</u>

The time interval of contact between the billiard ball and the wall is 0.100 s. Find the average net force acting on the wall during the collision.



[1] Resultant force is also known as net force, or (vector) sum of forces.

3.2.4 Impulse

Definition of Impulse

The impulse of a force is the product of the **average force** and the time interval over which it is applied.

Mathematically, $\vec{J} = \langle \vec{F} \rangle \Delta t$

Notes

- Impulse is a **vector** quantity; its direction is the same as that of the average force *F*.
- The SI units for impulse are **N s**. They can be shown to be equivalent to **kg m s^{-1}**, the same as the units of momentum.

3.2.4.1 Relation between Impulse and Momentum

From Newton's second law,

$$\vec{F} = \frac{d\vec{p}}{dt}$$

Rearranging and integrating, we obtain

$$\int \vec{F} dt = \int d\vec{p}$$

The left-hand side of the equation, $\int \vec{F}\, dt$, is the **impulse** of the force.

The right-hand side, $\int d\vec{p} = \vec{p}_f - \vec{p}_i = \Delta \vec{p}$, is the **total change in momentum**.

This result, which links impulse to the change in momentum, is known as the **impulse-momentum theorem**.

Impulse-Momentum Theorem

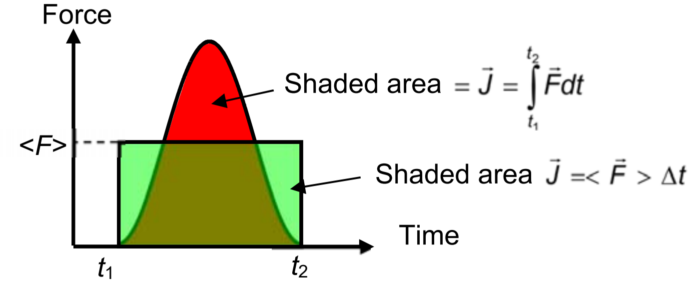
The impulse-momentum theorem states that the ***total change in*** **momentum** of a particle during a time interval **equals the impulse of the net force** that acts on the particle during that interval.

Mathematically, $\vec{J} = \int \vec{F} dt = \Delta \vec{p}$

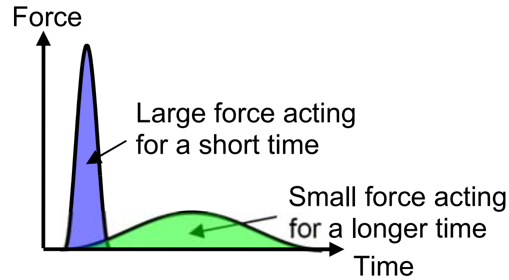
For **average** or **constant** forces, this simplifies to $\vec{J} = < \vec{F} > \Delta t = \Delta \vec{p}$.

3.2.4.2 Graphical Representation of Impulse

Since $\vec{J} = \int \vec{F}dt = \Delta\vec{p}$, impulse is represented by the **area under the *force-time* graph**. The same area also represents the **(total) change in the momentum** of the body.



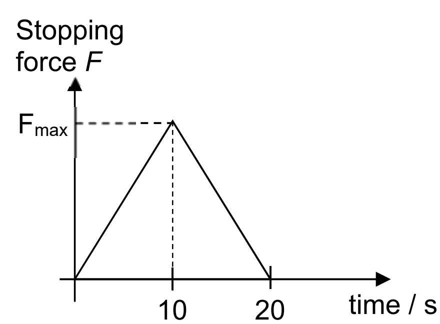
To illustrate further, study the *F-t* graphs below. Notice that, even though the two forces have different peak magnitudes, they both deliver the same impulse (i.e. same area under each graph). Hence, the two forces will cause the same change in momentum to the object they are applied on.



<u>Example 2 (N89/II/8 modified)</u>

In order to stop a car of mass 1500 kg travelling at 30 m s^{-1}, the driver applies his brakes so that F, the total stopping force, increases steadily to a maximum and then decreases to zero as shown in the figure.

Calculate
(a) the momentum of the car when it is travelling at 30 m s^{-1},
(b) the impulse due to the braking force,
(c) the magnitude of the average stopping force, $<F>$,
(d) the value of F_{max}.



Answer:

3.2.5 Newton's 2nd Law for Constant Mass

From Newton's 2nd Law, $F_{net} = \dfrac{dp}{dt} = \dfrac{d(mv)}{dt}$

For constant mass, $\vec{F}_{net} = m\dfrac{d\vec{v}}{dt}$

$$\vec{F}_{net} = m\vec{a}$$

Notes

- $\vec{F}_{net} = m\vec{a}$ is **not** a complete representation of Newton's second law. It only represents a special case of Newton's second law for constant mass.
- The product F_{net} represents the **resultant** of all the actual physical forces acting on a body. It is **not** a new or additional physical force. As such, we do not draw F_{net} as a separate force in free-body diagrams.
- Be clear about which is the cause (F_{net}), and which is the effect (acceleration).
  - A non-zero F_{net} *always results in* an acceleration (change in speed or direction).
  - If acceleration is detected, it *implies* F_{net} is non-zero. It does not result in a new force.

TIPS **Solving F_{net} = *ma* Problems**

Step 1 Draw a picture of the problem if no diagram is provided.
Step 2 Consider the expected direction of acceleration and resultant force.
(This is to facilitate setting up the $F_{net} = ma$ equation.)
Step 3 Draw the free-body diagram (see below) for the body (or bodies) of interest. Only include forces that act on the object you are drawing the diagram for.
Step 4 Establish the coordinate system, with **one axis parallel to the expected acceleration**, and a perpendicular axis if necessary.
Step 5 Resolve the forces into components parallel to your chosen axis/axes.
Step 6 Note additional constraints. For example, two objects moving in physical contact or with a taut rope connected in between must have the same acceleration.
Step 7 Apply $F_{net} = ma$ and solve for all unknowns.
(If there are n unknowns, you have to solve a system of *n* equations.)

3.2.5.1 Free-Body Diagram

A free-body diagram (FBD) is a simple representation of an object and a vector representation of all the forces (including all contact and non-contact forces) acting on the object.

TIPS **Drawing Free-Body Diagrams**

Step 1 Draw a simplified diagram if one is not provided.
Step 2 Identify and isolate the body/bodies (or system) of interest.
Step 3 Draw **all** the forces acting **on** the body/bodies (or system) using labelled arrows.
- The forces must be drawn from the point of application of the forces.
- Draw the forces to appropriate lengths, since the relative lengths of the arrows represent the relative magnitude of the forces.
- Label the arrows clearly with words or properly defined symbols.

Step 4 For two-dimensional problems, resolve the forces into two mutually perpendicular directions to simplify analysis.
- E.g. horizontal and vertical direction or parallel to a slope and perpendicular to the slope.
- Note: the resolved components should be drawn in **dotted** lines so as to distinguish them from the actual physical forces.

Applying F_{net} = *ma* to a system with forces in 1D - the importance of identifying the system

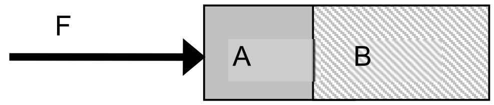
Example 3

A helicopter of mass *M* and weight *W* rises with vertical acceleration, *a*, due to the upward thrust *U* generated by its rotor. The crew and passengers of total mass *m* and total weight *w*, exerts a combined force *R* on the floor of the helicopter.

Draw an appropriate free-body diagram, and write down an equation for the motion of
(a) the helicopter, (b) the crew and passengers, (c) helicopter, crew and passengers

Example 4

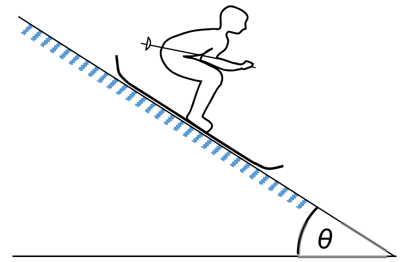
Two blocks A and B, of masses 2*M* and 4*M*, respectively, are pushed along a smooth horizontal surface by a force of *F* as shown in the diagram.
What is the magnitude of the force exerted by block A on block B during the acceleration?



Applying $F_{net} = ma$ to a system with forces in 2D

Example 5

A 75 kg skier is accelerating at 2.6 m s^{-2} down a slope at an angle θ = 30°. Find the friction *f* and the normal contact force *N* acting on the skier.

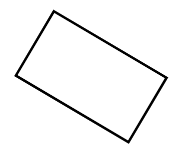


Step 1 Choose/Identify the system, in this case the skier, and draw a simple box to represent it.

Step 2 Draw the free-body diagram.

Step 3 For two-dimensional problems, resolve forces into two convenient, mutually perpendicular directions to simplify analysis.

- We choose the components *along* and *perpendicular to* the slope.
- The frictional force is parallel to the slope.
- The normal contact force is perpendicular to the slope.
- The weight has components along and perpendicular to the slope.



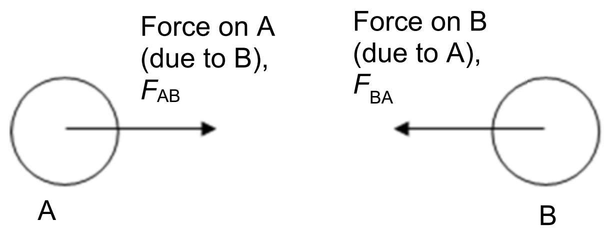
Step 4 Apply Newton's second law to the 2 perpendicular directions:

Don't know how to resolve the weight into the 2 components? Watch this:

3.2.6 Newton's 3rd Law

Statement of Newton's 3rd law

If body A exerts a force on body B, then body B will exert an equal and opposite force on body A.



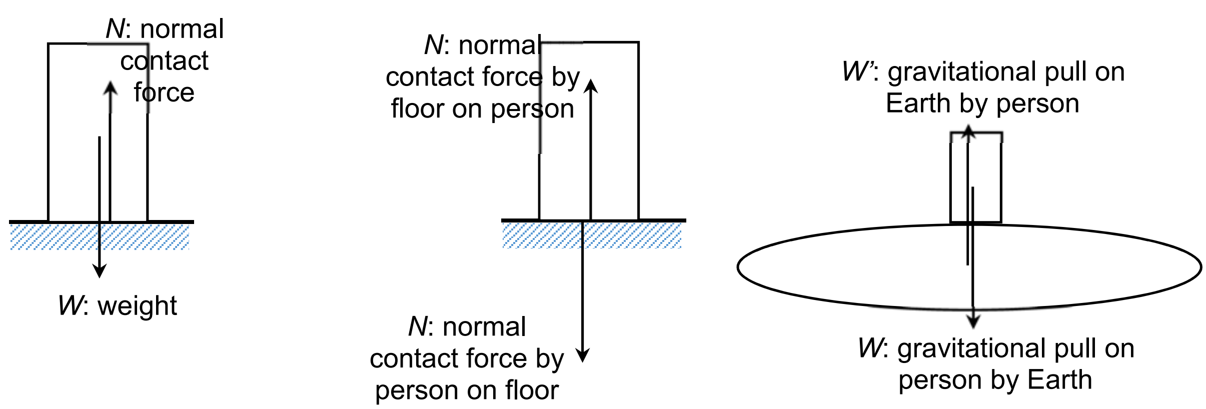
$$\vec{F}_{AB} = -\vec{F}_{BA}$$

It is easy to confuse action-reaction pairs with force pairs that happen to be pairs of "equal-but-opposite" forces. Performing the following "mental checklist" may help to detect "impostors".

For two forces to be Action-Reaction Pair, the two forces must be

- ☐ Equal in magnitude
- ☐ Opposite in direction
- ☐ Acting on different bodies
- ☐ Of the same nature (e.g. gravity-gravity, friction-friction, tension-tension)

To illustrate, consider the forces when a person is standing on the floor.



Forces acting on the person | 1st third-law pair | 2nd third-law pair

Example 6 (Man in the lift)

An 80 kg man weighs himself by standing on a weighing scale inside a lift. What does the scale read if the lift

(a) is at rest,

(b) moves with an upward acceleration of 1.8 m s^{-2},

(c) the lift is moving upwards with a constant velocity of 2.2 m s^{-1},

(d) the lift slows from its velocity in (c) to rest at rate of 1.9 m s^{-2}.

Answer:

This YouTube video sheds more light on apparent weight and true weight:
https://www.youtube.com/watch?v=AbNJv1VNWu8

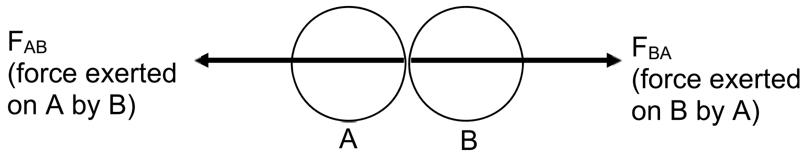
Example 7:

A hovering jetpack with nozzles of total cross-sectional area *A* expels water at constant velocity *v* relative to the jetpack. Show that the force exerted by the expelled water on the jetpack is $F = \rho A v^2$, where ρ is the density of the water jet.

Answer:

3.3 Principle of Conservation of Linear Momentum

Momentum is an important concept because, momentum is a **conserved** quantity. Consider, for example, the head-on collision of two billiard balls, A and B.



Assuming **no net *external* force act on the two bodies**, the only forces experienced by the two bodies A and B during the collision will be the mutual normal contact forces, F_{AB} and F_{BA}, respectively.

By Newton's 3rd law, the two forces must be equal but opposite:

$$\vec{F}_{AB} = -\vec{F}_{BA}$$

Since the entire interaction is simultaneous and over the same duration, the impulses of the two forces must also be equal but opposite:

$$\int \vec{F}_{AB}\, dt = \int -\vec{F}_{BA}\, dt$$
$$\vec{J}_{AB} = -\vec{J}_{BA}$$

It follows that the momentums exchanged between A and B must also be equal but opposite:

$$(\Delta\vec{p})_A = -(\Delta\vec{p})_B$$

If we see the two billiard balls as making up one system, then the **total momentum of the system does not change** in the collision:

$$\begin{aligned}\sum\Delta\vec{p} &= (\Delta\vec{p})_A + (\Delta\vec{p})_B \\ &= (\Delta\vec{p})_A + -(\Delta\vec{p})_A \\ \sum\Delta\vec{p} &= 0\end{aligned}$$

The above derivation can be extended to include any number of interacting objects in three-dimensional space, involving forces of any nature (magnetic, electric, gravitational, etc.), as long as the condition of **no resultant external force** holds. The logic is surprisingly straight-forward:

$$\sum\vec{F} = 0 \quad \Rightarrow \quad \sum\vec{J} = 0 \quad \Rightarrow \quad \sum\Delta\vec{p} = 0$$

Thus the general statement of the principle of conservation of momentum is

<u>Statement of the Principle of Conservation of Linear Momentum</u>

The **total linear momentum** of a system **remains unchanged** if ***no net external force*** acts on the system. Mathematically, $\sum m_n\vec{u}_n = \sum m_n\vec{v}_n$

The principle is **<u>NOT restricted to collisions!</u>**
Sadly, every year, many students get zero marks during examinations when asked to state the principle of conservation of linear momentum, because they write about momentum before and after collisions and stuff.

Notes:

- By a system, we mean a set of objects that we choose.
- Some textbooks refer to the system as being *isolated*. An isolated system is one in which the only (significant) forces are those between the objects in the system. The sum of all these "internal" forces within the system will be zero because of Newton's 3rd Law.
- If there is a net external force – then the total momentum of the system will *not* be conserved. E.g., if we take as our system a rock falling under gravity, the momentum of this system is *not* conserved because an *external* force, the gravitational pull by the Earth is acting on it. However, if we include the Earth in the system, the total momentum of rock plus Earth *is* conserved.

Example 8: (Freedman)

An open-topped freight car with mass 24 000 kg is coasting along without friction along a level track. It is raining very hard and the rain is falling vertically downward. Originally the car is empty and moving with a speed of 4.00 m/s. What is the speed of the car after it has collected 3000kg of rainwater?

Answer:

3.3.1 Two-body Systems

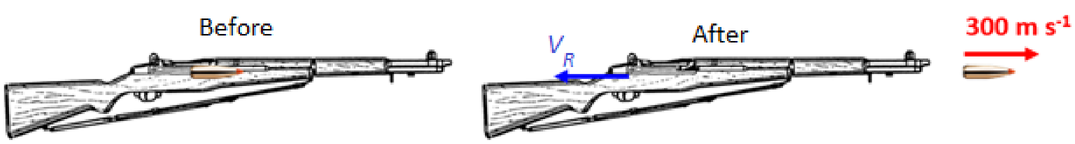
For a **two-body system**, the principle of conservation of momentum can be mathematically simplified as follows:

$$(\Delta\vec{p})_A = -(\Delta\vec{p})_B$$
$$(\vec{p}_f - \vec{p}_i)_A = -(\vec{p}_f - \vec{p}_i)_B$$
$$m_A(\vec{v}_A - \vec{u}_A) = m_B(\vec{u}_B - \vec{v}_B)$$
$$m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B$$

Example 9

A marksman holds a rifle of mass M = 3.00 kg loosely in his hands, so as to let it recoil freely when fired. He fires a bullet of mass m = 5.00 g horizontally with a velocity of v_B = 300 m s^{-1}.
What is the recoil velocity v_R of the rifle?

Answer:

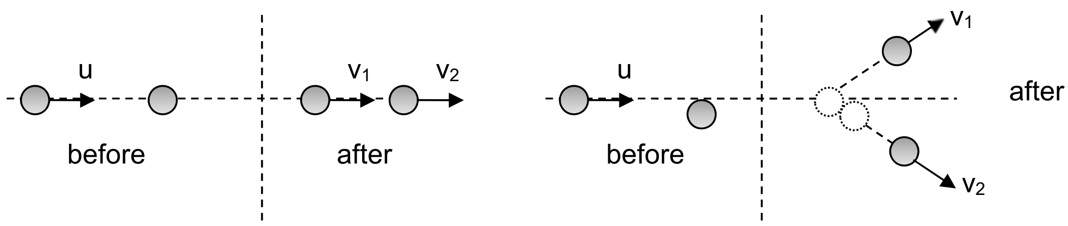


Note that since both the rifle and bullet are **stationary** before firing, the total initial momentum of the system is **zero**. Since momentum is a vector quantity, in order for the final total momentum to be zero, the **rifle must move in the direction opposite to the bullet** (recoil).

3.3.2 Types of Collisions

A **collision** is an isolated event in which two or more interacting bodies exert **relatively strong** forces on each other for a relatively short time. Note that the two bodies do not necessarily have to be in any physical contact, as interaction may involve a force at a distance.

3.3.2.1 Head-on vs Glancing Collisions

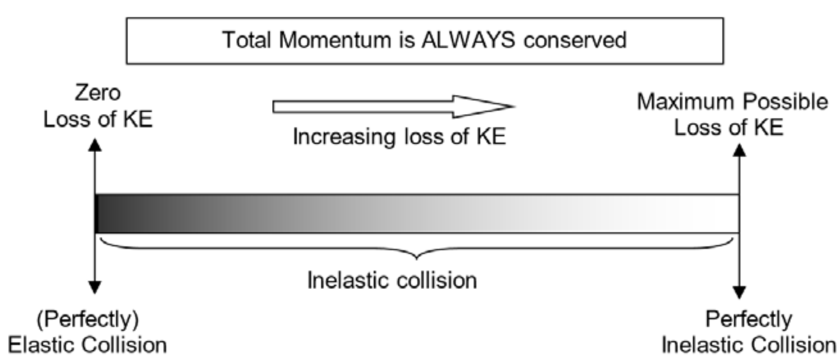


A **head-on** collision is one in which the directions of motion before and after the collision are along the same line of motion.

3.3.2.2 Elastic vs Inelastic Collisions

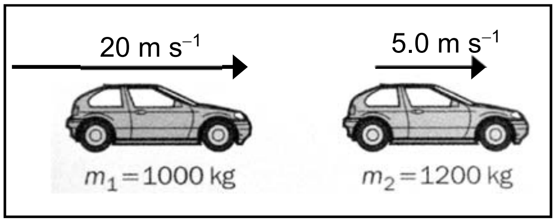
The total *kinetic* energy of the colliding bodies is *not* necessarily conserved during a collision. (This is unlike the total momentum, which is *always* conserved during a collision.) Some kinetic energy is usually converted into vibrational energy of the atoms, causing a heating effect, sound, and deformation energy. How much of the original kinetic energy is conserved after the collision depends on the **elasticity** of the collision.

- An **elastic** collision is one where 100% of the kinetic energy is conserved, e.g. two diamonds bouncing off each other. Note that *during* an elastic collision, some of the kinetic energy can temporarily be converted to potential energy.
- An **inelastic** collision is one that is *not* elastic, i.e. some or all of the kinetic energy is converted into vibrational energy, sound, deformation energy, etc.
- A **perfectly inelastic** collision is one where the 2 objects coalesce (stick together) and move with **common velocity** after the collision, e.g. two lumps of clay that don't bounce at all, but stick together after the collision. This represents the **maximum possible loss** of kinetic energy without violating the principle of conservation of energy; it does *not* necessarily mean all the kinetic energy is lost.

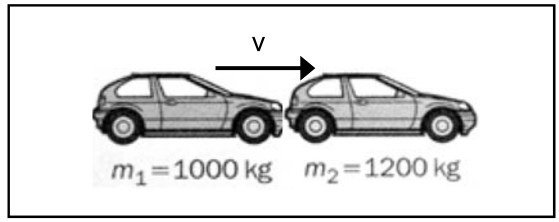


Note that **momentum** is **always** conserved, regardless of whether a collision is elastic or inelastic. In most exam questions, you will have to apply the principle of conservation of linear momentum.

Example 10 (Perfectly Inelastic collision)
A car of mass 1000 kg moving at 20.0 m s^{-1} collides with a car of mass 1200 kg, moving at 5.0 m s^{-1} in the same direction.
(a) If the collision is perfectly inelastic, determine their final velocity after the collision.
(b) Calculate also the ratio of (total final kinetic energy) to (total initial kinetic energy).



before

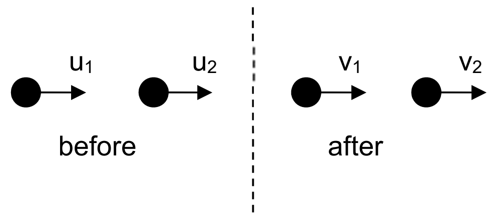


after

Answer:

3.3.2.3 Relative Speed of Approach/Separation during Elastic Collisions

Consider a two-body **elastic** head-on collision.



By the principle of conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

Rearranging, we get

$$m_1(u_1 - v_1) = m_2(v_2 - u_2) \text{ ---------------------------------------(1)}$$

Since the kinetic energy is conserved during an elastic collision,

$$\frac{1}{2}m_1u_1^{\ 2} + \frac{1}{2}m_2u_2^{\ 2} = \frac{1}{2}m_1v_1^{\ 2} + \frac{1}{2}m_2v_2^{\ 2}$$

Rearranging, we get

$$m_1(u_1^{\ 2} - v_1^{\ 2}) = m_2(v_2^{\ 2} - u_2^{\ 2})$$

$\Rightarrow$ $$m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2) \text{ ---------------------------(2)}$$

Dividing question (2) by equation (1),

$$(u_1 + v_1) = (v_2 + u_2)$$

Rearranging, we get

$$(u_1 - u_2) = (v_2 - v_1)$$

The $(u_1 - u_2)$ term represents the relative speed of approach before the collision.

The $(v_2 - v_1)$ term represents the relative speed of separation after the collision.

We thus come to the following conclusion:

<u>Statement of Relative Speed of Approach/Separation</u>

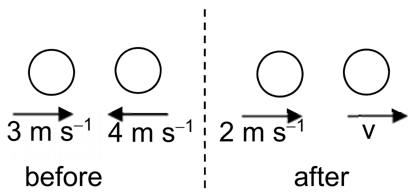
For **elastic** collisions, the relative speed of approach is equal to the relative speed of separation.

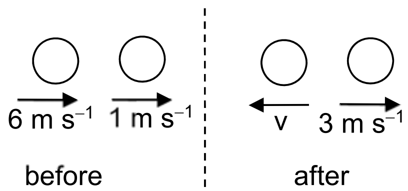
Mathematically, $u_1 - u_2 = v_2 - v_1$

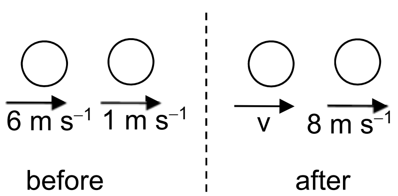
This result can only be applied for a 2-body **elastic** collision. During examinations, the above statement must be **made** before it can be applied mathematically in solving problems.

Example 11

Assuming the following are **head-on elastic collisions**, solve for the unknown velocity *v* in each case. Note that the masses of the objects are not (necessarily) equal.







TIPS **Solving 2-body Collision Problems**

Step 1 Indicate velocity vectors for each of the bodies before and after the collision. Choose a convenient positive direction, usually to the right. You can assume the unknown velocities are all positive. If the solutions turn out to be negative later, it means these velocities are in the negative direction.

Step 2 Set up the equation for the principle of conservation of linear momentum. The positive direction chosen in step 1 helps us to define the signs of the (known) velocities in the equations: velocities in the positive direction get a "+" and those in the negative direction get a "–".

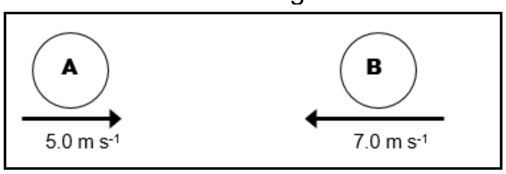
Step 3 **IF** the collision is perfectly inelastic, use one variable to represent the final velocities of both bodies (i.e. make $v_1 = v_2 = v$).

IF the collision is elastic, you will probably have two unknown velocities and need another equation. (You need as many independent equations as there are unknowns to solve them uniquely.) Use the equation for relative speed of approach/separation (preferred) of that for the conservation of kinetic energy (which is more tedious, as it is a quadratic equation).

Step 4 Solve for the unknowns, e.g. mass, initial or final velocities.

Example 12 (Elastic collision)
Two spheres, A and B, of mass 4.0 kg and 6.0 kg, respectively, collide head-on. Their initial velocities are 5.0 m s^{-1} and 7.0 m s^{-1} in opposite directions, as shown in the diagram.
Assuming the collision is **elastic**, determine their final velocities.

Answer:
The directions of motion of the spheres after the collision are not given in the question. Hence, we will need to draw the "after collision" diagram to indicate the directions used in the analysis. It is ok if the directions chosen are incorrect as the final answer will tell us if the sphere are really moving in the predicted direction in the "after collision" diagram.

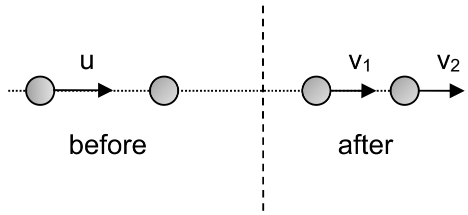


Before

After

3.3.2.4 Elastic Collision, One Body Initially at Rest *(Good to know)*

Let's study a head-on elastic collision between two bodies, one of which is initially at rest.



By the principle of conservation of linear motion:

$$m_1 u = m_1 v_1 + m_2 v_2 \quad \text{----------------} \ (1)$$

For an elastic collision, relative speed of approach equals relative speed of separation:

$$u - 0 = v_2 - v_1$$

Substitute $v_2 = u + v_1$ into (1):

$$m_1 u = m_1 v_1 + m_2 u + m_2 v_1$$

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} u$$

Substitute $v_1 = v_2 - u$ into (1):

$$m_1 u = m_1 v_2 - m_1 u + m_2 v_2$$

$$v_2 = \frac{2m_1}{m_1 + m_2} u$$

Case 1: $m_1 = m_2$

$v_1 = 0$, $v_2 = u$

E.g. during a billiard shot, the cue ball comes to a complete rest after hitting the target ball (of the same mass), passing all the momentum to the target ball.

Case 2: $m_1 \ll m_2$

$v_1 \approx -u$, $v_2 \approx 0$

E.g. a ping-pong ball lands on the table and bounces back with the same speed as before.

Case 3: $m_1 \gg m_2$

$v_1 \approx u$, $v_2 \approx 2u$

E.g. After hitting a bowling pin, the bowling ball continues with no noticeable drop in speed, while the pin is sent flying forward at about twice the ball's speed.

Obviously, these results are only valid for head-on elastic collisions between two bodies, one initially at rest. During examinations, you are *not* allowed to quote and use these results without derivation. However, they may come handy as "sanity checks" and in multiple-choice questions.

Appendix A: Static and Kinetic Friction

Static Friction

The magnitude of *static* friction between two surfaces can be expressed as

$$f_s \leq \mu_s N$$

where μ_s is the coefficient of static friction between the two surfaces in question and N is the magnitude of the normal contact force between the two surfaces.

Static friction thus has a maximum magnitude, called the limiting friction, $f_{\max} = \mu_s N$.

Kinetic Friction (or sliding friction or dynamic friction)

The magnitude of *kinetic* friction between two surfaces can be expressed as

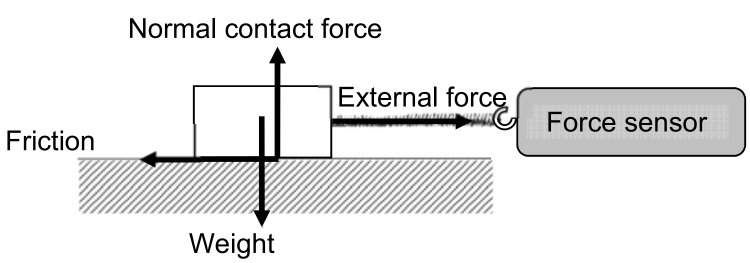
$$f_k = \mu_k N$$

where μ_k is the coefficient of kinetic friction between the two surfaces in question and N is the magnitude of the normal contact force between the two surfaces.

Unlike the static friction, kinetic friction is a constant value. It is also independent of the relative speed of motion.

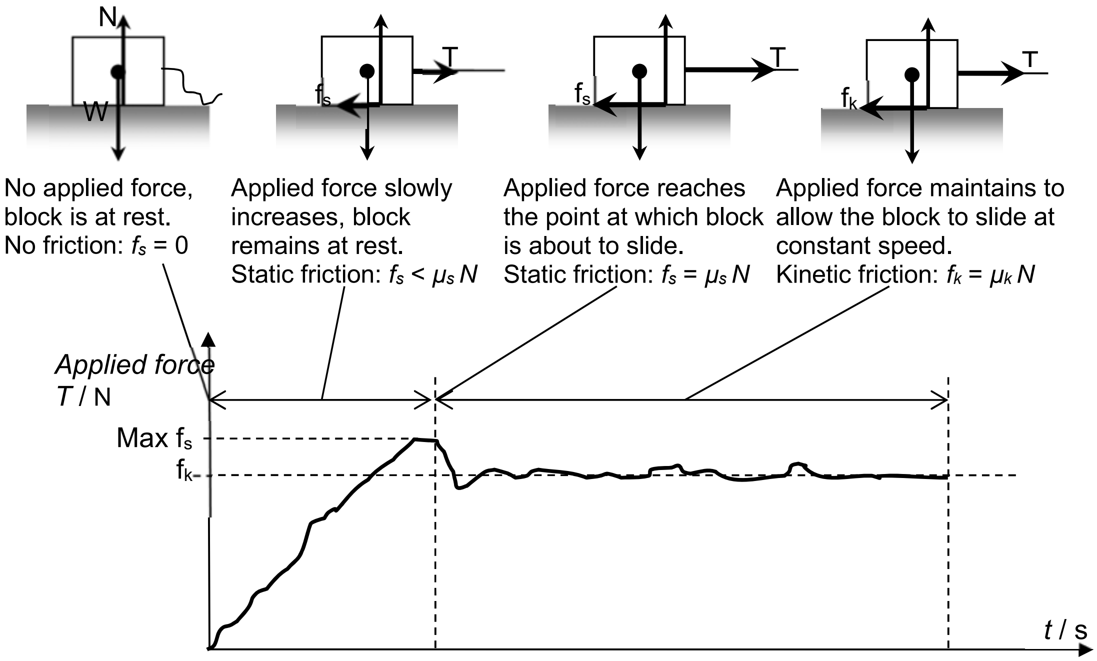
Both μ_s and μ_k are empirically found to be always less than 1 (friction is always less than the normal contact force). μ_k is found to be always less than μ_s (kinetic friction is always smaller than the limiting static friction).

To better understand the two, consider the results of the following experiment.



An external force is applied on the block and is increased very gradually until the block starts moving and continues to move at a constant speed. Since the block is always either stationary or moving at constant speed, the magnitude of the frictional force can be inferred from the applied force as measured by the force sensor.

The graph of the applied force against time is shown below in relation to the magnitudes of friction, both static and kinetic:



Appendix B: Newton's Law in Car Safety

Using Newton's laws, discuss how each of the following features in vehicles increases the safety of its passengers:

a) seat belts,
b) head restraints,
c) air bags.

(a) Seat belt exerts a retardation force on the passenger, without which he would have continued in his state of uniform motion in a straight line into the windscreen. (Newton's 1st Law).

If a moving car crashes into another (stationary) car, it comes to an abrupt stop. If the passenger continues to move forward, he will crash into the windscreen and likely suffer from severe injuries, or even death. Without the seat belt, the only retardation force on the passenger is the frictional force between the seat and the passenger's bum. But this force is limited. If the vehicle decelerates too fast, the frictional force is not large enough to decelerate the passenger at the same rate as the vehicle.

When passengers crash into the wind screen, it is due to their inertia: they are already moving forward and continue to do so. It is not due to some mysterious "forward force" that pushes them into the wind screen

(b) If a car is hit from the rear, the head restraint provides a forward force on the passenger's head so that the head is accelerated at the same rate as the rest of his body. If the passenger's body accelerates forward much faster than his head, the passenger may suffer from whiplash (a kind of neck injury). The muscles in the passenger's neck do not provide enough force to accelerate his head as fast as the rest of his body. (Newton's 2nd Law)

(c) A collision with the car's wind screen would bring the passenger to rest in a very short period of time. Airbags exert a relatively small force over a relatively long duration of time, safely bringing the passenger to rest by reducing the passenger's momentum at a relatively slower rate. (Newton's 2nd Law)

Tutorial 3: Dynamics

Self-Review Questions

Part 1: Newton's Laws, Inertia, Force, Momentum, Impulse

S1 When a force of 4 N acts on a mass of 2 kg for a time of 2 s, what is the rate of change of momentum?

A 1 kg m s^{-2} **B** 2 kg m s^{-2} **C** 4 kg m s^{-2} **D** 8 kg m s^{-2} **E** 16 kg m s^{-2}

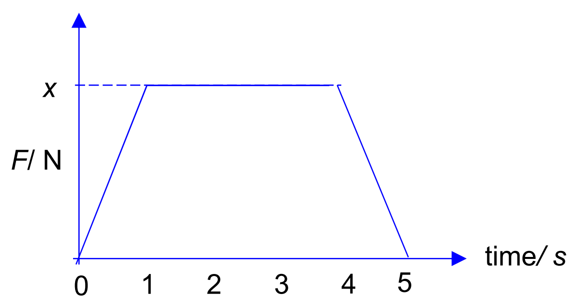
S2 On an aircraft carrier, an F-18 Super Hornet fighter aircraft that is being launched can be catapulted from 0 to 270 km h^{-1} in 2.0 s.

For a jet having a mass of 20 000 kg, what is the average force experienced by it?

S3 <u>**[N83/P2/2]**</u>

When a force, *F*, varying as shown below is applied to a mass of 10 kg, the gain in momentum in 5 s is 40 kg m s^{-1}.

What is the value of *x*?



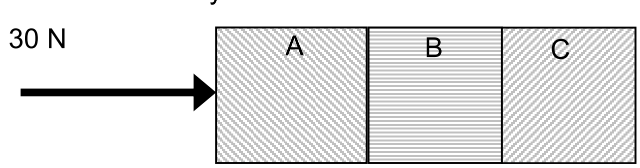
A 4 **B** 8 **C** 10 **D** 15

S4 <u>**[N84/P3/Q3]**</u>

A man is parachuting at constant speed towards the surface of the Earth. The force which, according to Newton's 3rd law, makes an action-reaction pair with the gravitational force on the man is

A The tension in the harness of the parachute.
B The viscous force of the man and his parachute on the air.
C The gravitational force on the Earth due to the man.
D The viscous force of the air on the man and his parachute.
E The tension in the fabric of the parachute.

S5 Three blocks A, B and C, of masses 2.0 kg, 4.0 kg and 3.0 kg, respectively, are pushed along a smooth horizontal surface by a force of 30 N as shown in the diagram.

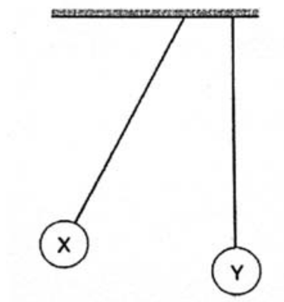


a) Calculate the net force experienced by each of the blocks, A, B and C.
b) What is the force exerted by block B on (i) block C? (ii) block A?

Part 2: Conservation of Linear Momentum/Collisions

S6 **[N02/P1/Q5]**

Two steel balls X and Y are suspended on a string. Ball X is pulled to one side as shown. After ball X is released, the balls collide.

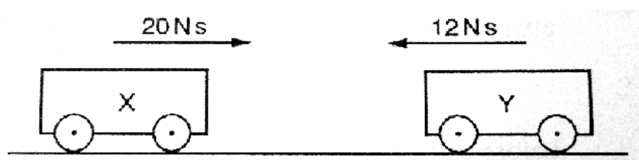


Which quantities must be conserved in the collision?

A Kinetic energy, total energy and momentum
B Kinetic energy and momentum only
C Kinetic energy and total energy only
D Total energy and momentum only

S7 **[N98/P1/Q4]**

The diagram shows two trolleys, X and Y, about to collide and gives the momentum of each trolley before the collision. After the collision, the directions of the motion of both trolleys are reversed and the magnitude of the momentum of X is then 2 N s.

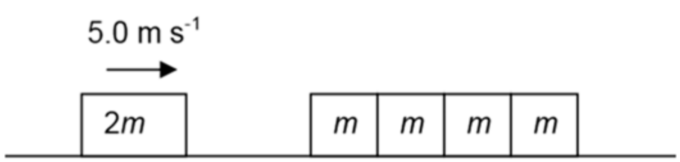


What is the magnitude of the corresponding momentum of Y?

A 6 N s **B** 8 N s **C** 10 N s **D** 30 N s

S8 **[Promo2007/I/6]**

Four identical railway trucks, each of mass *m*, are coupled together and are at rest on a smooth horizontal track. Initially, a fifth truck of mass 2*m* is moving at 5.0 m s^{-1}. It collides and moves off together with all the stationary trucks.



Calculate the speed of the trucks after impact.

A 0.83 m s^{-1} **C** 1.3 m s^{-1}

B 1.0 m s^{-1} **D** 1.7 m s^{-1}

S9 During a rescue operation, a 5500 kg helicopter hovers above a fixed point. The rotating blades of the helicopter's rotor send air downwards with a speed of 60.0 m s^{-1} relative to the helicopter, producing enough upward force for helicopter to hover.

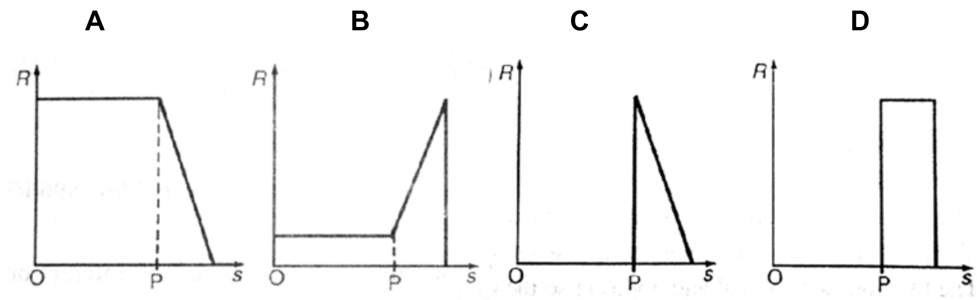
a) What is the net force acting on the helicopter?
b) What is the upward force produced by the helicopter's rotor?
c) What mass of air must pass through the blades every second to produce enough thrust for the helicopter to hover?

Discussion Questions

Part 1: Newton's Laws, Inertia, Force, Momentum, Impulse

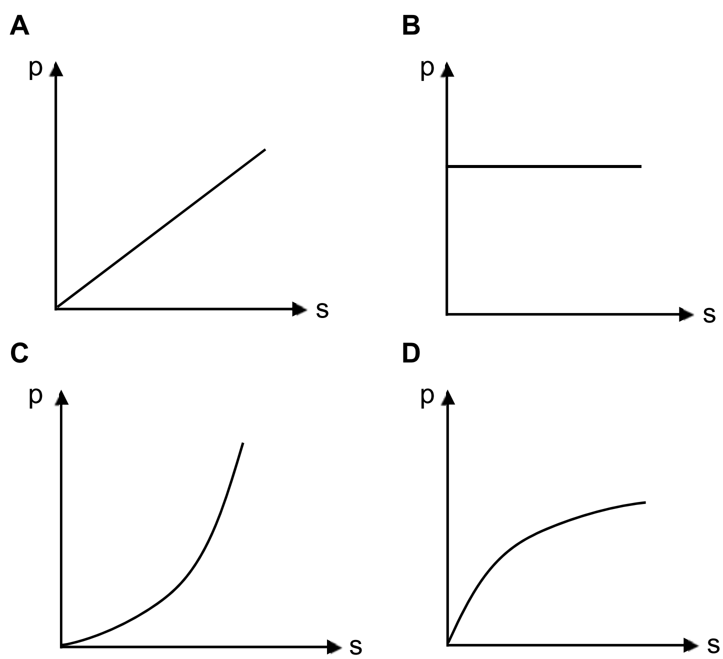
D1 **J88/P1/Q5**

An object falls vertically through air at a constant velocity and then strikes soft ground in which it becomes embedded. Its deceleration during impact is constant. If P represents the point of impact, which of the following graph best represents the variation of the total force R on the object with distance s?



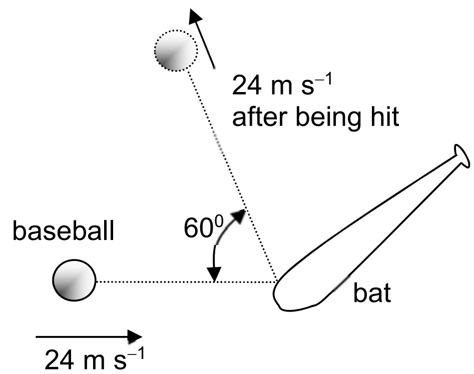
D2 **TJC/2008/P1/Q7**

A railway carriage starts from rest and is driven along a straight horizontal track by a motor which exerts a constant force. The effects of friction and air resistance can be neglected.
Which of the graphs below best represents the variation of the momentum p with distance travelled s?

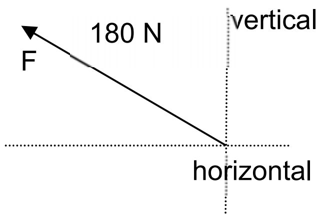


D3 **<u>HCI/2007/P1/Q8</u>**

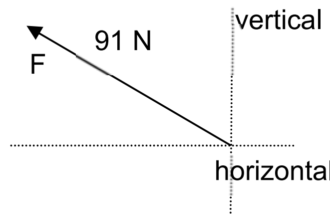
A baseball of mass 0.11 kg is thrown with a horizontal velocity of 24 m s^{-1} towards a batter. After the ball is struck by the bat, it has a speed of 24 m s^{-1} in the direction 60° above the horizontal as shown on the right.



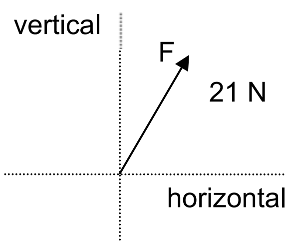
If the ball and bat are in contact for 0.025 s, the magnitude and direction of the average force exerted on the ball by the bat is



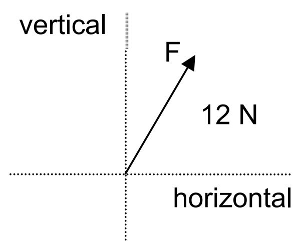
A



B

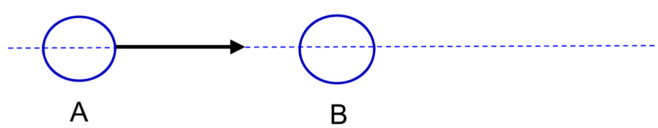


C



D

D4 Two identical billiard balls A and B collides, with B being at rest before the collision.



(top view of the two billiard balls)

Explain why in order for the initial speed of ball B after the collision to be in the same direction as the initial direction of travel of A, it must be a head-on collision.

D5 **<u>[2017 P1 Q4]</u>**

An object of mass 3.0 kg is falling vertically from rest. Air resistance R, in newtons, is given by the empirical equation

$$R = 0.60v$$

where v is the velocity in metres per second.

What is the maximum velocity of the object and what is the acceleration of the object when its velocity is 12 m s^{-1}.

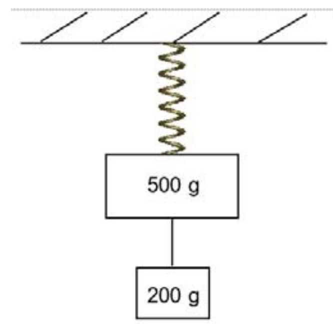
D6 **[J80/II/2&3, adapted]**

A car of mass 1000 kg pulls a caravan of mass 1000 kg. The total resistance to motion has a constant value of 4000 N. One quarter of this resistance acts on the caravan. At first, the acceleration of the car and the caravan is 2.0 m s^{-2}, but eventually they move at a constant speed of 6.0 m s^{-1}.

a) Calculate the force exerted on the car by the tow bar when the acceleration is 2.0 m s^{-2}.
b) Calculate the force exerted on the car by the tow bar when the car and caravan are moving at a constant speed of 6.0 m s^{-1}.

D7 **HCI/2007C2-BT2/P1/Q4**

The diagram below shows a system in static equilibrium in which two masses, joined by a light thread, are suspended from a light vertical spring. If the thread is cut, what is the value of the acceleration of the 500 g mass immediately after the thread is cut?

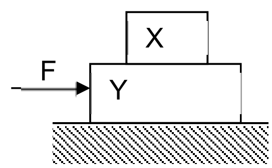


A g **B** 0.4g **C** 0.2g **D** zero

D8 **HCI/2008Promo/P1/Q5**

Two blocks, X and Y, of masses m and 3m, respectively, are accelerated along a smooth horizontal surface by a force F applied to block Y as shown.

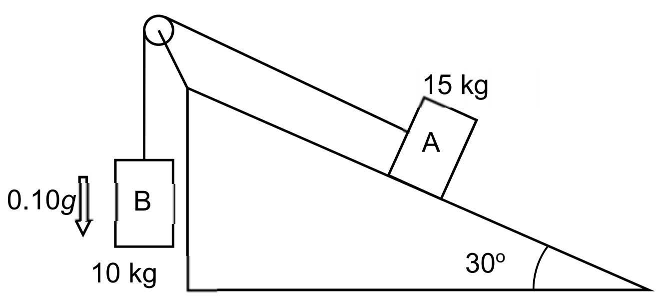
The interface between block X and Y is rough and friction is present. If block X does not slide on block Y, what is the frictional force acting on block X?



D9 **HCI/2007BT2/P1/Q4**

Two blocks, A and B, of mass 15 kg and 10 kg respectively, are connected by a light cord passing over a light, free-running pulley as shown in the diagram below. When released, block B accelerates downwards at 0.10 g.

What is the frictional force experienced by block A?

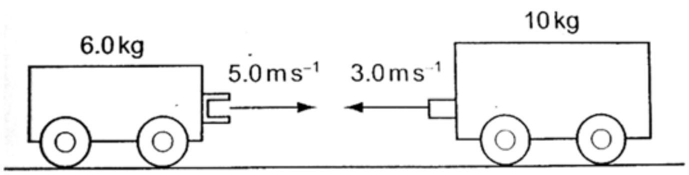


A 0 N **B** 15 N **C** 50 N **D** 90 N

Part 2: Conservation of Linear Momentum/Collisions

D10 **<u>H2 2008/P1/Q6</u>**

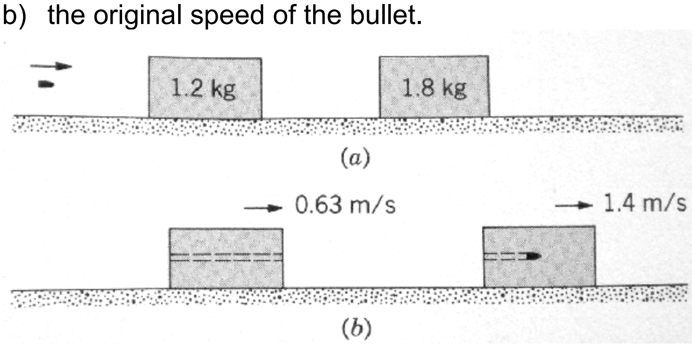
A trolley of mass 6.0 kg travelling at a speed of 5.0 m s^{-1} collides head-on and locks together with another trolley of mass 10 kg which is travelling in the opposite direction at a speed of 3.0 m s^{-1}. The collision lasts for 0.20 s.



What is the total momentum of the two trolleys before the collision and the average force acting on each trolley during this collision?

	Total momentum before the collision/ kg m s^{-1}	Average force on each trolley / N
A	0	300
B	60	150
C	0	150
D	60	300

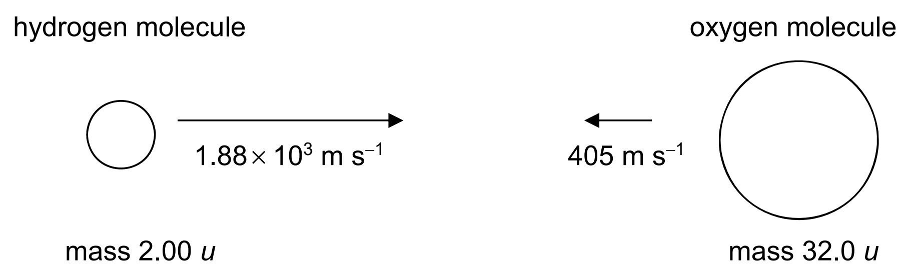
D11 A 3.5 g bullet is fired horizontally at two blocks resting on a smooth table top, as shown in the figure. The bullet passes through the first block, with mass 1.2 kg, and embeds itself in the second, with mass 1.8 kg.
The final speeds of the blocks are 0.63 m s^{-1} and 1.4 m s^{-1} respectively.
Neglecting the mass removed from the first block by the bullet, find
a) the speed of the bullet immediately after emerging from the first block and
b) the original speed of the bullet.



D12 **<u>N2000/III/1(part)</u>**

a) (i) Define linear momentum. [1]
(ii) State the principle of conservation of momentum. [2]

b) In a gas a hydrogen molecule, mass 2.00 *u* and velocity 1.88×10^3 m s^{-1}, collides elastically and head-on with an oxygen molecule, mass 32.0 *u* and velocity 405 m s^{-1}, as illustrated in the figure below.



In qualitative terms, what can be stated about the subsequent motion as a result of knowing that
(i) the collision is elastic,
(ii) the collision is head-on.
[3]

c) Using your answers to b),
(i) determine the velocity of separation of the two molecules after the collision.

Since collision is elastic,
(ii) apply the law of conservation of momentum to the collision.
(iii) determine the velocity of both molecules after the collision. [6]

D13 **N2007/P3/Q1 (modified)**

A tritium nucleus moves towards a deuterium nucleus as illustrated in Fig. 1.1.

Fig. 1.1

The nuclei initially have the same speed *v*. The tritium nucleus consists of two neutrons and a proton. The deuterium nucleus consists of a neutron and a proton. The proton and the neutron have the same mass *m*.

The two nuclei having like charges, repel one another.

a) Explain why it is **not** possible for the nuclei to stop at the same instant. [2]

b) At one instant during the interaction between the nuclei, they are both travelling **in the same direction** with the same speed. Calculate this speed, in terms of *v*. [2]

c) Fig. 1.2 is a velocity-time sketch graph showing how the velocity of each nucleus varies. The interaction between the nuclei is elastic.

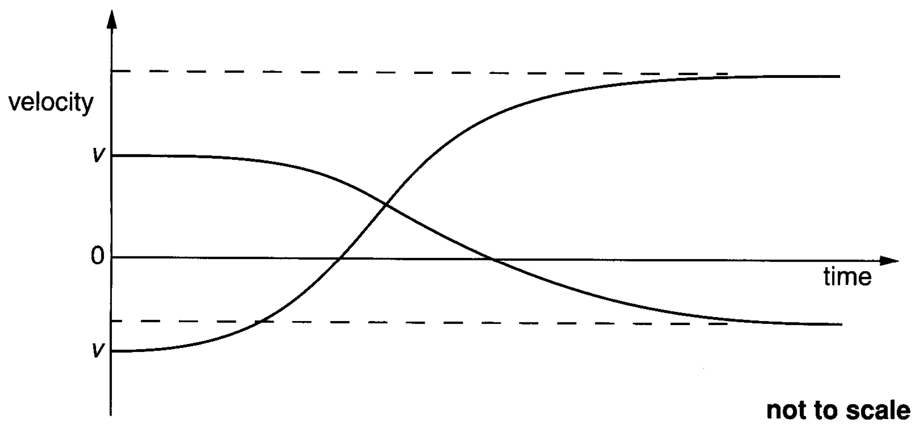


Fig. 1.2

(i) Label the graph to show

1. which curve is for the tritium nucleus,
2. the times at which each nucleus stops,
3. the time at which they are at their distance of closest approach. [3]

(ii) Determine the final speed of each nucleus in terms of *v*. [4]

D14 **HCI BT2010/II/3(b)**

The rocket engines of the space shuttle can exert a force of 28.6 MN at lift-off when the total mass is 2.00×10^6 kg.

(i) Determine the acceleration of the rocket on vertical lift off. [2]

(ii) Suppose a 65.0 kg astronaut in the space shuttle were to stand on a bathroom weighing scale that indicates "weight" in kilograms during lift-off.
Determine the reading on the scale. [2]

(iii) The rocket engine ejects gas at a rate of 2.00×10^4 kg s^{-1} on lift-off. Determine the speed of the ejected gas relative to the rocket engine. [2]

D15 **H2 2011/P3/Q1**

A toy car with a rocket engine moves along a horizontal track, as shown in Fig. 1.1.

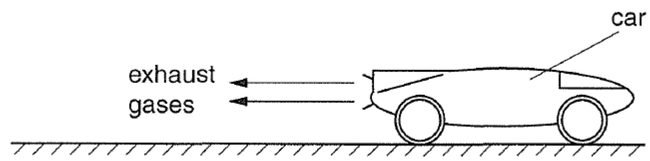


Fig 1.1

The rocket engine produces a constant forward force of 4.6 N. The car loses mass continuously as exhaust gases are produced by the rocket.

a) Use momentum consideration to explain why the rocket produces a forward force on the car.[3]

b) The variation with time *t* of the speed *v* of the car is shown in Fig. 1.2.

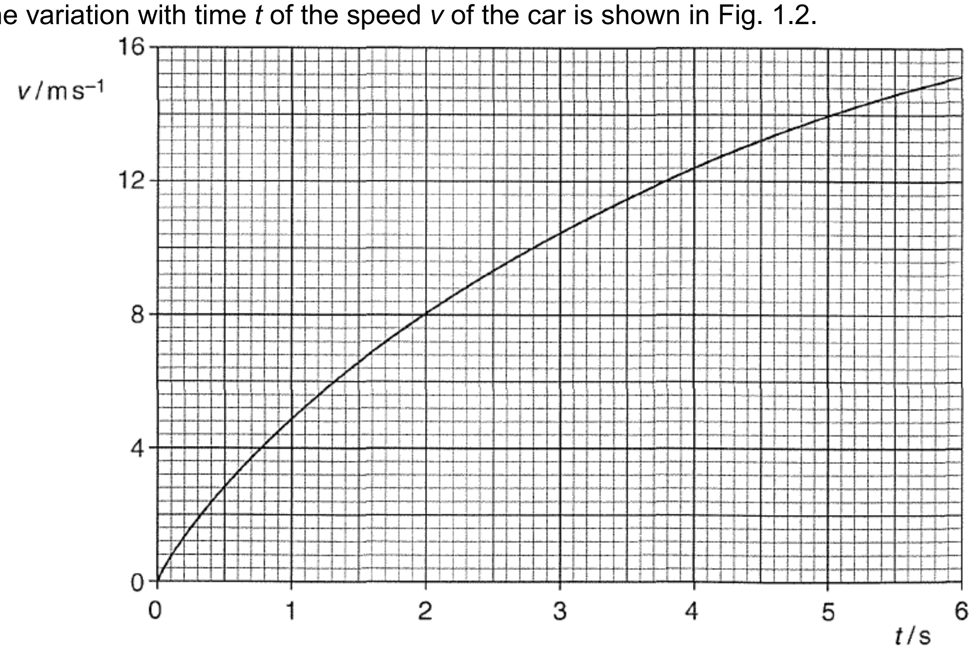


Fig. 1.2

At time *t* = 2.0 s, the mass of the car is 440 g.

(i) For the time *t* = 2.0 s,

1. use Fig. 1.2 to determine the acceleration of the car, [2]
2. use your answer in (i) part 1 to determine the magnitude of the resistive force acting on the car. [2]

(ii) Explain how it can be deduced that the resistive forces acting on the car increase with increase of speed. [2]

c) The toy car is now re-fuelled and then rotated so that it is pointing upwards. It is suggested that the rocket engine produces sufficient force to propel the car vertically. By considering the acceleration of the car at time *t* = 0 in Fig. 1.2, comment on this suggestion. [3]

Answers to Tutorial 3: Dynamics

Self-Review
S1 C **S2** 750 kN
S3 C **S4** C
S5 a) 6.7 N, 13 N, 10 N b) (i) 10 N (ii) 23.3 N
S6 D **S7** C **S8** D
S9 0N, 54N, 900 kg/s

Discussion
D5 49 m s^{-1}; 7.4 m s^{-2}
D6 a) 3000 N b) 1000 N
D8 *F*/4
D11 a) 721 m s^{-1} b) 937 m s^{-1}
D12 c) (i) 2285 m s^{-1} (iii) 136 m s^{-1}, 2420 m s^{-1} both towards the left.
D13 b) 0.20v c) (ii) $v_{deuterium}$ = 1.4v, $v_{tritium}$ = 0.60v
D14 4.49 ms^{-1}, 94.8 kg, 1430 ms^{-1}
D15 2.76 ms^{-2}, 3.4N