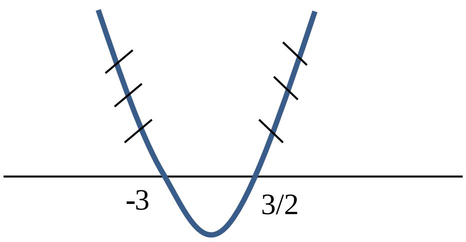
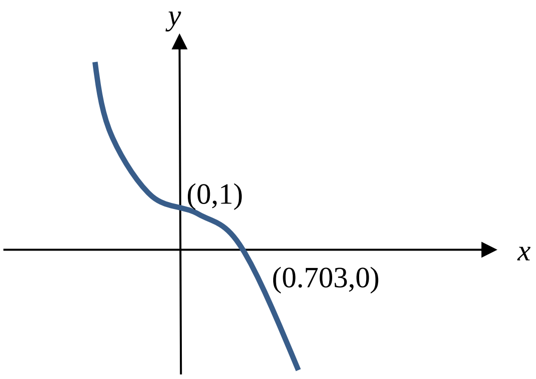
2016 H1 A Level Mathematics Solution

Section A

Q1.	(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(2\ln\left(3x^2+4\right)\right)=\frac{2}{\left(3x^2+4\right)}(6x)=\frac{12x}{\left(3x^2+4\right)}$ (i) $\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{1}{2(1-3x)^2}\right)=\frac{1}{2}(-2)(-3)(1-3x)^{-3}=\frac{3}{(1-3x)^3}$
Q2.	Substituting $u=e^x$ into inequality $2e^{2x}\geq 9-3e^x$ $2e^{2x}\geq 9-3e^x$ becomes $2u^2\geq 9-3u$ $2u^2+3u-9\geq 0$ $(u+3)(2u-3)\geq 0$ $u\leq -3$ or $u\geq\frac{3}{2}$ $e^x\leq -3$ or $e^x\geq\frac{3}{2}$  Since $e^x>0$ for $x\in\mathbb{R}$, $e^x\leq -3$ is rejected. Hence, $e^x\geq\frac{3}{2}$ $x\geq\ln\left(\frac{3}{2}\right)$
Q3.	(i) 

	(ii) From GC, gradient of C when x = 0.5 is -1.61 (3 sig. fig.) (iii) <u>**OUT of SYLLABUS**</u> when x = 0.5, gradient of the normal to C at the point = $-\left(\frac{1}{-1.606531}\right) = 0.6224591994 = 0.622$ (3 sig.fig.) At point (0.5, 0.3565307) Equation of the normal C when $x = 0.5$ $y - 0.35653 = 0.62246(x - 0.5)$ $y = 0.622x + 0.0453$ (3 s.f) (iv) $\int_0^k \left(e^{-x} - x^2\right) dx$ $= \left[-e^{-x} - \frac{x^3}{3}\right]_0^k$ $= \left(-e^{-k} - \frac{k^3}{3}\right) - \left(-e^0 - \frac{0}{3}\right)$ $= 1 - e^{-k} - \frac{k^3}{3}$
Q4.	(i) $y = 1 + 6x - 3x^2 - 4x^3$ $\frac{dy}{dx} = 6 - 6x - 12x^2$ For stationary point, $\frac{dy}{dx} = 0$ $6 - 6x - 12x^2 = 0$ $x = -1$ or $x = 0.5$ When $x = -1$, $y = -4$ When $x = 0.5$, $y = 2.75$ Coordinates of stationary points on the curve are (-1, -4) and (0.5, 2.75). (ii) $\frac{d^2y}{dx^2} = -6 - 24x$ For $x = -1$

$$\frac{d^2y}{dx^2} = -6 - 24(-1) = 18 \ (> 0)$$

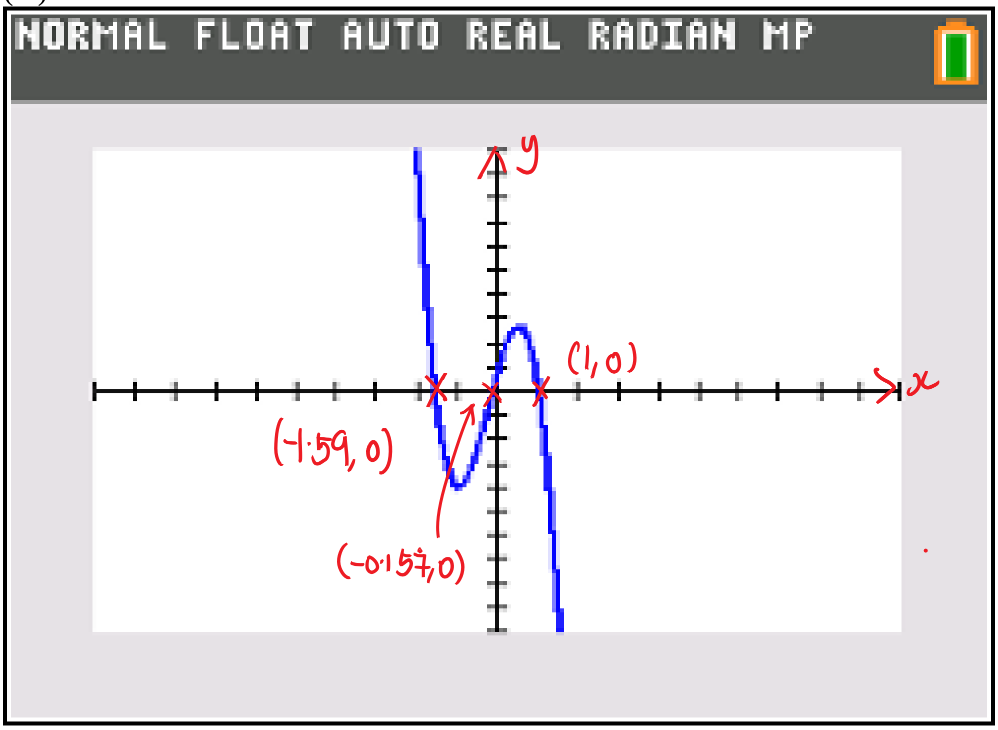
(-1, -4) is a minimum point.

For $x = 0.5$

$$\frac{d^2y}{dx^2} = -6 - 24(0.5) = -18 \ (< 0)$$

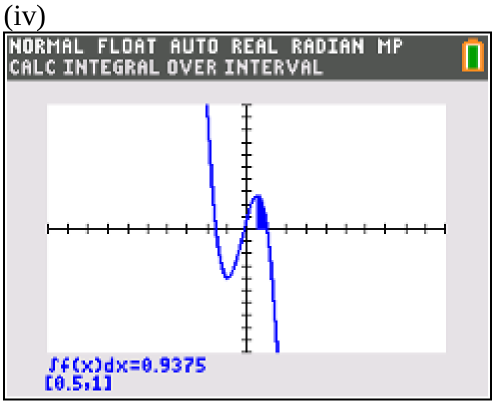
(0.5, 2.75) is a maximum point.

(iii)



The coordinates of the points where the curve crosses the x-axis are (-1.59, 0) , (-0.157, 0) and (1, 0).

(iv)

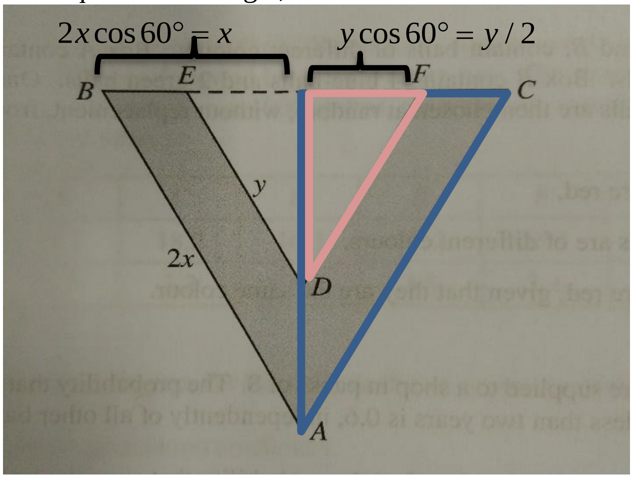


Numerical value of the area under C between x = 0.5 and x =1

$$=\int_{0.5}^{1} 1+6x-3x^2-4x^3 \text{ d}x = 0.9375 \text{ unit}^2$$

Q5.

Since DEF and is an equilateral triangle, $\measuredangle EFD = 60^o$



Area of the remaining shape *ABEDFCA*

Area of triangle = ½ absinC

= Area of triangle ABC – Area of triangle EDF

$$=\frac{1}{2}(2x)^2\sin 60° - \frac{1}{2}(y)^2\sin 60° \left(\text{properties of equilateral triangles}\right)$$

$$=\sqrt{3}x^2 - \frac{\sqrt{3}}{4}y^2$$

<u>Alternative</u>

Height of Triangle ABC = $\sqrt{(2x)^2 - x^2} = \sqrt{3}x$ [Pythagoras Theorem]

Height of Triangle EDF= $\sqrt{(y)^2 - \left(\frac{y}{2}\right)^2} = \frac{\sqrt{3}y}{2}$

Area of the remaining shape *ABEDFCA*

Area of triangle = ½ (base)(height)

= Area of triangle ABC – Area of triangle EDF

$$=\frac{1}{2}(2x)(\sqrt{3}x) - \frac{1}{2}(y)(\frac{\sqrt{3}y}{2})$$

$$=\sqrt{3}x^2 - \frac{\sqrt{3}}{4}y^2$$

Given that Area of the remaining shape *ABEDFCA* = $2\sqrt{3}$ cm^2.

	$\sqrt{3}x^2 - \frac{\sqrt{3}}{4}y^2 = 2\sqrt{3}$ $x^2 - \frac{1}{4}y^2 = 2$ $4x^2 - y^2 = 8\,(\text{shown})$ [Equation 1] (ii) Given that perimeter of the shape *ABEDFCA* = 10 cm $2(2x) + 2(y) + 2x - y = 10$ $4x + 2y + 2x - y = 10$ $6x + y = 10$ Substitute $y = 10 - 6x$ into Equation 1 $4x^2 - (10 - 6x)^2 = 8$ $4x^2 - (100 - 120x + 36x^2) = 8$ $-32x^2 + 120x - 108 = 0$ $32x^2 - 120x + 108 = 0$ $x = 1.5$ or $x = 2.25$ For $x = 1.5$, $y = 1$ For $x = 2.25$, $y = -3.5$ (reject as length y cannot be less than 0) The values of x and y are $x = 1.5$ and $y = 1$

Section B

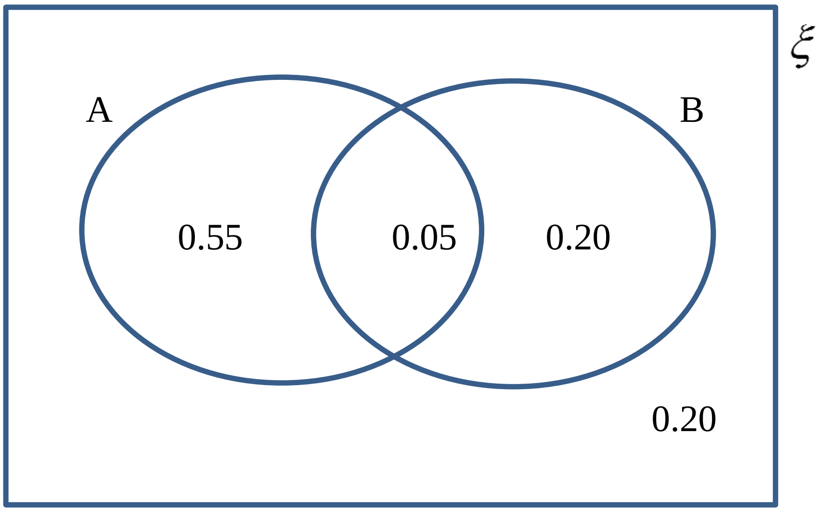
Q6.	(i) **OUT of SYLLABUS** Select the number of male and female students according to the table below. Within each category of students, use simple random sampling. For example in the category of male students, number the male students from 0001 to 1260, using 4-digit random numbers to generate a sample of 42 students.

Gender	Sample size
Male	$\frac{1260}{2400} \times 80 = 42$
Female	$\frac{1140}{2400} \times 80 = 38$

	(ii) **<u>OUT of SYLLABUS</u>** Stratified sampling method gives a sample that is more representative of the students expenditure on music annually compared to simple random sampling method. (iii) Unbiased estimate for population mean, $\overline{x} = \dfrac{312}{80} = 3.9$ Unbiased estimate for population variance, $s^2 = \dfrac{1}{79}\left[1328 - \dfrac{(312)^2}{80}\right]$ $= 1\dfrac{161}{395} = 1.41$ (3 s.f)

Q7.

(i)



(iia) P (at least one of A or B occurs) = $\mathrm{P}(\mathrm{A} \cup \mathrm{B}) = \mathrm{P}(\mathrm{A}) + \mathrm{P}(\mathrm{B}) - \mathrm{P}(\mathrm{A} \cap \mathrm{B})$

$= 0.6 + 0.25 - 0.05 = 0.8$

(iib) P (exactly one of A or B occurs) = 0.55 + 0.20 = 0.75

(iii) $\mathrm{P}(\mathrm{A}|\mathrm{B}') = \dfrac{\mathrm{P}(\mathrm{A} \cap \mathrm{B}')}{\mathrm{P}(\mathrm{B}')} = \dfrac{0.55}{1-0.25} = \dfrac{11}{15}$

Q8.

(i) P(both balls are red) = $\dfrac{1}{2} \times \dfrac{3}{10} \times \dfrac{2}{9} = \dfrac{1}{30}$

(ii) P(both balls are different)

$= P$(2 diff coloured Balls are fm Box A)+ P(2 diff coloured Balls are fm Box B)

= P(Box A & BR +BG+ RG + GR+GB+RB) + P(Box B & BG +GB)

$$= \frac{1}{2} \times \left(\frac{5}{10} \times \frac{3}{9} + \frac{5}{10} \times \frac{2}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{2}{10} \times \frac{5}{9} + \frac{3}{10} \times \frac{5}{9} \right) + \frac{1}{2} \times \left(\frac{4}{6} \times \frac{2}{5} + \frac{2}{6} \times \frac{4}{5} \right)$$

$$= \frac{11}{18}$$

<u>Alternative Method</u>

P(both balls are different)

= 1 – P(both balls are same colour)

= 1 – P(Both are Red) – P(Blue balls fm Box A) – P(Green balls fm Box A) –
P(Blue balls fm Box B) – P(Green balls fm Box B)

$$= 1 - \frac{1}{30} - \left(\frac{1}{2} \times \frac{5}{10} \times \frac{4}{9} \right) - \left(\frac{1}{2} \times \frac{2}{10} \times \frac{1}{9} \right) - \left(\frac{1}{2} \times \frac{4}{6} \times \frac{3}{5} \right) - \left(\frac{1}{2} \times \frac{2}{6} \times \frac{1}{5} \right)$$

$$= \frac{11}{18}$$

(iii) P(both balls are red given that that they are the same different)

$$= \frac{\mathrm{P}(\text{both balls are red})}{\mathrm{P}(\text{both balls are the same colour})} = \frac{\frac{1}{30}}{1 - \frac{11}{18}} = \frac{3}{35}$$

Q9.	Let X be the r.v. number of batteries that have a lifetime of less than two years out of 8 batteries. $X \sim B(8, 0.6)$ (ia) Required probability = $P(X = 8) = 0.01679616 = 0.0168$ (ib) Required probability = $P(X \geq 4) = 1 - P(X < 4) = 1 - P(X \leq 3) = 0.8263296 = 0.826$ Let Y be the r.v. number of packs of batteries that have at least half of the batteries have a lifetime of less than two years out of 4 packs of batteries. $Y \sim B(4, 0.8263296)$ (ii) Required probability = $P(Y \leq 2) = 0.1417924285 = 0.142$ (iii) **<u>OUT Of SYLLABUS</u>** Let T be the r.v. number of batteries that have a lifetime of less than two years out of 80 batteries. $T \sim B(80, 0.6)$ Since n = 80 is large, np = 80(0.6) = 48 > 5, nq = 80(0.4) = 32 > 5, $T \sim N(48, 19.2)$ approximately E(T) = 48, Var(T) = 19.2 (iii) Required probability = $P(T \geq 40) \xrightarrow{cc} = P(T \geq 39.5) = 0.974$
Q10.	(i) Let X be the random variable denoting the mean top speed of cheetahs in km/h and μ be the population mean top speed of cheetahs. $H_0 : \mu = 95$ $H_1 : \mu \neq 95$ Under H_0, $\bar{X} \sim N(95, \frac{4.1^2}{40})$. Using a two tailed z-test at 5% level, $\bar{x} = 96.3$ gives $z_{cal} = 2.01$ and p value =0.0449 < 0.05 We reject Ho and conclude that there is sufficient evidence at 5% level of significance that the mean top speed of cheetahs is not 95 km/h. (ii) $H_0 : \mu = 95$ $H_1 : \mu > 95$ Under H_0, $\bar{X} \sim N(95, \frac{4.1^2}{40})$. $Z = \frac{\bar{X} - \mu}{\sqrt{\frac{\sigma^2}{n}}} \sim N(0,1)$ Using a one tailed z-test at 5% level. Critical value = 1.64485 Critical region: $z \geq 1.64485$ $z_{cal} = \frac{\bar{x} - 95}{\sqrt{\frac{4.1^2}{40}}}$ Since H_0 is not rejected, $z_{cal} < 1.64485$

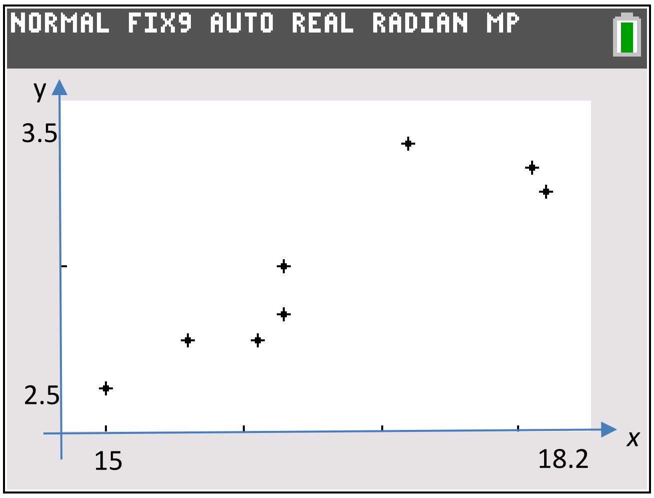
$$\frac{\bar{x}-95}{\sqrt{\frac{4.1^2}{40}}}<1.64485$$

$$\bar{x}<96.1$$

The required set of values of mean top speed is $\left\{\bar{x}\in\mathbb{R}^+ \mid \bar{x}<96.1\right\}$

Q11

(i) **<u>NOT tested in 2020 'A' level Exam</u>**



(ii) The product moment correlation coefficient, r is 0.903

(iii) The required equation is $y=0.294x-1.89$

(iv) When $x=16.9$, $y=3.08$. Therefore the estimated time is 3.08 minutes.
This is a reliable estimate as r is close to 1 and that x = 16.9 is within the given data range (interpolation).

(v) The new product moment correlation coefficient, r is 0.568.

(vi) (ii) is more likely to represent the correlation between swimming and running times for all the members of the club as r = 0.903 is closer to 1 which is supported by the the scatter diagram which shows an increaing trend; as x increases, y increases. The timings of the new member are likely to be outliers.

Q12

Let X be the random variable denoting the mass of a biscuit.

$X \sim N\left(20, 1.1^2\right)$

(i) Required probability = $P\left(X<19\right)=0.182$

Let T be the random variable denoting the mass of a box.

$T \sim N\left(5, 0.8^2\right)$

	$Y = X_1 + \cdots + X_{12} + T$ $\mathrm{E}(Y) = 20\times12 + 5 = 245$ $\mathrm{Var}(Y) = 1.1^2\times12 + 0.8^2 = 15.16$ $Y \sim N(245, 15.16)$ (ii) Required probability = $P(Y > 248) = 0.221$ Cost of a box of biscuits, $C = 0.6\times(X_1 + \cdots + X_{12}) + 0.2\times T$ $\mathrm{E}(C) = 0.6\times240 + 0.2\times5 = 145$ $Var(C) = 0.6^2\times1.1^2\times12 + 0.2^2\times0.8^2 = 5.2528$ $C \sim N(145, 5.2528)$ (iii) Required probability = $P(142 < C < 149) = 0.864$