Anglo - Chinese School

(Independent)

FINAL EXAMINATION 2020

YEAR 3 INTEGRATED PROGRAMME

CORE MATHEMATICS
PAPER 2

WEDNESDAY   **7th October 2020**   **1 hour 30 minutes**

ADDITIONAL MATERIALS:

Answer Paper (7 sheets)
Graph Paper (1 sheet)

INSTRUCTIONS TO STUDENTS

Do not open this examination paper until instructed to do so.
A calculator is required for this paper.
Answer all the questions on the answer sheets provided.
At the end of the examination, fasten the answer sheets together.
Unless otherwise stated in the question, all numerical answers must be given exactly
or correct to three significant figures. Answers in degrees are to be given to one decimal place.

INFORMATION FOR STUDENTS
The maximum mark for this paper is 80.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for correct method, provided this is shown by written working. You are therefore advised to show all working.

Answer all the questions on the answer sheets provided. Begin each question on a new page.

1. *[Maximum mark: 6]*

(a) Evaluate $\dfrac{(0.3578)^2 \times \sqrt{7.647}}{\sqrt[4]{43.96}}$, leaving your answer correct to 3 significant figures. [2]

(b) Express $\dfrac{1}{x+2} + \dfrac{1}{2-x} + \dfrac{10+3x}{x^2-4}$ as a single fraction in its simplest form. [4]

2 *[Maximum mark: 7]*

(a) Solve the equation $2x - \dfrac{9}{2x} = 3$, giving your answers correct to two decimal places. [4]

(b) Expand and simplify $\left(p^{\frac{2}{3}} + q^{\frac{1}{3}} \right) \left(q^{\frac{2}{3}} - q^{\frac{1}{3}} p^{\frac{2}{3}} + p^{\frac{4}{3}} \right)$. [3]
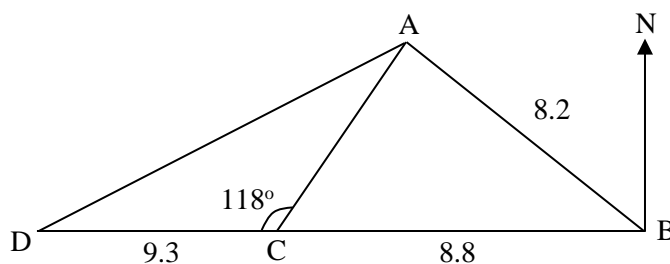
3 *[Maximum mark: 9]*

(a) Simplify $\dfrac{\left(2x^{-2}\right)^3 \times 4x^{12}}{(4x^{-4})^2 \left(x^3\right)^4}$, expressing your answer in positive indices. [3]

(b) Given that $\dfrac{16^{x+1} + 2(2^{2x})^2}{2^{x-2}8^{x+1}} = k$, find the value of k . [3]

(c) Find the value of $\dfrac{1}{\log_p pqr} + \dfrac{1}{\log_q pqr} + \dfrac{1}{\log_r pqr}$. [3]

4   *[Maximum mark: 15]*

The diagram shows four towns A, B, C and D. Given that Town C and Town D lie west of B, AB = 8.2 km, BC = 8.8 km, CD = 9.3 km and $\angle ACD = 118^o$, calculate

(a) $\angle ABC$,   [4]

(b) the bearing of B from A,   [2]

(c) the distance AD.   [3]

A cyclist starts from D at 1030 and travels towards A at a constant speed of 15 km/h.

(d) Find the time, to the nearest minute, when he will be nearest to Town C.   [6]

5   *[Maximum mark: 13]*

(a) The equation of a curve is $y = 3qx^2 - 3px + 12q$ where p and q are positive integers. Show

that the x − axis is the tangent to the curve if $\dfrac{p}{2q} = 2$.   [4]

(b) Explain why the line $y = kx - 2$ will always intersect the curve $y = \dfrac{3}{x-3}$.   [4]

(c) The roots of $(x-1)(3-x) = m$ are α and β.

   (i)   Find the value of $\alpha + \beta$.   [2]

   (ii)   State the value of $\alpha\beta$ in terms of m.   [1]

   (iii)   Given that $\dfrac{1}{\alpha} + \dfrac{1}{\beta} = \dfrac{4}{(\alpha\beta)^2}$, find the value(s) of m.   [2]

6 *[Maximum mark: 17]*

(a) Evaluate $\log_3 2 + \log_2 3$. [2]

(b) Find the value of k if $e^{e^{k+2}} = 5$. [3]

(c) $2^{3x-5} = 7^{x-2}$ can be expressed as $\left(\dfrac{7}{a}\right)^x = \dfrac{49}{32}$ where a is a positive integer.

   (i) Find the value of a. [3]

   (ii) Hence, find the value of x. [2]

(d) When a cup of hot water is left in a room, its temperature T ºC at time t minutes is given by
    $T = 67e^{-0.2t} + 27$.

   (i) Kenneth claimed that the initial temperature of the water is 100ºC. Explain clearly
         whether Kenneth is correct. [2]

   (ii) Find the temperature when $t = 5$ minutes. [2]

   (iii) How long will it take for the temperature of the water to drop to 30ºC? [3]

7 *[Maximum mark: 13]*

Answer the whole of this question on a sheet of graph paper.

The variables x and y are connected by the equation $y = 2^{2-x}$. Some corresponding values of x and y are given in the following table.

x	-2	-1	0	1	2	3	4	5
y	16	a	4	2	1	0.50	b	0.125

(a) State the value of a and of b. [2]

(b) Taking 2 cm to represent 1 unit on the horizontal axis and 1 cm to represent 1 unit on the vertical
    axis, draw the graph of $y = 2^{2-x}$ for $-2 \leq x \leq 5$. [4]

(c) Use your graph to solve the equation $2^{2-x} = 5$. [2]

(d) Find the gradient of the tangent at $x = 0$. [2]

(e) By drawing a suitable straight line on the graph, find the solution of the equation
    $2^{x-2}(5-2x) = 1$. [3]

---END OF PAPER---