Worksheet 1 – Vectors

1. Answer

   (a) magnitude of $\overrightarrow{AB} = \left|\overrightarrow{AB}\right| = \sqrt{3^2 + 4^2} = 5$

   (b) magnitude of $\overrightarrow{BC} = \left|\overrightarrow{BC}\right| = \sqrt{(-5)^2 + (-12)^2} = 13$

   (c) magnitude of $\overrightarrow{CD} = \left|\overrightarrow{CD}\right| = \sqrt{3^2 + (-5)^2} = 5.83$ (3.s.f)

2. Answer

   (a) $\overrightarrow{OA} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}$

   $\overrightarrow{OB} = \begin{pmatrix} -1 \\ y-2 \end{pmatrix}$

   (b) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix} -4 \\ 6 \end{pmatrix} + \begin{pmatrix} 4 \\ -10 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

   $C = (0, -4)$

   (c) $\overrightarrow{AO} = -\overrightarrow{OA} = \begin{pmatrix} 4 \\ -6 \end{pmatrix}$

   (d) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = \begin{pmatrix} 4 \\ -6 \end{pmatrix} + \begin{pmatrix} -1 \\ y-2 \end{pmatrix} = \begin{pmatrix} 3 \\ y-8 \end{pmatrix}$

   $\overrightarrow{AB} = k\overrightarrow{AC}$

   $\begin{pmatrix} 3 \\ y-8 \end{pmatrix} = k\begin{pmatrix} 4 \\ -10 \end{pmatrix}$

   $\begin{pmatrix} 3 \\ y-8 \end{pmatrix} = \begin{pmatrix} 4k \\ -10k \end{pmatrix}$

   $3 = 4k$

   $k = \frac{3}{4}$

   $y - 8 = -10\left(\frac{3}{4}\right)$

   $y = -7.5 + 8 = \frac{1}{2}$

3. Answer

   (a) $\left|\overrightarrow{PQ}\right| = \sqrt{3^2 + 4^2} = 5$

   (b) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$\overrightarrow{OP} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

$\overrightarrow{OQ} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} -3 \\ 7 \end{pmatrix}$

$Q = (-3, 7)$

(c) $\overrightarrow{OR} = \begin{pmatrix} x \\ 0 \end{pmatrix}$

$\overrightarrow{PR} = \overrightarrow{PO} + \overrightarrow{OR} = -\begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} x \\ 0 \end{pmatrix} = \begin{pmatrix} x \\ -3 \end{pmatrix}$

(d) $\overrightarrow{PQ} = k\overrightarrow{PR}$

$\begin{pmatrix} -3 \\ 4 \end{pmatrix} = k\begin{pmatrix} x \\ -3 \end{pmatrix}$

$\begin{pmatrix} -3 \\ 4 \end{pmatrix} = \begin{pmatrix} xk \\ -3k \end{pmatrix}$

$-3 = xk$

$4 = -3k$

$k = -\frac{4}{3}$

(e) $-3 = xk$

$-3 = x\left(-\frac{4}{3}\right)$

$x = \frac{9}{4}$

$\overrightarrow{OR} = \begin{pmatrix} 2\frac{1}{4} \\ 0 \end{pmatrix}$

4. Answer

(a) $\overrightarrow{OA} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= -\begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -5 \\ -2 \end{pmatrix} + \begin{pmatrix} 6 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

$\left|\overrightarrow{AB}\right| = \sqrt{1^2 + (-5)^2} = 5.10$ units

(b) m of $AB = \frac{-3-2}{6-5} = -5$

$y = mx + c$

$2 = -5(5) + c$

$c = 27$

$y = -5x + 27$

(c) $\overrightarrow{AB} = 2\overrightarrow{AC} = 2\overrightarrow{BC}$

$\frac{1}{2}\overrightarrow{AB} = \overrightarrow{AC} = \overrightarrow{BC}$

$\overrightarrow{AC} = \frac{1}{2}\begin{pmatrix} 1 \\ -5 \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ -\frac{5}{2} \end{pmatrix}$

$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix} 5 \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{1}{2} \\ -\frac{5}{2} \end{pmatrix} = \begin{pmatrix} 5\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}$

$C = (5.5, -0.5)$

5. $\overrightarrow{QX} = \overrightarrow{QO} + \overrightarrow{OX}$

$\overrightarrow{QO} = -\overrightarrow{OQ} = -q$

$\overrightarrow{OX} = \frac{3}{5}\overrightarrow{OP} = \frac{3}{5}p$

$\overrightarrow{QX} = -q + + \frac{3}{5}p$

6. Answer

(a) Answer

(i) $\overrightarrow{OR} = \overrightarrow{OP} + \overrightarrow{PR}$

$\overrightarrow{PR} = 2\overrightarrow{PQ} = 2q$

$\overrightarrow{OR} = p + 2q$

(ii) $\overrightarrow{TR} = \overrightarrow{TO} + \overrightarrow{OR}$

$\overrightarrow{TO} = 3\overrightarrow{PO} = -3\overrightarrow{OP} = -3p$

$\overrightarrow{TR} = -3q + p + 2q = -2p + 2q$

(iii) $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$

$\overrightarrow{QR} = q$ (since Q is midpoint of PR, $PQ = QR$)

$\overrightarrow{RS} = \frac{1}{6}\overrightarrow{RT} = -\frac{1}{6}\overrightarrow{TR} = -\frac{1}{6}(-2p + 2q)$

$\overrightarrow{QS} = q - \frac{1}{6}(-2p + 2q)$

$= q + \frac{2}{6}p - \frac{2}{6}q$

$= \frac{2}{6}p + \frac{4}{6}q$

$= \frac{1}{3}(p + 2q)$

(b) $\overrightarrow{QS} = k\overrightarrow{OR}$

$\overrightarrow{QS} = \frac{1}{3}\overrightarrow{OR}$

Since they are a factor of one another, they are parallel.

7. Answer

(a) (i) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 4\mathbf{y} - 8\mathbf{x}$

(ii) Since $\overrightarrow{AB}$ is parallel to $\overrightarrow{OC}$,

$\overrightarrow{AD} = \frac{3}{2}\overrightarrow{AB} = \frac{3}{2}(4\mathbf{y}) = 6\mathbf{y}$

$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = 8\mathbf{x} + 6\mathbf{y}$

(b) Triangle OCE is similar to triangle OAD.

$\therefore$ Ratio of area of $\frac{OCE}{OAD} = \frac{4\mathbf{y} \times 4\mathbf{x}}{6\mathbf{y} \times 8\mathbf{x}} = \frac{1}{3}$

8. Answer

(a) ΔOKL

(b) (i) $\frac{\text{Area of } \Delta OKP}{\text{Area of } \Delta OKL}$

$= \frac{\frac{1}{2} \times KP \times \text{height}}{\frac{1}{2} \times KL \times \text{height}}$

$= \frac{KP}{KL}$

$= \frac{1}{1+3}$

$= \frac{1}{4}$

(ii) $\frac{\text{Area of }\Delta OKL}{\text{Area of }\Delta OMN} = \left(\frac{OK}{OM}\right)^2$

$= \left(\frac{4}{8}\right)^2$

$= \frac{1}{4}$

9. Answer

(a) $|\overrightarrow{PQ}| = \sqrt{(-3)^2 + 4^2} = 5$ units

(b) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ}$

$= \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} -3 \\ 4 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 5 \end{pmatrix}$

$\therefore$ Coordinates of Q are $(-2, 5)$.

(c) $\overrightarrow{PQ} \parallel \overrightarrow{SR}$

$\therefore \overrightarrow{SR} = h\overrightarrow{PQ}$, where h is constant.

$\begin{pmatrix} k \\ -12 \end{pmatrix} = h\begin{pmatrix} -3 \\ 4 \end{pmatrix}$

Equating the corresponding elements, we have

$4h = -12$

$h = -3$

$\therefore k = -3h$

$= -3(-3)$

$= 9$

Worksheet 2 – Vectors

1. Answer

(a) **(i)** $\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \mathbf{a} + \mathbf{b}$

(ii) $\overrightarrow{CD} = \overrightarrow{CB} + \overrightarrow{BD}$

$= -2\overrightarrow{AB} + \overrightarrow{OB}$

$= -2\mathbf{b} + \mathbf{a} + \mathbf{b}$

$= \mathbf{a} - \mathbf{b}$

(iii) $\overrightarrow{OD} = \overrightarrow{OB} + \overrightarrow{BD}$

$= 2\overrightarrow{OB}$

$= 2\mathbf{a} + 2\mathbf{b}$

(iv) $\overrightarrow{OE} = \overrightarrow{OD} + \overrightarrow{DE}$

$= 2\mathbf{a} + 2\mathbf{b} + 2(\mathbf{a} - \mathbf{b})$

$= 4\mathbf{a}$

(b) OE is 4 times the length of OA.

O, A and E are collinear.

2. Answer

(a) **(i)** $\overrightarrow{QP} = \overrightarrow{QR} + \overrightarrow{RP}$

$= 3\mathbf{a} + 2\mathbf{b}$

(ii) $\overrightarrow{QT} = \overrightarrow{QR} + \overrightarrow{RT}$

$= 3\mathbf{a} + \mathbf{b}$

$\overrightarrow{QV} = \frac{1}{2}\overrightarrow{QT} = \frac{1}{2}(3\mathbf{a} + \mathbf{b})$

(iii) $\overrightarrow{PV} = \overrightarrow{PQ} + \overrightarrow{QV}$

$= -3\mathbf{a} - 2\mathbf{b} + \frac{3}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= -\frac{3}{2}\mathbf{a} - \frac{3}{2}\mathbf{b}$

$= -\frac{3}{2}(\mathbf{a} + \mathbf{b})$

(b) $\overrightarrow{PU} = \overrightarrow{PQ} + \overrightarrow{QU}$

$= -3\mathbf{a} - 2\mathbf{b} + \mathbf{a}$

$= -2(\mathbf{a} + \mathbf{b})$

$\overrightarrow{PU} = \frac{4}{3}\overrightarrow{PV}$

Since P, U and V are collinear and $PU > PV$

produced passes through U. (shown)

(c) (i) $\dfrac{\text{Area of } \Delta PQS}{\text{Area of } \Delta PVS} = \dfrac{\frac{1}{2}(QS)\text{ height}}{\frac{1}{2}(VS)\text{ height}} = \dfrac{4}{3}$

(ii) $\dfrac{\text{Area of } \Delta PVS}{\text{Area of } PQRS} = \dfrac{\text{Area of } \Delta PVS}{\text{Area of } \Delta PQS} \times \dfrac{1}{2}$

$= \dfrac{3}{4} \times \dfrac{1}{2}$

$= \dfrac{3}{8}$

3.

(a) (i) $\overrightarrow{OP} = 3\,\overrightarrow{OA} = 3\mathbf{a}$

(ii) $\overrightarrow{OM} = 2\,\overrightarrow{OB} = 2\mathbf{b}$

(iii) $\overrightarrow{AQ} = \overrightarrow{AO} + \overrightarrow{OQ} = -\mathbf{a} + 4\mathbf{b}$

(iv) $\overrightarrow{MP} = \overrightarrow{MO} + \overrightarrow{OP} = -2\mathbf{b} + 3\mathbf{a}$

(v) $\overrightarrow{MX} = \dfrac{1}{5}\overrightarrow{MP} = \dfrac{1}{5}(3\mathbf{a} - 2\mathbf{b})$

(b) $\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OM} + \overrightarrow{MX}$

$= -\mathbf{a} + 2\mathbf{b} + \dfrac{3}{5}\mathbf{a} - \dfrac{2}{5}\mathbf{b}$

$= -\dfrac{2}{5}\mathbf{a} + \dfrac{8}{5}\mathbf{b}$

$= \dfrac{2}{5}(-\mathbf{a} + 4\mathbf{b})$

$= \dfrac{2}{5}\overrightarrow{AQ}$

Since A, X and Q are collinear and $AX > AQ$.

AX when produced will pass through Q. (shown)

(c) $\dfrac{AX}{AQ} = \dfrac{2}{5}$

$\therefore \dfrac{AX}{XQ} = \dfrac{2}{3}$

$\therefore AX : XQ = 2 : 3$

(d) **(i)** $\dfrac{\text{Area of } \Delta PMQ}{\text{Area of } \Delta POQ} = \dfrac{\frac{1}{2}(MQ)\text{ height}}{\frac{1}{2}(OQ)\text{ height}} = \dfrac{1}{2}$

$\therefore$ Area of $\Delta PMQ = \dfrac{1}{2} \times 30 = 15 \text{ cm}^2$

(ii) $\dfrac{\text{Area of } \Delta PQX}{\text{Area of } \Delta PMQ} = \dfrac{\frac{1}{2}(PX)\text{ height}}{\frac{1}{2}(PM)\text{ height}} = \dfrac{4}{5}$

$\therefore$ Area of $\Delta PQX = \dfrac{4}{5} \times 15 = 12 \text{ cm}^2$

(iii) $\dfrac{\text{Area of } \Delta OAB}{\text{Area of } \Delta OPQ} = \dfrac{\frac{1}{2}(OA)(OB)\sin\angle AOB}{\frac{1}{2}(OP)(OQ)\sin\angle POQ}$

$= \dfrac{OA}{OP} \times \dfrac{OB}{OQ}$

$= \dfrac{1}{3} \times \dfrac{1}{4}$

$= \dfrac{1}{12}$

$\therefore$ Area of $\Delta OAB = \dfrac{1}{12} \times 30 = 2.5 \text{ cm}^2$

4. Answer

(a) Answer

(i) $\overrightarrow{AC} = \overrightarrow{AD} + \overrightarrow{DC}$

$= 4q + \frac{3}{5}\overrightarrow{AB}$

$= 4q + \frac{3}{5}(5p)$

$= 4q + 3p$

(ii) $\overrightarrow{BX} = \overrightarrow{BA} + \overrightarrow{AX}$

$= -5p + \frac{5}{8}\overrightarrow{AC}$

$= -5p + \frac{5}{8}(4q + 3p)$

$= -5p + \frac{20q}{8} + \frac{15p}{8}$

$= \frac{5}{2}q - \frac{25p}{8}$

$= \frac{5}{8}(4q - 5p)$

(iii) $\overrightarrow{XD} = \overrightarrow{XA} + \overrightarrow{AD}$

$= -\frac{5}{8}\overrightarrow{AC} + 4q$

$= -\frac{5}{8}(4q + 3p\ ) + 4q$

$= \frac{3}{2}q - \frac{15}{8}p$

$= \frac{3}{8}(4q - 5p)$

(b) $\overrightarrow{BX} = k\overrightarrow{XD}$

$\frac{5}{8}(4q - 5p) = \frac{3}{8}k(4q - 5p)$

$\frac{5}{8} = \frac{3}{8}k$

$k = \frac{5}{3}$

$\overrightarrow{BX} = \frac{5}{3}\overrightarrow{XD}$

Thus *B, X* and *D* lie on the same straight line.

5. Answer

(a) Answer

(i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\begin{pmatrix}-1\\5\end{pmatrix} + \begin{pmatrix}2\\-3\end{pmatrix} = \begin{pmatrix}3\\-8\end{pmatrix}$

(ii) $\sqrt{3^2 + 8^2} = 8.54$ units (3.s.f.)

(iii) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix}-1\\5\end{pmatrix} + 3\overrightarrow{AB} = \begin{pmatrix}-1\\5\end{pmatrix} + 3\begin{pmatrix}3\\-8\end{pmatrix} = \begin{pmatrix}8\\-19\end{pmatrix}$

$C = (8, -19)$

(b) Answer

(i) $\overrightarrow{OQ} = \overrightarrow{OP} + \overrightarrow{PQ} = \begin{pmatrix}4\\-2\end{pmatrix} + \begin{pmatrix}-8\\12\end{pmatrix} = \begin{pmatrix}-4\\10\end{pmatrix}$

$m = \frac{10-(-2)}{-4-(-4)} = \frac{12}{-8} = -\frac{3}{2}$

$y - 10 = -\frac{3}{2}(x + 4)$

$y = -\frac{3}{2}x + 4$

(ii) $3x + 2y = 11$

$y = -\frac{3}{2}x + 5\frac{1}{2}$

Since $m1 = m2$, they are parallel and will never intersect.

6. Answer

(a) $\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$

$= -\begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix}$

$= \begin{pmatrix}6\\2\end{pmatrix}$

(b) $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD}$

$= \begin{pmatrix}-2\\3\end{pmatrix} + \frac{1}{2}\overrightarrow{BC}$

$= \begin{pmatrix}-2\\3\end{pmatrix} + \frac{1}{2}\begin{pmatrix}6\\2\end{pmatrix}$

$= \begin{pmatrix}1\\4\end{pmatrix}$

$\left|\overrightarrow{AD}\right| = \sqrt{1^2 + 4^2} = 4.12$ units (3.s.f.)

(c) $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}4\\5\end{pmatrix} = \begin{pmatrix}5\\6\end{pmatrix}$

$\overrightarrow{PC} = \overrightarrow{PO} + \overrightarrow{OC} = -\begin{pmatrix}3\\9\end{pmatrix} + \begin{pmatrix}5\\6\end{pmatrix} = \begin{pmatrix}2\\-3\end{pmatrix}$

$\overrightarrow{AB} = -\overrightarrow{PC}$ → *AB* // *PC*

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = \begin{pmatrix}1\\1\end{pmatrix} + \begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix}$

$\overrightarrow{PB} = \overrightarrow{PO} + \overrightarrow{OB} = -\begin{pmatrix}3\\9\end{pmatrix} + \begin{pmatrix}-1\\4\end{pmatrix} = \begin{pmatrix}-4\\-5\end{pmatrix}$

$\overrightarrow{AC} = -\overrightarrow{PB}$ → *AC* // *PB*

Thus *ABPC* is a parallelogram as there are two pairs of parallel lines.

7. Answer

(a) Answer

(i) $\overrightarrow{WM} = \overrightarrow{WX} + \overrightarrow{XM}$

$= 6p + 3q + \frac{3}{5}\overrightarrow{XY}$

$= 6p + 3q + \frac{3}{5}(10p - 5q)$

$= 6p + 3q + 6p - 3q$

$= 12p$

(ii) $\overrightarrow{ZM} = \overrightarrow{ZW} + \overrightarrow{WM}$

$= -(10p - 5q) + 12p$

$= -10p + 5q + 12p$

$= 2p + 5q$

(b) Answer

(i) area of triangle WMX : area of $WXYZ$ = $\frac{1}{2}(XM)(h) : (WZ)(h)$

$= \frac{1}{2}(XM) : (XY)$

$= \frac{1}{2}\left(\frac{XM}{XY}\right) \quad \rightarrow \quad \frac{XM}{XY} = \frac{3}{5}$

$= \frac{1}{2}\left(\frac{3}{5}\right)$

$= \frac{3}{10}$

(ii) 3 units → 8

1 unit → $\frac{8}{3}$

10 units → $\frac{80}{3}$ units

(iii) $\triangle NWZ \sim \triangle NXM$

$\frac{XM}{WZ} = \frac{NX}{NW} = \frac{3}{5}$

Thus $WX : XN$ = 3 : 2

$\overrightarrow{WN} = \frac{5}{2}\overrightarrow{WX} = \frac{5}{2}(6p + 3q) = \frac{15}{2}(2p + q)$

8. Answer

(a) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -\begin{pmatrix}-3\\3\end{pmatrix} + \begin{pmatrix}7\\-13\end{pmatrix} = \begin{pmatrix}10\\-16\end{pmatrix}$

(b) x = 10 (adj)

y = 16 (opp)

$\tan a = \frac{16}{10}$

$a = \tan^{-1}\frac{8}{5} = 58.0°$ (1.d.p)

$Hint: Draw\ vector$

(c) $\overrightarrow{AB} = k\begin{pmatrix}2m\\-n\end{pmatrix}$ or $\overrightarrow{AB} = k\begin{pmatrix}-2m\\n\end{pmatrix}$ → take note negative can be given to either *m* or *n* as we do not know their value. *k* should be there as gradient would have been simplified.

9. Answer

(a)

$$\overrightarrow{OB} = \overrightarrow{OA} - \overrightarrow{BA}$$

$$\overrightarrow{OB} = \begin{pmatrix} -2 \\ 5 \end{pmatrix} - \begin{pmatrix} -4 \\ 8 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

$$B = (2,\ -3)$$

(b)

$$\left|\overrightarrow{AB}\right| = \sqrt{(-4)^2 + (8)^2} = 8.94$$

10. Answer

(a) Answer

(i) $\overrightarrow{CE} = \overrightarrow{CO} + \overrightarrow{OE}$

$= -2\overrightarrow{OB} + \frac{3}{2}\overrightarrow{OA}$

$= -2b + \frac{3}{2}(2a)$

$= 3a - 2b$

(ii) $\overrightarrow{CD} = \frac{3}{7}\overrightarrow{CE}$

$= \frac{3}{7}(3a - 2b)$

(iii) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA}$

$= -b + 2a$

$= 2a - b$

(iv) $\overrightarrow{OF} = \overrightarrow{OB} + \overrightarrow{BF}$

$= b + \frac{1}{2}\overrightarrow{BA}$

$= b + \frac{1}{2}(2a - b)$

$= b + a - \frac{1}{2}b$

$= a + \frac{1}{2}b$

$= \overrightarrow{OA} + \overrightarrow{AF}$

$= 2a - \frac{1}{2}\overrightarrow{BA}$

$= 2a - \frac{1}{2}(2a - b)$

$= 2a + a + \frac{1}{2}b$

or $= \frac{1}{2}(2a + b)$

(v) $\overrightarrow{FD} = \overrightarrow{FO} + \overrightarrow{OD}$

$\overrightarrow{OD} = \overrightarrow{OE} + \overrightarrow{ED}$

$$= \frac{3}{2}\overrightarrow{OA} - \frac{4}{7}\overrightarrow{CE}$$

$$= \frac{3}{2}(2a) - \frac{4}{7}(3a - 2b)$$

$$= 3a - \frac{12}{7}a + \frac{8}{7}b$$

$$= \frac{9}{7}a + \frac{8}{7}b$$

$$\overrightarrow{FD} = -a - \frac{1}{2}b + \frac{9}{7}a + \frac{8}{7}b$$

$$= \frac{2}{7}a + \frac{9}{14}b$$

$$= \frac{1}{14}(4a + 9b)$$

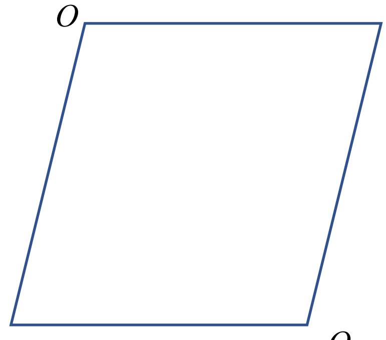
(b) Find

(i) $$\frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OBE} = \frac{\frac{1}{2}(OA)(h)}{\frac{1}{2}(OE)(h)} = \frac{OA}{OE} = \frac{2}{3}$$

(ii) $$\frac{\text{Area of } \triangle OBA}{\text{Area of } \triangle OCE} = \frac{\frac{1}{2}(OA)(OB)\sin BOA}{\frac{1}{2}(OE)(OC)\sin BOA} = \frac{(OA)(OB)}{(OE)(OC)} = \frac{OA}{OE} \times \frac{OB}{OC} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

Worksheet 3 – Vectors

1. Answer



(a) $\overrightarrow{OP} = \overrightarrow{RQ} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$\overrightarrow{PQ} = \overrightarrow{OR} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

$\overrightarrow{RP} = \overrightarrow{RQ} + \overrightarrow{QP} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \end{pmatrix} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(b) $\overrightarrow{RP} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

$\overrightarrow{PJ} = m\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} m \\ -2m \end{pmatrix}$

$\overrightarrow{OJ} = \overrightarrow{OP} + \overrightarrow{PJ} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} m \\ -2m \end{pmatrix} = \begin{pmatrix} 3+m \\ 2-2m \end{pmatrix}$ (shown)

2. Answer

(a) Answer

(i) $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC} = c - a$

(ii) $\overrightarrow{BF} = \overrightarrow{BC} + \overrightarrow{CF} = \overrightarrow{AO} + \overrightarrow{OC} = c - a$

(iii) $\overrightarrow{OD} = \overrightarrow{OC} + \overrightarrow{CD} = c + \frac{1}{2}\overrightarrow{OA} = c + \frac{1}{2}a = \frac{1}{2}(2c + a)$

(iv) $\overrightarrow{OE} = \overrightarrow{OF} + \overrightarrow{FE} = 2\overrightarrow{OC} - \frac{2}{3}\overrightarrow{BF}$

$= 2c - \frac{2}{3}(c - a)$

$= 2c - \frac{2}{3}c + \frac{2}{3}a$

$= \frac{2}{3}a + \frac{4}{3}c$

$= \frac{2}{3}(a + 2c)$

(b) $\overrightarrow{OE} = k\overrightarrow{OD}$

$\frac{2}{3}(a + 2c) = \frac{1}{2}k(2c + a)$

$\frac{1}{2}k = \frac{2}{3}$

$k = \frac{4}{3}$

$\overrightarrow{OE} = \frac{4}{3}\overrightarrow{OD}$

O, E and *D* lies on the same straight line (collinear).

(c) Answer

(i) $\frac{Area\ \triangle ODF}{Area\ \triangle OEF} = \frac{\frac{1}{2}(OD)(h)}{\frac{1}{2}(OE)(h)} = \frac{OD}{OE} = \frac{3}{4}$

(ii) $\frac{Area\ \triangle OCD}{Area\ OABC} = \frac{\frac{1}{2}(CD)(h)}{(CB)(h)} = \frac{1}{2}\left(\frac{CD}{CB}\right) = \frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$

(iii) $BF//AC$ → based on vector

$\triangle\ FBC \equiv \triangle\ ACB$ ($\angle FBC = \angle ACB, BC = CB$ and $BF = CA$)

$Area\ \triangle\ FBC = Area\ \triangle\ ACB = \frac{1}{2}Area\ OABC$

$\frac{Area\ \triangle OCD}{Area\ OABF} = \frac{Area\ \triangle OCD}{Area\ OABC + \triangle FBC} = \frac{1}{4+2} = \frac{1}{6}$

3. Answer

(a) Answer

(i) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -2a + b$

(ii) $\overrightarrow{OC} = \overrightarrow{OB} + \overrightarrow{BC} = b + \frac{3}{2}(2a) = 3a + b$

(iii) $\overrightarrow{OX} = \frac{2}{5}\overrightarrow{OC} = \frac{2}{5}(3a + b)$

(iv) $\overrightarrow{AX} = \overrightarrow{AO} + \overrightarrow{OX} = -2a + \frac{2}{5}(3a + b)$

$= -2a + \frac{6}{5}a + \frac{2}{5}b$

$= \frac{2}{5}b - \frac{4}{5}a$

$= \frac{2}{5}(b - 2a)$

(b) $\overrightarrow{AX} = \frac{2}{5}\overrightarrow{AB}$

Thus, they are collinear.

(c)

(i) $\overrightarrow{BY} = \frac{1}{2}\overrightarrow{BC} = \frac{3}{4}(2a) = \frac{3}{2}a$

$\overrightarrow{OY} = \overrightarrow{OB} + \overrightarrow{BY} = b + \frac{3}{2}a$

$\overrightarrow{AY} = \overrightarrow{AO} + \overrightarrow{OY} = -2a + b + \frac{3}{2}a = b - \frac{1}{2}a$

$\overrightarrow{AZ} = h\overrightarrow{AY} = h\left(b - \frac{1}{2}a\right) = \frac{1}{2}h(2b - a)$

(ii) $\overrightarrow{OZ} = k\overrightarrow{OB} = kb$

(iii) $\overrightarrow{AZ} = \overrightarrow{AO} + \overrightarrow{OZ} = -2a + kb$

$\frac{1}{2}h(2b - a) = -2a + kb$

$bh - \frac{1}{2}ah = -2a + kb$

$-\frac{1}{2}ah = -2a$

$h = 4$

$bh = kb$

$h = k = 4$

(d) Answer

(i) $\frac{Area\ of\ \triangle OAX}{Area\ of\ \triangle OAC} = \frac{\frac{1}{2}(OX)(h)}{\frac{1}{2}(OC)(h)} = \frac{OX}{OC} = \frac{2}{5}$

(ii) $\frac{Area\ of\ \triangle OBX}{Area\ of\ \triangle BXC} = \frac{\frac{1}{2}(OX)(h)}{\frac{1}{2}(OC)(h)} = \frac{2}{5} = \frac{6}{15}$

$\frac{Area\ of\ \triangle BXC}{Area\ of\ \triangle CAX} = \frac{\frac{1}{2}(BX)(h)}{\frac{1}{2}(AX)(h)} = \frac{3}{2} = \frac{15}{10}$ → $AX : AB = 2 : 5$

$\frac{Area\ of\ \triangle OBX}{Area\ of\ \triangle ABC} = \frac{6}{25}$

4. Answer

$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = \begin{pmatrix}-3\\4\end{pmatrix} + \begin{pmatrix}12\\5\end{pmatrix} = \begin{pmatrix}9\\9\end{pmatrix}$

$|\overrightarrow{PR}| = \sqrt{9^2 + 9^2} = \sqrt{162} = 12.7$ units

PQ is not perpendicular to PS!

Length PR ≠ length PQ + length QR!

5. Answer

(a) Answer

(i) $\overrightarrow{BA} = \overrightarrow{BO} + \overrightarrow{OA} = -6b + a = a - 6b$

(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} - \frac{2}{5}\overrightarrow{BA} = a - \frac{2}{5}(a - 6b) = \frac{3}{5}(a + 4b)$

(iii) $\overrightarrow{CB} = \overrightarrow{CO} + \overrightarrow{OB} = -3a + 6b = 3(2b - a)$

(iv) $\overrightarrow{AE} = \overrightarrow{AC} + \overrightarrow{CE} = 2a + \frac{4}{9}\overrightarrow{CB} = 2a + \frac{4}{9}(6b - 3a) = \frac{2}{3}(a + 4b)$

(b) $\overrightarrow{OD} = \frac{3}{5}(a + 4b)$

$\overrightarrow{AE} = \frac{2}{3}(a + 4b)$

$\overrightarrow{OD} = \frac{9}{10}\overrightarrow{AE}$

Thus OD and AE are parallel lines

(c) $\frac{Area\ triangle\ CAE}{Area\ triangle\ AOD} = \frac{\frac{1}{2}(CA)(AE)\sin CAE}{\frac{1}{2}(OA)(OD)\sin AOD} = \frac{AE}{OD} \times \frac{CA}{OA} = \frac{10}{9} \times \frac{2}{1} = \frac{20}{9}$

(d) $\frac{Area\ triangle\ CAE}{Area\ triangle\ AOB} = \frac{Area\ triangle\ CAE}{Area\ triangle\ AOD} \times \frac{Area\ triangle\ AOD}{Area\ triangle\ AOB}$

$$= \frac{20}{9} \times \frac{\frac{1}{2}(h)(AD)}{\frac{1}{2}(h)(AB)}$$

$$= \frac{20}{9} \times \frac{2}{5}$$

$$= \frac{8}{9}$$

6. Answer

(a) $\overrightarrow{BC} = \overrightarrow{ED}$, so BC and ED are parallel.

$\angle BCM = \angle DEM$ (alternate angles, $BC \parallel ED$)

$BC = DE$ (since $\overrightarrow{BC} = \overrightarrow{ED}$)

$\angle CBM = \angle EDM$ (alternate angles, $BC \parallel ED$)

Therefore, triangles BMC and DME are congruent (ASA).

Alternative method:

$\overrightarrow{BC} = \overrightarrow{ED}$, so BC and ED are parallel.

$\angle BMC = \angle DME$ (vertically opposite angles)

$\angle BCM = \angle DEM$ (alternate angles, $BC \parallel ED$)

$BC = DE$ (since $\overrightarrow{BC} = \overrightarrow{ED}$)

Therefore, triangles BMC and DME are congruent (ASA).

(b) (i) From triangle ABC,

$\overrightarrow{AC} = \mathbf{p} + 3\mathbf{q}$

(b) (ii) From triangle ABD,

$\overrightarrow{BD} = 5\mathbf{q} - \mathbf{p}$

(b) (iii) Since triangles BMC and DME are congruent,

$BM = DM$

$\overrightarrow{BM} = \frac{1}{2}\overrightarrow{BD}$

$$\begin{aligned}\overrightarrow{AM} &= \overrightarrow{AB} + \overrightarrow{BM}\\ &= \mathbf{p} + \frac{1}{2}\overrightarrow{BD}\\ &= \mathbf{p} + \frac{1}{2}(5\mathbf{q} - \mathbf{p})\\ &= \frac{1}{2}\mathbf{p} + \frac{5}{2}\mathbf{q}\end{aligned}$$

(c) (i)

$$\begin{aligned}\overrightarrow{CD} &= \overrightarrow{AD} - \overrightarrow{AC}\\ &= 5\mathbf{q} - (\mathbf{p} + 3\mathbf{q})\\ &= 2\mathbf{q} - \mathbf{p}\end{aligned}$$

$$\begin{aligned}\overrightarrow{CF} &= \frac{2}{7}\overrightarrow{CD}\\ &= \frac{2}{7}(2\mathbf{q} - \mathbf{p})\end{aligned}$$

$$\begin{aligned}\overrightarrow{AF} &= \overrightarrow{AC} + \overrightarrow{CF}\\ &= \mathbf{p} + 3\mathbf{q} + \frac{2}{7}(2\mathbf{q} - \mathbf{p})\\ &= \frac{5}{7}\mathbf{p} + \frac{25}{7}\mathbf{q}\\ &= \frac{10}{7}\left(\frac{1}{2}\mathbf{p} + \frac{5}{2}\mathbf{q}\right)\\ &= \frac{10}{7}\overrightarrow{AM}\end{aligned}$$

This shows that $\overrightarrow{AF}$ and $\overrightarrow{AM}$ are parallel.
Also, $\overrightarrow{AF}$ and $\overrightarrow{AM}$ have the common point A.
Therefore, the points A, M and F lie on a straight line.

(c) (ii) area of triangle AME : area of triangle $EMD = 2:3$

area of triangle EMD : area of triangle $CMD = 1:1$

area of triangle CMD : area of triangle $FMD = 7:5$

area of triangle AME : area of triangle $FMD = 14:15$

7.

(a) Find $\overrightarrow{BC}$.

$$\overrightarrow{BC} = \overrightarrow{BA} + \overrightarrow{AC}$$

$$= -\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 8 \end{pmatrix} \quad \textbf{M1}$$

$$= \begin{pmatrix} -6 \\ 9 \end{pmatrix} \quad \textbf{A1}$$

Answer .... $\begin{pmatrix} -6 \\ 9 \end{pmatrix}$ [2]

(b) Hence, or otherwise, show that $\angle BAC = 108.4°$.

Answer [2]

"Hence" method

$|\overrightarrow{BC}| = \sqrt{117}$

$|\overrightarrow{AB}| = \sqrt{26}$

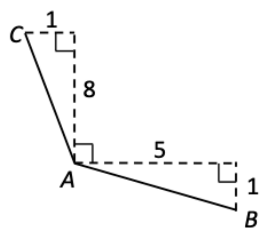
$|\overrightarrow{AC}| = \sqrt{65}$

$117 = 26 + 65 - (\sqrt{26})(\sqrt{65})cos\angle BAC$ **M1**

$cos\angle BAC = -0.3162277$

$\angle BAC = 108.4°$ **A1**

"Otherwise" method



$\angle BAC = 90 + tan^{-1}\frac{1}{5} + tan^{-1}\frac{1}{8}$ **M1**

$= 108.4°$ **A1**

(c) Hence, calculate the area of ΔABC.

Area $= \frac{1}{2}(\sqrt{26})(\sqrt{65})\sin 108.4$ **M1** length of AB and AC

M1 formula

$= 19.5$ units^2 **A1**

If students calculate the length in part (b), award them the M1 too.

Answer ....19.5.......... units^2

[3]

8. Answer

	(a)(i)	$\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$ $= -9\boldsymbol{q} + \frac{1}{3}\overrightarrow{OA}$ $= -9\boldsymbol{q} + 4\boldsymbol{p}$ [B1]	
	(a)(ii)	$\overrightarrow{DA} = \overrightarrow{DO} + \overrightarrow{OA}$ $= \frac{1}{3}\overrightarrow{BO} + 12\boldsymbol{p}$ [M1] $= -3\boldsymbol{q} + 12\boldsymbol{p}$ [A1]	
	(b)	$\frac{\text{area of triangle } ODE}{\text{area of triangle } ODA} = \frac{1}{4}$ $\frac{0.5 \text{ x ED x h}}{0.5 \text{ x AD x h}} = \frac{1}{4}$ [Shared same perpendicular height][M1] $\frac{\text{ED}}{\text{AD}} = \frac{1}{4}$ $\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$ $= 12\boldsymbol{p} + \frac{3}{4}\overrightarrow{AD}$ $=12\boldsymbol{p} + \frac{3}{4}(3\boldsymbol{q} - 12\boldsymbol{p})$ [M1] $= 3\boldsymbol{p} + \frac{9}{4}\boldsymbol{q}$ [A1]	**[8]**

	(c)	$\frac{\text{area of triangle } ODE}{\text{area of triangle DEB}} = \frac{OD}{DB} = \frac{1}{2}$ $\frac{\text{area of triangle AEC}}{\text{area of triangle OEC}} = \frac{AC}{\text{OC}} = \frac{2}{1}$ M1 Triangle ODE: Triangle DEB: Triangle AEC : Triangle OEC = 1 : 2 : 2 : 1 $\frac{\text{area of triangle } BDE}{\text{area of quadrailateral } EDOC}$ $= \frac{2}{2}$ = 1 [A1]	