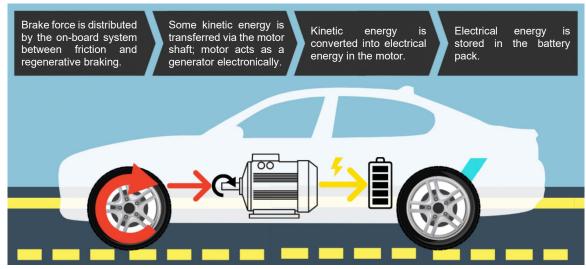


H2 Topic 17 – Electromagnetic Induction



In traditional vehicles, friction is applied at the wheels to slow down the vehicle. Essentially, all of the kinetic energy of a vehicle is converted into heat each time the vehicle comes to a stop. With regenerative braking, about 60% to 70% of kinetic energy can be turned back into useful work in accelerating the vehicle subsequently.

Content

Magnetic flux

Laws of electromagnetic induction

Learning Objectives:

Candidates should be able to:

- (a) define magnetic flux as the product of an area and the component of the magnetic flux density perpendicular to that area
- (b) recall and solve problems using $\Phi = BA$
- (c) define magnetic flux linkage
- (d) infer from appropriate experiments on electromagnetic induction:
 - i. that a changing magnetic flux can induce an e.m.f.
 - ii. that the direction of the induced e.m.f. opposes the change producing it
 - iii. the factors affecting the magnitude of the induced e.m.f.
- (e) recall and solve problems using Faraday's law of electromagnetic induction and Lenz's law
- (f) explain simple applications of electromagnetic induction

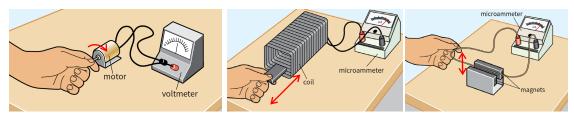


17.0 Introduction

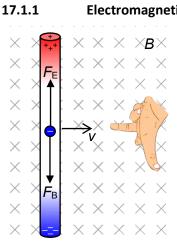
Earlier in electromagnetism, we learnt that (i) an electric current can produce a magnetic field and (ii) a magnetic field can exert a force on moving charges or current-carrying conductors. As nature is often symmetric, it was later found that magnetic fields can in turn generate electric current.

17.1 E.m.f. from Motion

In electromagnetism, we observe electrical energy converted into kinetic energy using an electric motor. In electromagnetism induction, we can observe, explain, and calculate the electrical energy that is converted from mechanical motion.



In each of the above, an induced e.m.f. or induced current can be observed via the voltmeter or microammeter.



Electromagnetic Induction of straight conductor

To explain why **relative motion** of a conductor and a magnetic field induces an e.mf., consider a thin straight conductor of length L moving at a constant speed v in a region of uniform magnetic flux density B directed normally into the plane of paper as shown. By Fleming's left-hand rule, electrons initially experience a downward magnetic force.

Negative charges accumulate at the lower end, which results in a corresponding region of positive charge at the upper end.

- *Inside* the conductor, a potential difference is induced and results in an electric field down the length of the conductor.
 - *Outside* the conductor, it can be regarded as an induced e.m.f. that can drive current around an external circuit.

Steady state is attained when the magnetic force and electric force are balanced. There will be no further charge separation and the induced e.m.f. ($\varepsilon_{induced}$) will remain constant:

$$F_{\rm B} = F_{\rm E}$$

$$B \not q v \sin q = \not q E$$

$$Bv = \frac{\Delta V}{d} = \frac{\varepsilon_{\rm induced}}{L}$$

$$\varepsilon_{\rm induced} = BLv$$

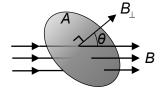
Note: The equation is true in general for a straight conductor of length L "cutting" magnetic flux lines but it is not one that we should recall and use. For most questions, we will need to start from basic principles and show how an equation is formed, before applying it.

From the equation, magnitude of $\mathcal{E}_{induced}$ depends on the magnitude of the external magnetic flux density, the length of conductor that is inside the magnetic field, and the speed at which the wire is moving across the field.



magnetic flux is the

product of an area and component of magnetic flux density perpendicular to that area



 $\boldsymbol{\varPhi}=\boldsymbol{B}_{\!\!\perp}\boldsymbol{A}=\boldsymbol{B}\boldsymbol{A}\cos\boldsymbol{\theta}$

where Φ represents magnetic flux,

- *B* represents magnetic flux density,
 - *A* represents area,
 - θ represents the angle between *B* and the normal of the area

We can visualise magnetic flux ϕ as magnetic field lines piercing an identified area at 90°:

The unit for magnetic flux ϕ is the Weber (Wb).

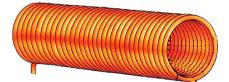
One Weber is the magnetic flux through an area of one squared metre when the magnetic flux density normal to the area is one tesla.

17.3 Magnetic Flux Linkage

Consider a solenoid of *N* number of turns.

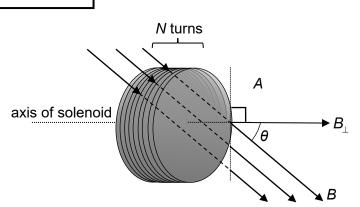
Magnetic flux linkage through a loop is product of magnetic flux through the loop and number of turns of wire in the loop.

Magnetic flux is the product of an area and component of magnetic flux density perpendicular to that area.



We can visualise magnetic flux linkage $(N\Phi)$ as *N* layers of *magnetic flux* stacked on each other.

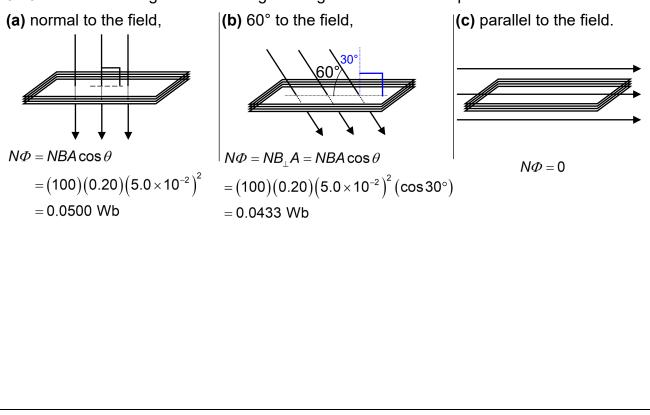
 $N\Phi = N(B_A) = NBA\cos\theta$



The *area* for *magnetic flux* need not be a filled area e.g. flat circular sheet of metal; it can be an area of free space that is outlined by a wire loop.

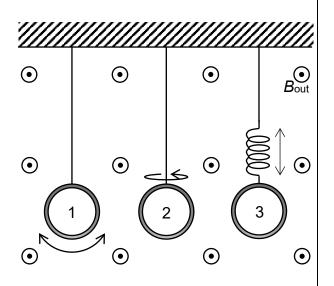


A 100-turn square coil of sides 5.0 cm is placed in a uniform magnetic field of flux density 0.20 T. Find the magnetic flux linkage through the coil when its plane is



Example 2

Three loops of wire shown below are placed in a region of uniform magnetic field. Loop 1 swings left and right like a pendulum. Loop 2 rotates about a vertical axis. Loop 3 oscillates vertically on the end of a spring. State which loops experiences a changing magnetic flux.



Solution:

Loop 2 experiences a changing magnetic flux; orientation relative to magnetic field changes with time as it rotates.



17.4 Laws of Electromagnetic Induction

Faraday's Law allows us to find the **magnitude** of the induced e.m.f. in a conductor. Lenz's Law gives the **direction** of the induced e.m.f. (or direction of induced current if there is a closed circuit).

Faraday's Law states that the

induced e.m.f. is directly proportional to the rate of change of magnetic flux linkage

Lenz's Law states that the

direction of induced e.m.f. produces effects to oppose the change causing it

 $E_{
m induced} \propto {{
m d}(N \Phi) \over {
m d}t}$

Constant of proportionality is 1, hence

$$E_{\text{induced}} = \frac{d}{dt} (NBA\cos\theta)$$

A negative sign is introduced as a result of Lenz's Law. As a result of both laws,

$$E_{\text{induced}} = -\frac{d(N\Phi)}{dt}$$
$$= -\frac{d}{dt}(NBA\cos\theta)$$

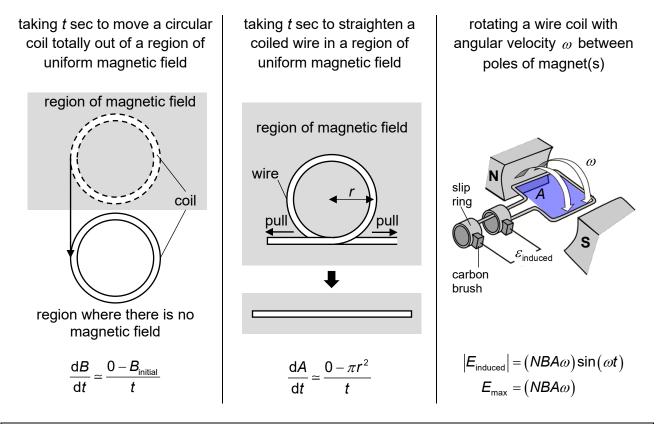
17.4.1 Laws of Electromagnetic Induction Calculations and Graphs

Consider a wire of length L moving with velocity v normal to a uniform magnetic field of flux density B. Determine the induced e.m.f.:

$$\begin{split} E_{\text{induced}} &= \frac{d}{dt} \{ N \Phi \} = N \frac{d}{dt} \{ \Phi \} \\ &= (1) \frac{d}{dt} \{ B A \} \qquad (\text{where A is the area "swept" by the wire}) \\ &= B \frac{d}{dt} \{ L (\text{width}) \} \\ &= B L v \end{split}$$

Note: This is the same equation, as it should be, when we considered the "microscopic" picture of an electron inside the wire being subject to a magnetic force and an electric force. The following scenarios provide a non-exhaustive list of how *B*, *A*, and θ can be changed across time to induce an e.m.f. in a conductor:





A square coil of side 5.0 cm lies initially at rest perpendicular to a magnetic field of flux density 4.0 T. The coil consists of 200 turns of wire.

(a) What is the magnetic flux linkage through the coil?

$$N\Phi = NBA\cos\theta$$

$$= 200(4.0)(0.050^2)\cos 0^\circ = 2.0$$
 Wb

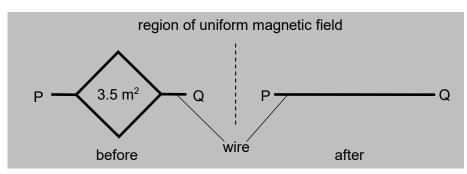
(b) The coil is rotated through an angle of 180° in 0.40 seconds. Calculate the magnitude of the average e.m.f. induced in the coil while it is being rotated.

$$\begin{aligned} \left| \boldsymbol{E}_{\text{induced}} \right| &= \frac{dN\Phi}{dt} \simeq N \left| \frac{\Delta(\Phi)}{\Delta t} \right| \\ &= N \left| \frac{\Delta(B_{\perp}A)}{\Delta t} \right| = NBA \left| \frac{\Delta(\cos\theta)}{\Delta t} \right| \\ &= (200)(4.0)(0.050^2) \frac{\left| \cos 180^\circ - \cos 0^\circ \right|}{0.40} = 10 \text{ V} \end{aligned}$$

Note: if (b) includes "constant angular velocity", we can find maximum instantaneous e.m.f.



A single-layer wire coil encloses an area of 3.5 m². it is placed perpendicular to a uniform magnetic field of flux density 2.0 T as shown. Points P and Q are pulled apart until the coil becomes a straight line over a duration of 7.0 s.



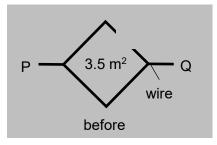
(a) Find the average e.m.f. induced in the coil.

$$\begin{aligned} \left| \boldsymbol{E}_{\text{induced}} \right| &= \left| \frac{\Delta(N\Phi)}{\Delta t} \right| \\ &= \left| 0 - \left(\frac{N\Phi}{t} \right)_{\text{initial}} \right| \\ &= \left| \left(\frac{NBA\cos\theta}{t} \right)_{\text{initial}} \right| \\ &= \left| \frac{(1)(2.0)(3.5)(\cos 0^{\circ})}{7.0} \right| = 1.0 \text{ V} \end{aligned}$$

(b) Find the average induced current in the coil if the resistance of the coil is 5.0 Ω .

$$I = \frac{E_{induced}}{R}$$
$$= \frac{1.0}{5.0}$$
$$= 0.20 \text{ A}$$

(c) Explain if there will still be induced e.m.f. and current if there is a break in the coil.



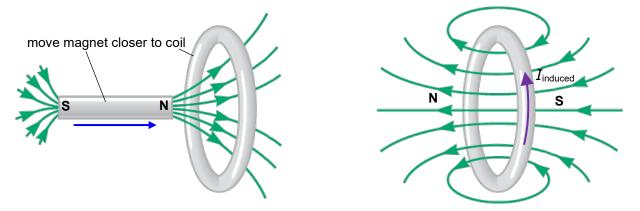
induced e.m.f. still present as there is a rate of change of <u>magnetic flux linkage</u> no induced current because circuit is open

Lenz's Law and the Principle of Conservation of Energy

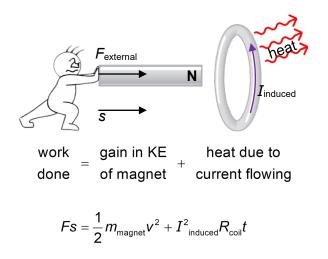


Lenz's Law gives the direction of induced e.m.f., thereby also giving the direction of induced current if there is a closed circuit around the induced e.m.f.. It is a consequence of principle of conservation energy.

Consider a permanent magnet initially at rest. We apply an external force on the magnet to move it nearer a flat circular coil. By Lenz's Law, since the coil is a closed circuit, the direction of induced current produces an effect to opposing the incoming magnet. The induced current should produce a magnetic field with a North pole on the left of the coil.



The flow of the induced current around the coil results in electrical heating of the coil. Demonstrating the principle of conservation of energy:



If the coil is absent, all of the external work done is converted into kinetic energy of the magnet.

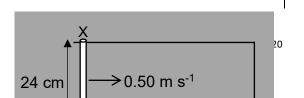
With coil, an opposing magnetic force acts on the magnet. *Some* of the external work done is converted into kinetic energy of the magnet, and the remaining is converted into heat dissipated by the coil when induced current flows ($I^2R_{coil}t$).

The principle of conservation of energy is valid since the total amount of energy is constant, and the various forms of energy are accounted for.

Conversely, Principle of Conservation of

Energy will be violated if we go against Lenz's Law:

- suppose a South pole is induced on the left side of the coil
- incoming magnet is attracted and accelerates towards the coil, gaining kinetic energy
- there are increases in both the kinetic energy of the magnet and the thermal output of the coil with much lesser input of energy; the Principle of Conservation of Energy is violated.





A straight metal rod XY of length 24 cm and resistance 20 Ω , is moved at a constant velocity of 0.50 m s⁻¹ along a U-shaped metal frame in a region of uniform magnetic field of flux density *B* = 0.35 T directed out of the plane of paper as shown. The metal frame has negligible resistance.

(a) Determine the current in the rod.

$$\begin{aligned} \left| E_{\text{induced}} \right| &= \frac{d(N\Phi)}{dt} = \frac{d}{dt} \{ NBA\cos\theta \} \qquad I = \frac{E_{\text{induced}}}{R} \\ &= BLv \\ &= (0.35)(0.24)(0.50) \\ &= 0.042 \text{ V} \end{aligned}$$

(b) State and explain the direction of the current within the rod.

Moving rod XY cuts magnetic flux.

by Faraday's Law, e.m.f. induced in XY

by Lenz's Law, induced current flows to give a magnetic field pointing out of plane of paper so as to oppose decrease in magnetic flux linkage through area enclosed by right hand grip rule, induced current flows anti-clockwise, hence from X to Y

(c) Determine the power dissipated in the rod.

$$P = I^2 R = (2.1 \times 10^{-3})^2 (20) = 8.82 \times 10^{-5} W$$

(d) Find the magnitude of external force needed to keep rod moving at constant velocity.

$$\begin{split} \left| F_{\text{ext}} \right| &= \left| F_{\text{B, on rod}} \right| = \textit{NBIL} \sin \theta = \textit{BIL} \\ &= (0.35) (2.1 \times 10^{-3}) (0.24) = 1.76 \times 10^{-4} \text{ N} \end{split}$$

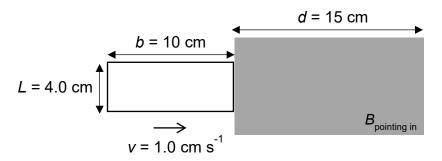
(e) Hence find the rate of work done by the external force.

$$P_{\text{ext}} = F_{\text{ext}} v = (1.76 \times 10^{-4})(0.50) = 8.82 \times 10^{-5} \text{ W}$$

Note: (c) and (e) are the same values because of principle of conservation of energy. Work done by the external force on the rod is converted into the electrical energy, which is then dissipated as heat in the rod through joule heating.

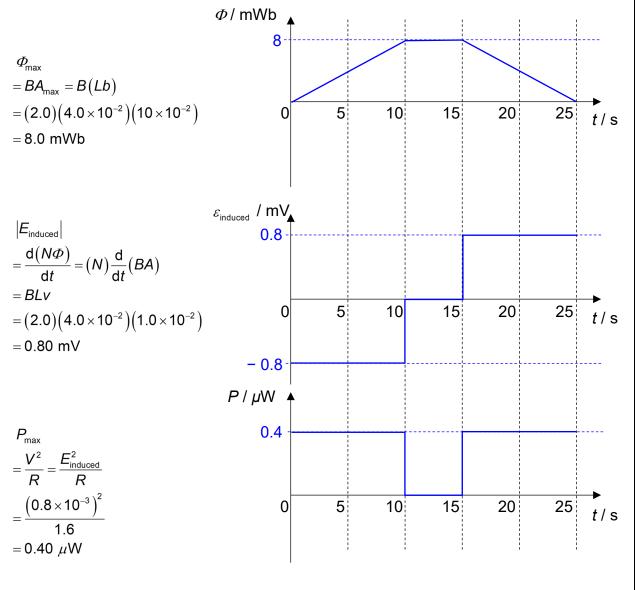


A rectangular metallic loop of resistance $R = 1.6 \Omega$, width L = 4.0 cm and length b = 10 cm is made to move at constant speed $v = 1.0 \text{ cm} \text{ s}^{-1}$ through a region of uniform magnetic field of flux density B = 2.0 T. The magnetic field spans a length of d = 15 cm. Sketch



- (a) the variation of magnetic flux Φ of the loop with time, where t = 0 corresponds to the moment that the loop is just about to enter the magnetic field as shown,
- (b) the variation of induced e.m.f. $E_{induced}$ with time,
- (c) the variation of the rate of heating in the loop with time t.

Solution:

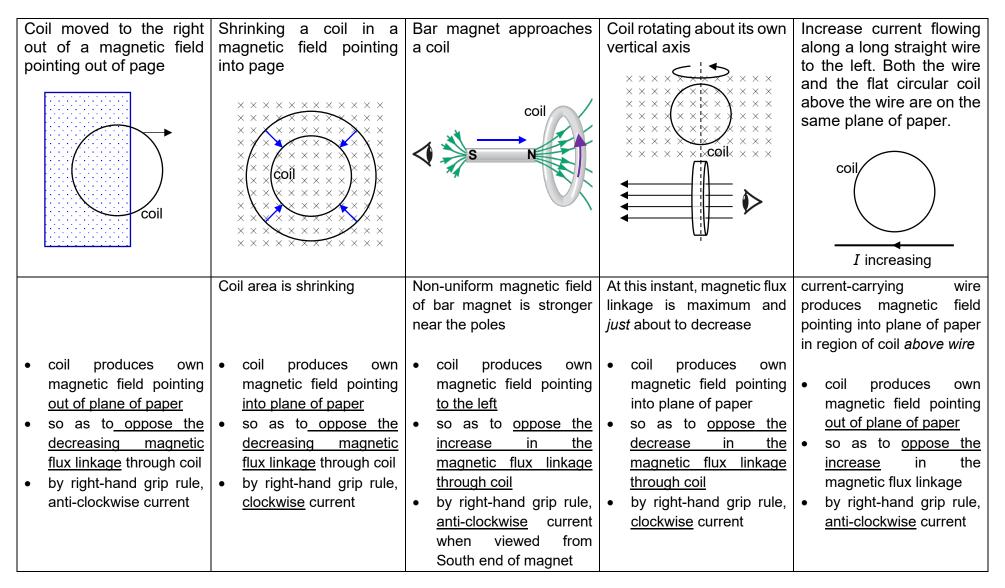




17.4.2 Laws of Electromagnetic Induction Explanations

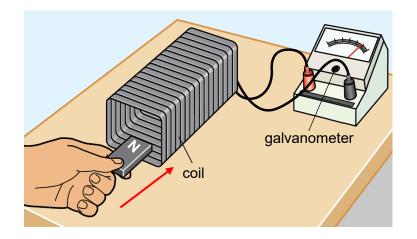
Example 7

Using Lenz's Law, state the direction of current induced in the circular coil:





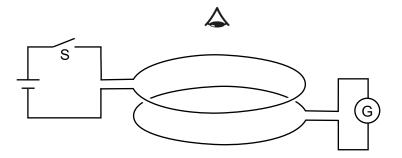
Using the laws of electromagnetic induction, explain the following observations:



experimental observation	explanation
deflection in galvanometer (sensitive ammeter) when bar magnet is moved towards coil	 coils cut magnetic flux of approaching magnet OR magnetic flux linkage through wire coils increases by Faraday's Law, there is an induced e.m.f. in the coil coil and galvanometer form a closed circuit, induced current flows
faster movement of bar magnet causes larger deflection in galvanometer	 rate of flux cutting by the coils increases OR magnetic flux linkage through wire coils increases at a higher rate by Faraday's Law, there is a larger induced e.m.f. in the coil resulting in a larger induced current so larger deflection
magnet is moved away from coil, resulting deflection in galvanometer is in opposite direction	 magnetic flux linkage through wire coils decreases as magnet moves away from coil induced current flows in direction to oppose the decrease in magnetic flux linkage current direction is opposite to when magnetic flux linkage was increasing deflection in galvanometer occurs in opposite direction



Using the laws of electromagnetic induction, explain the following observations:



experimental observation	explanation
when switch S is closed, galvanometer register a momentary deflection in a particular direction	 current flow increases from zero to steady state clockwise in upper coil produces an increasing downward magnetic field magnetic flux linkage through lower coil increases by Faraday's law, an e.m.f. in the lower coil is induced lower coil and galvanometer forms a closed circuit, induced current flows momentarily by Lenz's Law, lower coil produces own upward magnetic field so as to oppose the increase in magnetic flux linkage by right-hand grip rule, induced current in lower coil flows in an anticlockwise direction, as viewed from above induced current stops flowing when current reaches steady state in upper coil
when switch S is open, galvanometer register a momentary deflection in opposite direction	 clockwise current flow decreases from steady value to zero upper coil magnetic flux density decreases magnetic flux linkage through lower coil decreases by Faraday's law, an e.m.f. is induced in the lower coil by Lenz's Law, lower coil produces own magnetic field so as to oppose the decrease in magnetic flux linkage by right-hand grip rule, induced current in lower coil flows in clockwise direction current direction is opposite to that when magnetic flux linkage was increasing deflection in galvanometer occurs in opposite direction

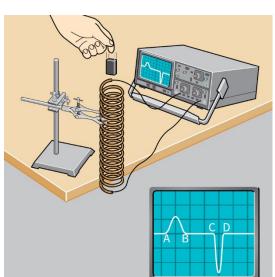


Be reminded that an e.m.f. is induced by changing magnetic flux linkage even if the circuit is open, although there will be no induced current flowing.

When a bar magnet is dropped vertically downwards through a long solenoid, the variation of the induced e.m.f. with time as the magnet accelerates downwards can be traced on an oscilloscope.

The laws of electromagnetic induction can explain why

- (a) there is an e.m.f. between A and B,
- (b) there isn't an e.m.f. between B and C,
- (c) the e.m.f. between C and D is
 - (i) opposite in polarity to A and B,
 - (ii) shorter in duration,
 - (iii) of larger peak value.



(a) E.m.f. between A and B

as magnet approaches, magnetic flux linkage through wire coils increases by Faraday's law, e.m.f. is induced in coil

(b) No e.m.f. between B and C

as magnet falls through within length of solenoid, no net change in flux linkage through solenoid

so no e.mf. induced

(c)(i) Opposite polarity of e.m.f. peaks

as magnet enters solenoid, by Lenz's Law, induced current in coil flows in direction to produce a magnetic field that opposes the falling magnet so as to oppose the increase in magnetic flux linkage through coils

as magnet leaves solenoid, induced current in coil flows in direction to produce a magnetic field that opposes the *exiting* magnet so as to oppose the decrease in magnetic flux linkage through coils

induced current flows in opposite direction compared to when magnet was entering solenoid

(ii)/(iii) Narrower but larger e.m.f. peak between C and D

change in magnetic flux linkage through solenoid is same for both entry and exit of magnet evident from the same area under graph for both peaks

magnet accelerated downwards during the fall has higher exit velocity the same change in magnetic flux linkage takes place within a shorter time by Faraday's Law, the higher rate of change of magnetic flux linkage results in larger peak

17.4.2.1 Eddy Currents and Resistive Forces/Damping

Consider a metal disc at the end of a rod that can swing freely. Without the magnets, the disc oscillates for a long time due to light damping from air resistance. When the magnets are present, the oscillations are damped and the amplitude of oscillations decay quickly.

Explain why an e.m.f. is induced in the disc

As the disc enters the magnetic field, the disc cuts magnetic flux. By Faraday's law, an e.m.f. is induced in the disc.

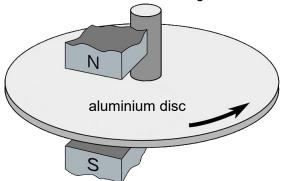
Explain why eddy currents are induced in the disc

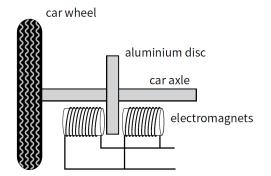
linear speed along the length of the rod is not the same and varies with distance from pivot. rate of cutting magnetic flux lines is not the same so different magnitudes of e.m.f. are induced in different parts of the disc disc is a conductor, eddy currents are induced in the disc

Explain why the disc comes to a rest after a few oscillations

eddy currents dissipate thermal energy in disc energy derived from oscillation of disc energy of disc depends on amplitude of oscillations

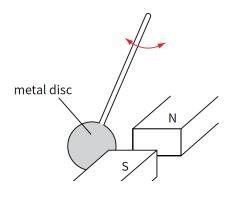
Eddy current damping is the principle behind electromagnetic braking. Electromagnets replace the permanent magnets (figure below, left) so that the spinning disc can be slowed down whenever current flows in the electromagnets.





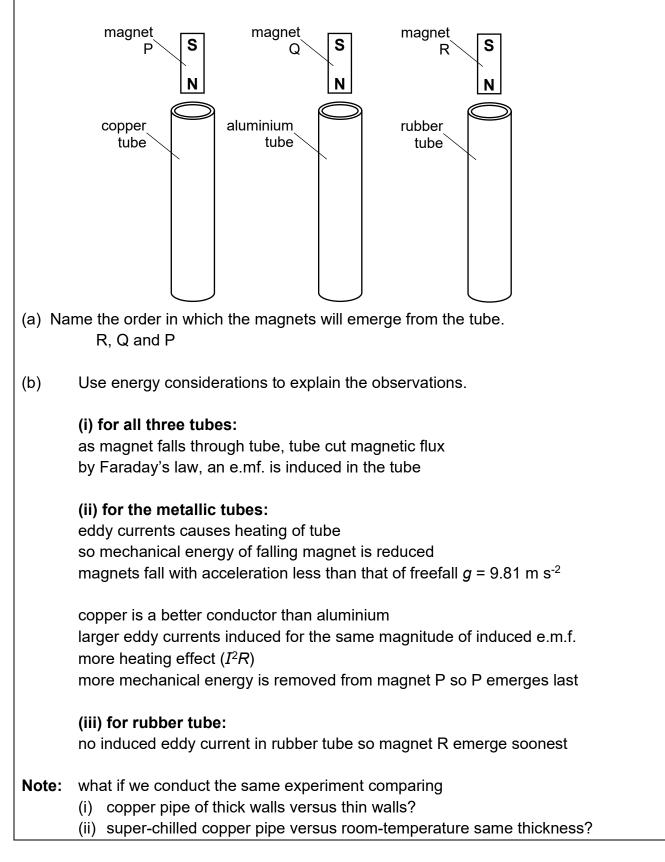
The advantage over conventional brakes is that there is no physical contact with the spinning disc and therefore no wear and tear of brake pads. However, the disadvantage is that induced eddy currents are smaller with lower rotational speeds and so braking is less efficient. Such braking is usually deployed alongside conventional brakes on vehicles such as high speed trains.

Note: when answering questions involving eddy current heating / braking / damping / resistive forces, account for the conversion of energy that is removed from the system.





Three vertical tubes, made from copper, aluminium and rubber respectively, have identical dimensions. Identical, strong, cylindrical magnets P, Q and R, are released simultaneously from the same distance above each tube.





17.5 Applications

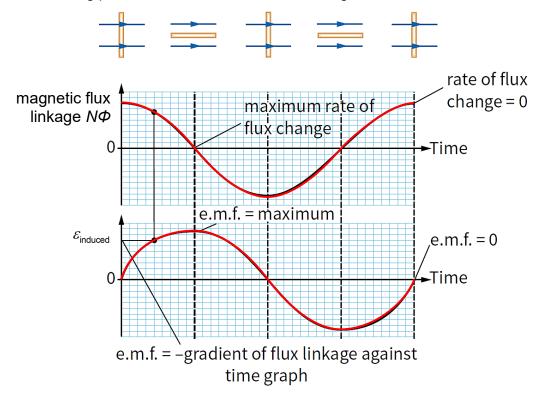
Generator

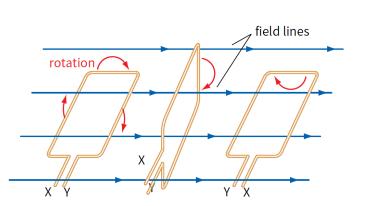
A wire coil spinning in a magnetic field generates electricity.

This is equivalent to using an electric motor backwards, which is how most electric vehicles perform regenerative braking.

The rate of change of magnetic flux linkage is maximum when the coil is moving through the horizontal position – one side is cutting flux lines rapidly downwards while the other is cutting rapidly upwards.

As the coil moves through the vertical position, the rate of change of magnetic flux linkage is zero as the sides of the coil are moving parallel to the flux lines and not cutting them.



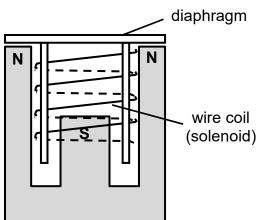




A wire coil attached to a diaphragm is placed near a stationary permanent magnet.

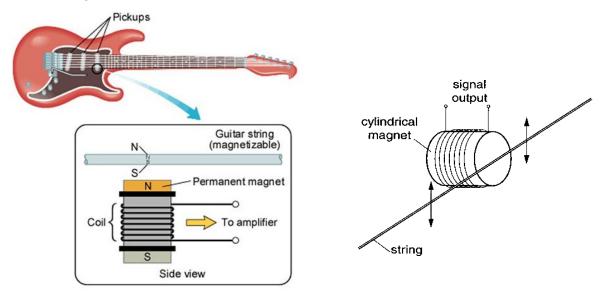
When sound wave strikes the microphone, pressure differences within the wave causes the diaphragm to oscillate. Wire coil moves at the same frequency to and fro from the magnet.

By Faraday's Law, <u>e.m.f. is induced</u> through coil as there is a rate of change of <u>magnetic flux linkage</u> through the coil. The induced e.m.f. signal can be transmitted and amplified to recreate the sound.



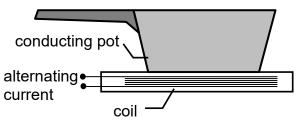
Guitar Pickup

An acoustic guitar depends on the acoustic resonance produced in the hollow body of the instrument by the oscillations of strings. An electric guitar is, on the other hand, a solid instrument, so there is no body resonance. Instead, the oscillations of the metal strings are sensed by electric "pickups" that send signals to an amplifier and a set of speakers.



For each pickup, wire is coiled around a small magnet and this wire is connected to an amplifier. The magnetic field of the permanent magnet induces a north and south pole in the section of the metal string above the magnet. This section of string is magnetized and has its own magnetic field.

When the magnetized string is plucked and made to oscillate, its motion relative to the coil causes a coil to cut the flux by the string. By Faraday's Law, e.m.f. is induced in the coil. Induced e.m.f. varies at the same frequency as the string's oscillations. Signal can be transmitted and amplified to recreate this frequency at the speakers.



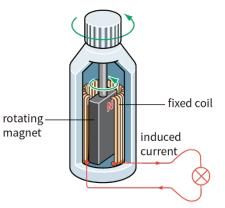
An alternating current in the coil within the induction cooker produces a changing magnetic field. Results in changing magnetic flux linkage through the base of the cooking pot.

By Faraday's law, an e.m.f. is induced which produces eddy currents in the base of the pot. Since the pot is metallic and conducting, the flow of eddy currents causes Joule heating and the heat is used for cooking.

Bicycle Dynamo

A bicycle dynamo is a mini generator which converts some of the tyre's kinetic energy into electricity, which is handy for powering the lights.

Instead of a coil rotating, most dynamos spin a permanent magnet around static wire coils instead to achieve the cutting of magnetic flux lines. By Faraday's law, an e.m.f. is induced.





17.6 Ending Notes

Electromagnetic induction is widely applied throughout our daily lives and in fact is responsible for the generation of the electricity we use. In the next topic we will delve deeper into how electricity is generated and transported from the power stations to households and industries.

