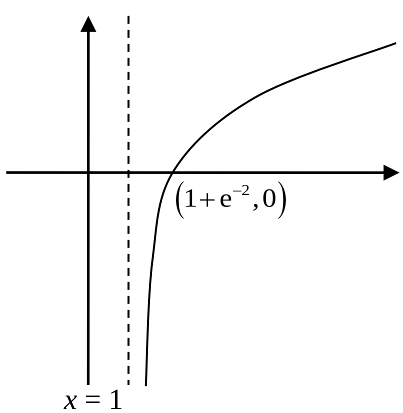
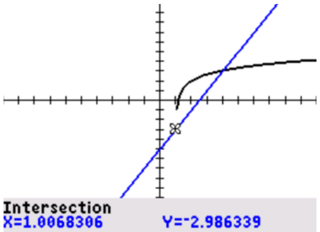
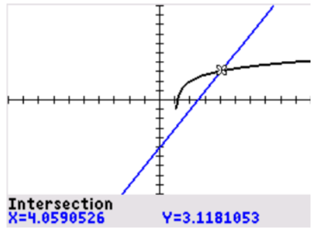
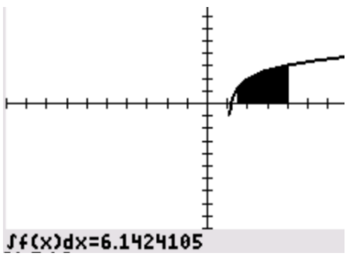
ACJC 2023 H1 Math JC1 Promo Solutions

1	$(\ln x)^2 - \ln\left(ex^2\right) > 7$ $(\ln x)^2 - \ln e - \ln x^2 - 7 > 0$ $(\ln x)^2 - \ln e - 2\ln x - 7 > 0$ $(\ln x)^2 - 2\ln x - 8 > 0$ Let $u = \ln x$ $(u)^2 - 2u - 8 > 0$ $(u-4)(u+2) > 0$ $u > 4$ or $u < -2$ $\ln x > 4$ or $\ln x < -2$ $x > e^4$ or $x < e^{-2}$ Since $x > 0$ for $\ln x$ to be defined Hence $x > e^4$ or $0 < x < e^{-2}$	Many students were not able to simplify $\ln\left(ex^2\right)$ as $\ln e + 2\ln x$. For those who did, majority of them wrote $-\ln\left(ex^2\right) = -\ln e + 2\ln x$ instead, resulting in wrong values of roots. The common **misconception** to solve $(u-4)(u+2) > 0$ is "splitting" as $(u-4) > 0$ or $(u+2) > 0$. Very few students can write $0 < x < e^{-2}$ as part of the answer.
2	**Method 1** $5x^2 - 5x + 2 = 5(x^2 - x + \frac{2}{5})$ $= 5((x - \frac{1}{2})^2 - \left(\frac{1}{2}\right)^2 + \frac{2}{5})$ $= 5((x - \frac{1}{2})^2 + \frac{3}{20})$ $= 5(x - \frac{1}{2})^2 + \frac{3}{4} > 0$ **Method 2** Discriminant of $5x^2 - 5x + 2 = 0$ is $5^2 - 4(5)(2) = -15 < 0$ (1) Hence $y = 5x^2 - 5x + 2$ does not intersect the x-axis . Also since the coefficient of x^2 >0, the curve $y = 5x^2 - 5x + 2$ has a minimum point. Hence $5x^2 - 5x + 2$ is positive for all real values of *x*.	Most students use completing the square to show that the expression is positive. A handful of them use discriminant but a few of them did D > 0 instead.
	$4x^2 + (12k-8)x - k(k+2) = 0$ Discriminant = $(12k-8)^2 + 4(4)(k^2 + 2k)$ $= 144k^2 + 64 - 192k + 16k^2 + 32k$ $= = 160k^2 - 160k + 64$ $= 32(5k^2 - 5k + 2)$ > 0 for all real values of *k* from above	Almost all students are able to begin with D > 0 in this part. However, only a few students are able to derive that "for all real values of *k*" as the final answer.

3a	$\ln\left(\dfrac{e^{-2x}}{\sqrt{2+5x}}\right)=\ln\left(e^{-2x}\right)-\dfrac{1}{2}\ln(2+5x)$ $=-2x-\dfrac{1}{2}\ln(2+5x)$ $\dfrac{d}{dx}\left[-2x-\dfrac{1}{2}\ln(2+5x)\right]=-2-\dfrac{5}{2(2+5x)}$ $=-2-\dfrac{5}{4+10x}$ $A=-2,\ B=-5,\ C=10$	Quite well done. A common error is to differentiate $\ln\left(\dfrac{e^{-2x}}{\sqrt{2+5x}}\right)$ directly, which lead to complicated wrong answers. Do note that product rule is not necessary in H1 Math syllabus.
b	$\int_{\frac{2}{3}}^{k}\dfrac{6}{\sqrt{2+3x}}\ \mathrm{dx}=4$ $6\left[\dfrac{(2+3x)^{1/2}}{\frac{1}{2}(3)}\right]_{\frac{2}{3}}^{k}=4$ $\left[\sqrt{2+3x}\right]_{\frac{2}{3}}^{k}=1$ $\sqrt{2+3k}-\sqrt{4}=1$ $\sqrt{2+3k}=3$ $k=\dfrac{7}{3}$	Majority were able to integrate. A handful of them could not obtain the answer due to careless mistakes.
4i		A common error was that the graph was truncated near x =1. A handful of students wrote y = 2 as the horizontal asymptote as well. Most of the students did not realise that "exact" coordinates is needed.

4ii	Rearrange the equation to $\ln(x-1)+2=2x-5$ add the $y=2x-5$ line.  From GC, the 2 solutions are 1.01 (3sf) and 4.06 (3sf)	Many students obtained 4.06 using the line $y=2x-5$. Very few students are able to obtain 1.01 as one of the answers.
4iii	 From GC, area = 6.14 units^2	Quite well done.
5i	$k^2-x^2=-kx+k^2$ $x^2-kx=0$ $x=0$ or $x=k$ Sub into equation to find *y*-coordinates. $A\ (0,k^2)$ and $B\ (k,0)$	Mostly well done.
5ii	$\int_0^k \left(k^2-x^2\right)-\left(-kx+k^2\right)\,dx$ $=\int_0^k \left(k^2-x^2+kx-k^2\right)\,dx$ $=\left[\frac{kx^2}{2}-\frac{x^3}{3}\right]_0^k$ $=\frac{k^3}{2}-\frac{k^3}{3}$ $=\frac{k^3}{6}$	Most students are able to begin with the first line for either methods. Some errors in integrating k^2. Many mistaken *k* as *x* in integrating with respect to *x*.

	Alternative method $\int_0^k \left(k^2 - x^2\right) dx -$ area of triangle $= \left[k^2 x - \frac{x^3}{3}\right]_0^k - \frac{1}{2}(k)\left(k^2\right)$ $= \frac{2k^3}{3} - \frac{k^3}{2}$ $= \frac{k^3}{6}$	
6i	$y = \frac{x^2+12}{\sqrt{x}} = x^{3/2} + 12x^{-1/2}$ $\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 6x^{-3/2}$ Let $\frac{dy}{dx} = 0 \Rightarrow \frac{3}{2}x^{1/2} = \frac{6}{x^{3/2}}$ $\Rightarrow x^{1/2}\left(x^{3/2}\right) = 6\left(\frac{2}{3}\right)$ $\Rightarrow x^2 = 4$ $\Rightarrow x = 2, \ -2(\text{na})$ Alternative method: Sub x = 2 into $\frac{dy}{dx} = \frac{3}{2}x^{1/2} - 6x^{-3/2} = \frac{3}{2}\sqrt{2} - 6(2)^{-3/2} = 0$ $y = \frac{x^2+12}{\sqrt{x}} = \frac{4+12}{\sqrt{2}} = \frac{16}{\sqrt{2}}$ Stationary point is $\left(2\ ,\ \frac{16}{\sqrt{2}}\right)$	Pretty well done as most of the students are aware that the expression requires some simplification before differentiating. A handful of them made some errors with the indices rules thus not able to obtain the correct answers.

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$$\frac{dy}{dx}=\frac{3}{2}x^{\frac{1}{2}}-6x^{-\frac{3}{2}}$$

1st derivative test:

<table>
<tr><td>x</td><td>1.99</td><td>2</td><td>2.01</td></tr>
<tr><td>$\frac{dy}{dx}$</td><td>−0.0213</td><td>0</td><td>+0.0211</td></tr>
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<u>Alternative method</u>:

$$\frac{d^2y}{dx^2}=\frac{3}{4}x^{-\frac{1}{2}}+9x^{-\frac{5}{2}}=2.12>0$$

Therefore $\left(2\ ,\ \frac{16}{\sqrt{2}}\right)$ is a minimum turning point.

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<td>A handful of them are not aware that computing values of $\frac{dy}{dx}$ is necessary in first derivative test.</td>
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<td>ii.</td>
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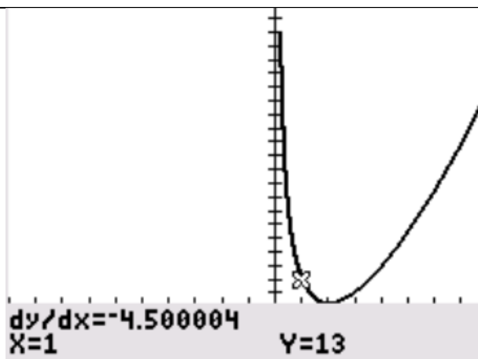

From GC, gradient $=-4.50$,
Coordinate is (1, 13)

Equation of the tangent is $y-13=-4.50(x-1)$
$y=-4.5x+17.5$ (2dp)

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<td>Most students are aware that they need to find equation of a straight line.

Some of the students obtained the wrong $\frac{dy}{dx}$ value due to the error made in the earlier parts.</td>
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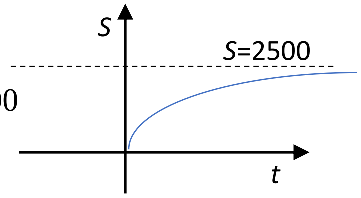
7i	$S = 2500 + \dfrac{-5000}{2+0.4t}$ $2+0.4t = 0 \Rightarrow t = -5$ When $S = 0,\ t = 0$ Horizontal asymptotes $S = 2500$  Or Horizontal asymptote is $S = \dfrac{1000}{0.4} = 2500$	Majority of the students are aware that the horizontal asymptote is S = 2500. A handful of them labelled as y = 2500. A number of them did not realise that $t \geq 0$.
ii	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = \dfrac{2000}{(2+0.4t)^2}$ When $t = 2$, $\dfrac{dS}{dt} = \dfrac{2000}{(2+0.4t)^2} = \dfrac{2000}{(2+0.8)^2} = 255\ \ (3\text{ sf})$ The sales of the company is **changing at the rate** of \$255000 per year in the year **2012**.	Many students did not include the word "rate". Many students mistaken that the value is meant for 2010 to 2012. In fact, it should just be 2012.
iii	In the long run the sales of the company increases and **tends towards \$2 500 000.**	Not many students use the similar phrasing.
iv	$\int_2^4 \dfrac{1000t}{2+0.4t}\mathrm{d}t = \int_2^4 (2500 - \dfrac{5000}{2+0.4t})\mathrm{d}t$ $=[2500t - \dfrac{5000}{0.4}\ln(2+0.4t)]_2^4$ $=[2500(4) - \dfrac{5000}{0.4}\ln(2+0.4(4)] - [2500(2) - \dfrac{5000}{0.4}\ln(2+0.4(2)]$ $= (2500(2) + 12500\ln\dfrac{2.8}{3.6})$ = 1859 (to the nearest integer) It represents the **total sales** in thousand dollars for the jewellery store from **2012 to 2014**.	A handful of them use G.C to integrate. Students are not aware that "Find algebraically" means G.C is not allowed in solving.

v	$C = t^3 - 12t^2 + (k+36)t$ $\dfrac{\mathrm{d}C}{\mathrm{d}t} = 3t^2 - 24t + (k+36)$ > 0 for all real t since x is an increasing function of t **<u>Method 1</u>**: Discriminant = $(24)^2 - 4(3)(k+36) < 0 \Rightarrow k > 12$ **<u>Method 2</u>**: $3t^2 - 24t + k + 36 = 3\left(t^2 - 8t + \dfrac{k}{3} + 12\right) = 3\left((t-4)^2 - 16 + \dfrac{k}{3} + 12\right) > 0$ for all real t if $-16 + \dfrac{k}{3} + 12 > 0$ $\Rightarrow \dfrac{k}{3} > 4$ $\Rightarrow k > 12$	Quite badly done as most students did D > 0 instead.