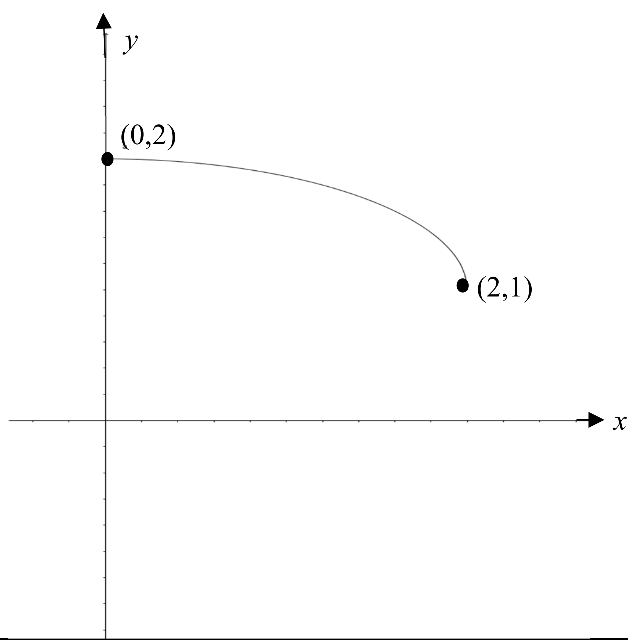
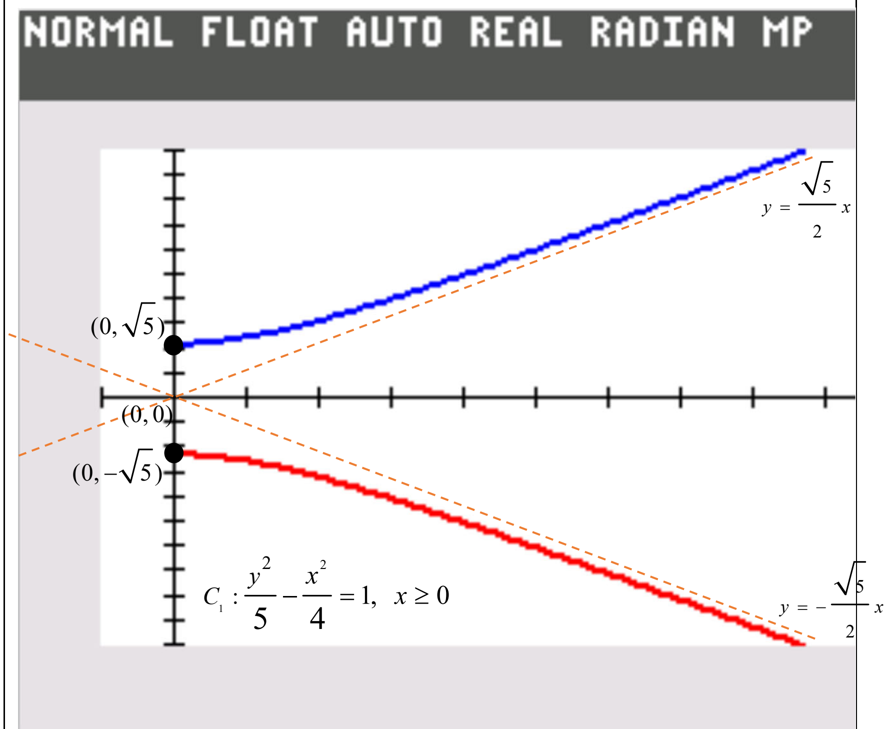
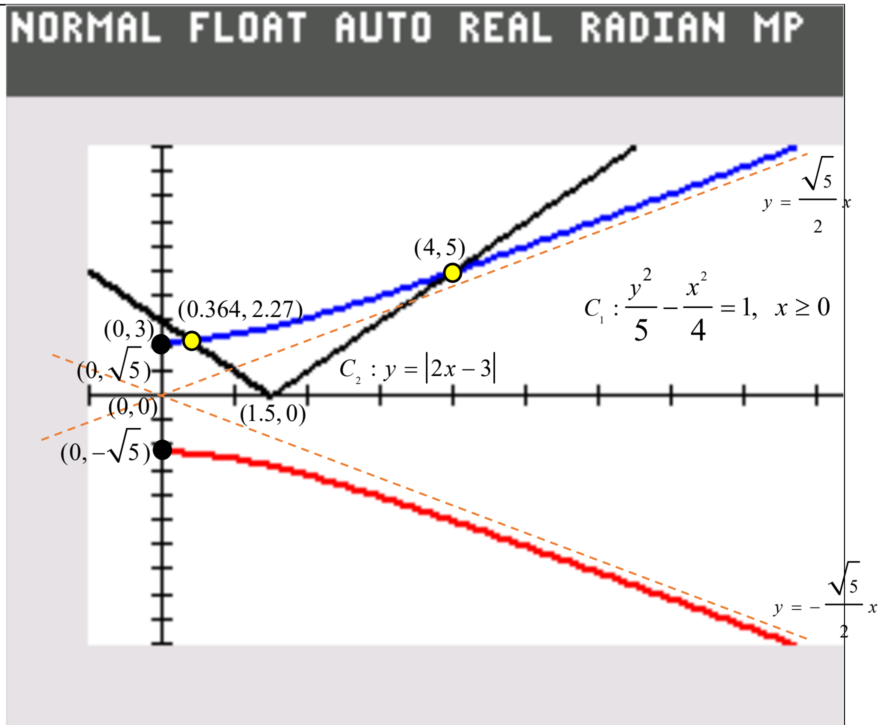
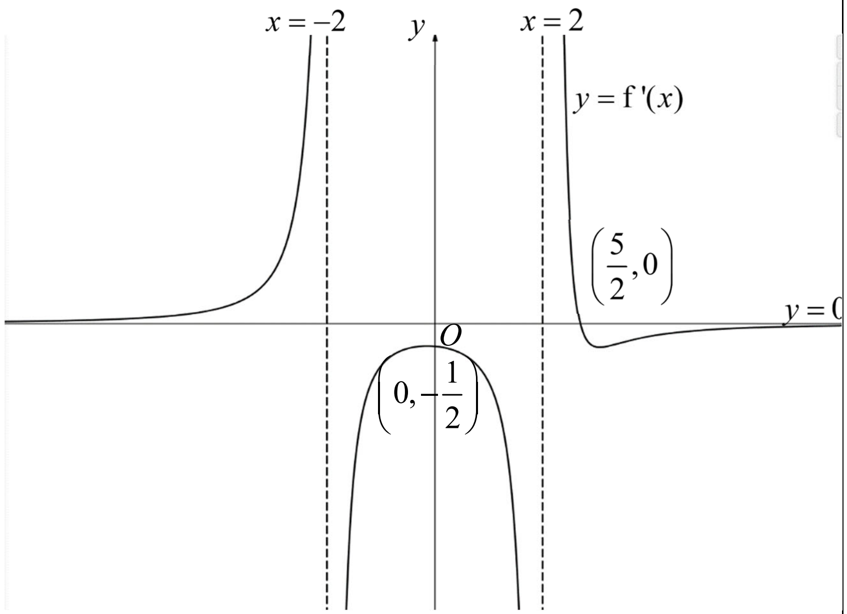
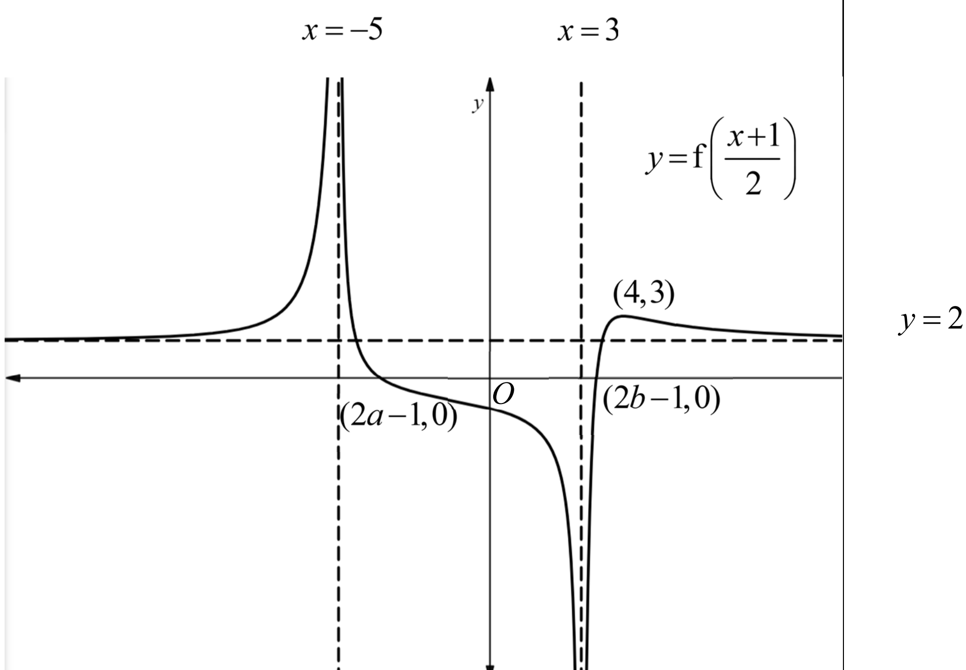
No	Solution
1 (i)	$S_n = \frac{5}{24} - \frac{n+5}{(n+4)!}$ As $n \to \infty$, $\frac{n+5}{(n+4)!} \to 0$ $\therefore S_n = \frac{5}{24} - \frac{n+5}{(n+4)!} \to \frac{5}{24}$, which is unique and finite. Hence, the series converges. Hence, $S_\infty = \frac{5}{24}$.
(ii)	$\sum_{r=1}^{\infty} u_{r+2} = \sum_{r=3}^{\infty} u_r$ (replace r with $r-2$) $= S_\infty - S_2$ $= \frac{5}{24} - \left(\frac{5}{24} - \frac{7}{6!}\right)$ $= \frac{7}{6!}$ $= \frac{7}{720}$
(iii)	For $n \geq 2$, $u_n = S_n - S_{n-1}$ $= \frac{5}{24} - \frac{n+5}{(n+4)!} - \left(\frac{5}{24} - \frac{n+4}{(n+3)!}\right)$ $= \frac{n+4}{(n+3)!} - \frac{n+5}{(n+4)!}$ $= \frac{1}{(n+4)!}\left[(n+4)^2 - n - 5\right]$ $= \frac{1}{(n+4)!}\left[n^2 + 8n + 16 - n - 5\right]$ $= \frac{n^2 + 7n + 11}{(n+4)!}$ $u_1 = S_1 = \frac{5}{24} - \frac{6}{5!} = \frac{19}{120}$

	For $n=1$, $RHS=\dfrac{1^2+7+11}{(1+4)!}=\dfrac{19}{120}$ $\therefore u_n=\dfrac{n^2+7n+11}{(n+4)!}$ where $n\in\mathbb{Z}^+$
2 (i)	$x=2\sqrt{t},\ y=1+\sqrt{1-t}$ $\dfrac{dx}{dt}=\dfrac{1}{\sqrt{t}},\ \dfrac{dy}{dt}=-\dfrac{1}{2\sqrt{1-t}}$ $\Rightarrow \dfrac{dy}{dx}=-\dfrac{1}{2}\sqrt{\dfrac{t}{1-t}}$ As $t\to 1$, $\dfrac{dy}{dx}\to-\infty$. The tangent to the curve approaches a vertical line.
2 (ii)	
(iii)	At point $T\,(2\sqrt{t},\,1+\sqrt{1-t})$, Equation of the tangent at T

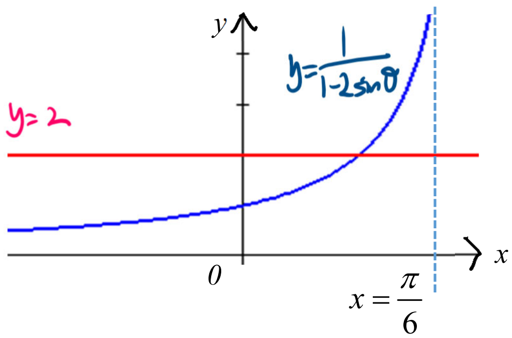
	$y-(1+\sqrt{1-t})=-\dfrac{\sqrt{t}}{2\sqrt{1-t}}(x-2\sqrt{t})$ $2\sqrt{1-t}\,y-2\sqrt{1-t}(1+\sqrt{1-t})=-\sqrt{t}x+2t$ $2\sqrt{1-t}\,y-2\sqrt{1-t}-2(1-t)=-\sqrt{t}x+2t$ $2\sqrt{1-t}\,y=-\sqrt{t}x+2+2\sqrt{1-t}$
(iv)	When $x=\sqrt{2}$, $2\sqrt{t}=\sqrt{2}$ $\sqrt{t}=\dfrac{1}{\sqrt{2}}$ $t=\dfrac{1}{2}$ When $t=\dfrac{1}{2}$, equation of tangent: $2\sqrt{\dfrac{1}{2}}y=-\dfrac{1}{\sqrt{2}}x+2+2\sqrt{\dfrac{1}{2}}$ $y=-\dfrac{1}{2}x+\sqrt{2}+1$ When tangent meets x-axis, $y=0\Rightarrow x=2(1+\sqrt{2})$ $\text{P}(2(1+\sqrt{2}),0)$ When tangent meets y-axis, $x=0\Rightarrow y=1+\sqrt{2}$ $\text{Q}(0,\ 1+\sqrt{2})$ Area of triangle OPQ $=\dfrac{1}{2}(2)(1+\sqrt{2})(1+\sqrt{2})$ $=3+2\sqrt{2}$ units^2
3(i)	$C_1: 4y^2-5x^2=20$ $\dfrac{y^2}{5}-\dfrac{x^2}{4}=1$ $\dfrac{y^2}{\left(\sqrt{5}\right)^2}-\dfrac{x^2}{2^2}=1$ [Vertical Hyperbola with centre at origin $(0,0)$] Asymptotes

	As $x \to \pm\infty$, $\frac{y^2}{5} \to \frac{x^2}{4}$ $y \to \pm\sqrt{\frac{5x^2}{4}}$ $\therefore$ Asymptotes: $y = \frac{\sqrt{5}}{2}x$ and $y = -\frac{\sqrt{5}}{2}x$  <u>Axial Intercepts of C_1 (From GC)</u> $(0, \sqrt{5})$ & $(0, -\sqrt{5})$ or $(0, 2.24)$ & $(0, -2.24)$
(ii)	$C_2 : y = \lvert 2x - 3 \rvert$

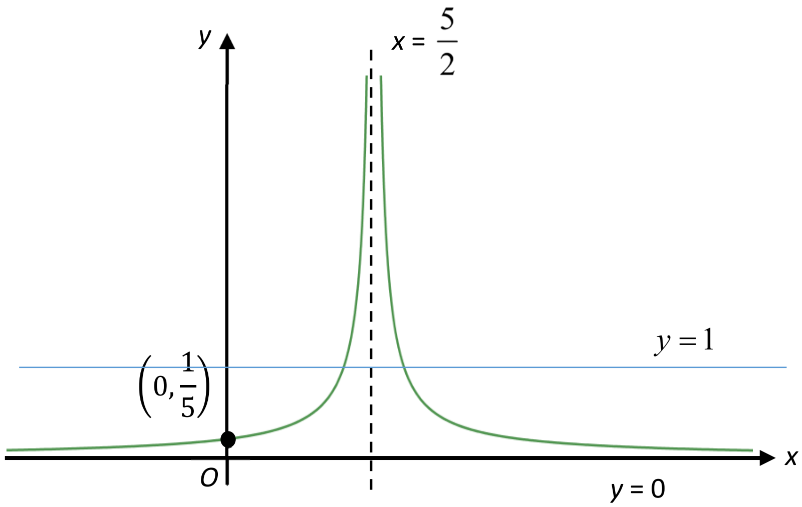
	 Points of Intersection between C_1 and C_2 (From GC) $(0.364, 2.27)$ & $(4,5)$
(iii)	Note that $C_1:\frac{y^2}{5}-\frac{x^2}{4}=1 \Rightarrow C_1: y=\pm\sqrt{5\left(1+\frac{x^2}{4}\right)}$ $\lvert 2x-3\rvert \geq \sqrt{5\left(1+\frac{x^2}{4}\right)}$ $\quad (C_2 \geq \text{Upper Half of } C_1)$ From the graphs in (ii), $0\leq x\leq 0.36364(5\text{sf})$ or $x\geq 4$ $0\leq x\leq 0.364(3\text{sf})$ or $x\geq 4$
4(a) (i)	

	
(ii)	
(b)	$y=\frac{1}{4ax-x^2}=\frac{1}{4a^2-(x-2a)^2}$ $\xrightarrow{x\to x+2a} y=\frac{1}{4a^2-x^2} \xrightarrow{y\to -y} y=\frac{1}{x^2-4a^2}$ Sequence of transformations: (order is not important)

	1. Translation of $2a$ units in the negative x-direction. 2. Reflection about the x-axis. Alternative Method: $y=\dfrac{1}{4ax-x^2}=\dfrac{1}{x(4a-x)}$ $\xrightarrow{x\to x+2a} y=\dfrac{1}{(x+2a)(2a-x)} \xrightarrow{y\to -y} y=\dfrac{1}{x^2-4a^2}$ Sequence of transformations: (order is not important) 1. Translation of $2a$ units in the negative x-direction. 2. Reflection about the x-axis.
5(a)	For sum to infinity to exist, $\lvert 2\sin\theta\rvert<1---(*)$ $-1<2\sin\theta<1$ $-\dfrac{1}{2}<\sin\theta<\dfrac{1}{2}$ $-\dfrac{\pi}{6}<\theta<\dfrac{\pi}{6}---(1)$ $\dfrac{1}{1-2\sin\theta}>2---(\#)$ $0<1-2\sin\theta<\dfrac{1}{2}$ since $\lvert 2\sin\theta\rvert<1$ $\sin\theta>\dfrac{1}{4}\Rightarrow\theta>0.253---(2)$ Since $-\dfrac{\pi}{6}<\theta<\dfrac{\pi}{6}$, therefore $0.253<\theta<0.526$ (final answer)

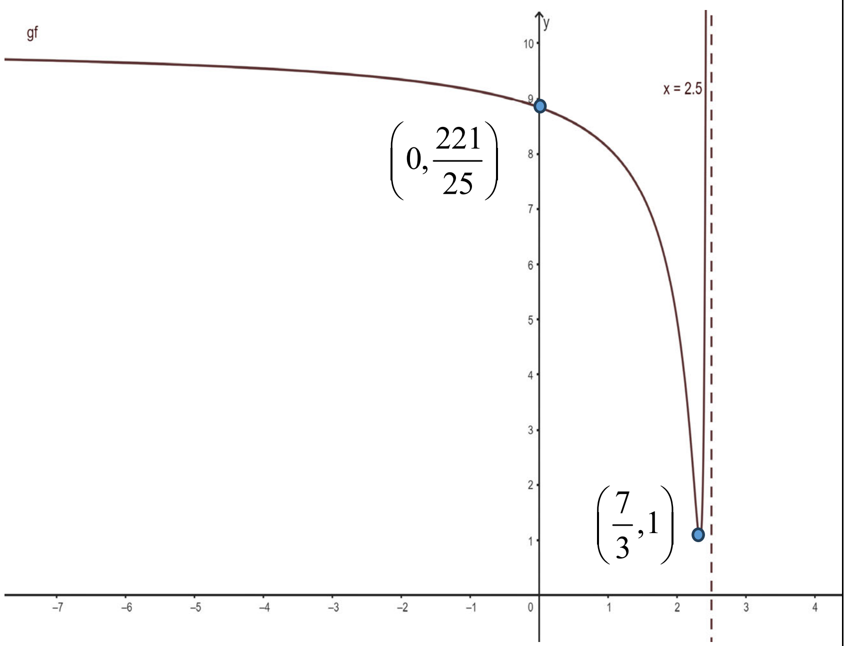
	Alternatively, students can graph $y=\dfrac{1}{1-2\sin\theta}$  From the graph, $\dfrac{1}{1-2\sin\theta}>2$ $\Rightarrow\ 0.253<\theta<0.526$ (final answer)
(b)	Let a denote the first term and r be the common ratio of the geometric progression respectively. Likewise, let b and d denote the first term and common difference of the arithmetic progression. $\therefore$ $ar^4 = b + 6d$ …(1) $ar^8 = b + 24d$ …(2) $ar^{10} = b + 49d$ …(3) (2) – (1): $ar^8 - ar^4 = 18d$ …(4) (3) – (2): $ar^{10} - ar^8 = 25d$ …(5) Eq(5)/Eq(4): $\dfrac{ar^8(r^2-1)}{ar^4(r^4-1)}=\dfrac{25d}{18d}$ $\dfrac{r^4(r^2-1)}{(r^2+1)(r^2-1)}=\dfrac{25}{18}$ $\dfrac{r^4}{(r^2+1)}=\dfrac{25}{18}$ $18r^4=25r^2+25$ $18r^4-25r^2-25=0$ – – – (@) $r^2=\dfrac{25\pm\sqrt{(-25)^2-4(18)(-25)}}{2(18)}=\dfrac{25\pm\sqrt{2425}}{36}$

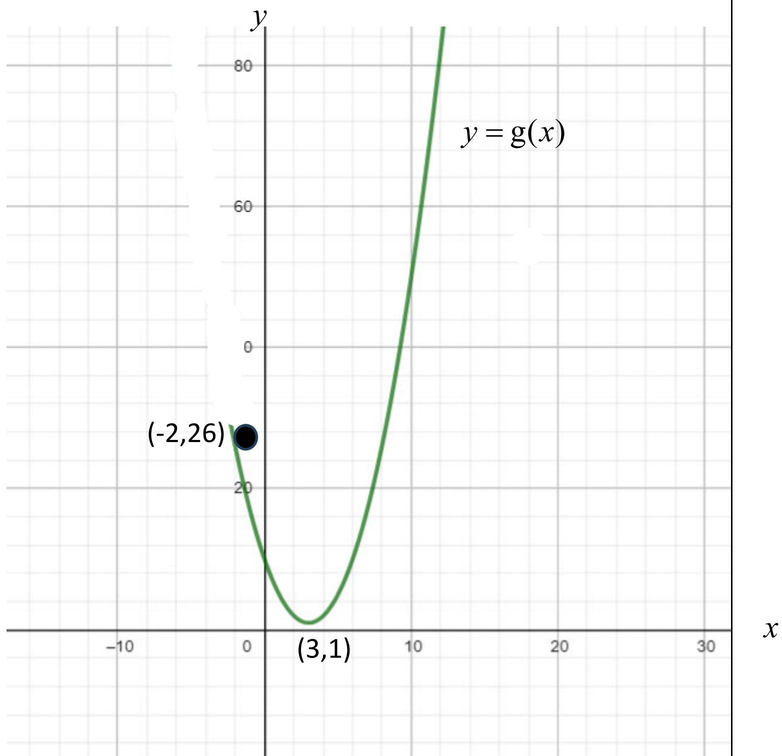
<table>
<tr><td></td><td>$r=\pm\sqrt{\dfrac{25+\sqrt{2425}}{36}}$
$=1.436$ or -1.436
Since for both values of r, $|r|=1.436>1$, the geometric progression is not convergent.

Alternatively, students can use GC to solve
$18r^4-25r^2-25=0$
The 4 roots are $1.436,\ -1.436,\ 0.821\mathrm{i},\ -0.821\mathrm{i}$
However we only need to consider real roots.
Hence $r=1.436$ or -1.436 .</td></tr>
<tr><td>6(i)</td><td>
Since the line $y=1$ cuts the graph $y=\mathrm{f}(x)$ at two points, f is not a one-to-one function. Therefore f does not have an inverse.</td></tr>
<tr><td>(ii)</td><td>Maximum $k=\dfrac{5}{2}$.

Let $y=\mathrm{f}(x)$.
For $\mathrm{D_f}=\left(-\infty,\dfrac{5}{2}\right)$,
$y=-\dfrac{1}{2x-5}$</td></tr>
</table>

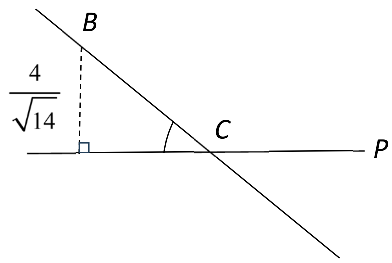
	$2x-5=-\frac{1}{y}$ $2x=5-\frac{1}{y}$ $x=\frac{5}{2}-\frac{1}{2y}$ $f^{-1}(y)=\frac{5}{2}-\frac{1}{2y}$ Replacing y by x, $f^{-1}(x)=\frac{5}{2}-\frac{1}{2x}$ $D_{f^{-1}}=R_f=(0,\infty)$ $f^{-1}:x\mapsto\frac{5}{2}-\frac{1}{2x}$, where $x\in\mathbb{R}, x>0$.
(iii)	$R_f=(0,\infty)$ $D_g=[-2,\infty)$ Since $R_f\subseteq D_g$, gf exists.
(iv)	$gf(x)=g\left(-\frac{1}{2x-5}\right)$ $=\left(-\frac{1}{2x-5}-3\right)^2+1$ $=\left(\frac{1}{5-2x}-3\right)^2+1$ $=\left[\frac{1-3(5-2x)}{5-2x}\right]^2+1$ $=\left(\frac{6x-14}{5-2x}\right)^2+1$, where $a=6,\ b=5$.

	$D_{gf} = D_f = \left(-\infty, \frac{5}{2}\right)$  From above graph, $R_{gf} = [1, \infty)$.

	Alternative method By sketching the graph of g and letting the new domain of g be $R_f = (0,\infty)$,  $D_f = \left(-\infty, \frac{5}{2}\right) \to R_f = \text{new } D_g = (0,\infty) \ \to \text{new } R_g = [1,\infty)$ $\therefore R_{gf} = [1,\infty)$.
7(i)	$L\text{: } \mathbf{r} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \lambda \in \mathbb{R}$ *C* lies on *L* $\Rightarrow \overrightarrow{OC} = \begin{pmatrix} -2+\lambda \\ -2\lambda \\ 1 \end{pmatrix}$ for some $\lambda \in \mathbb{R}$ $\therefore \overrightarrow{BC} = \begin{pmatrix} -3+\lambda \\ -1-2\lambda \\ -1 \end{pmatrix}$

	$\overrightarrow{BC} \perp L$ $\Rightarrow \begin{pmatrix} -3+\lambda \\ -1-2\lambda \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = 0$ $-3+\lambda+2+4\lambda=0$ $\lambda=\frac{1}{5}$ $\therefore C\left(-\frac{9}{5}, -\frac{2}{5}, 1\right)$
(ii)	$\overrightarrow{AD} = \begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix}$ A normal vector is $\begin{pmatrix} 1 \\ 4 \\ 2 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$ Eqn of $\prod$: $\mathbf{r} \cdot \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$ $\therefore 2x+y-3z=-7$
(iii)	**Method 1: Use of projection formula:** $\overrightarrow{DB} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$ shortest distance between B and $\prod$ $= \frac{\left\| \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix} \right\|}{\sqrt{4+1+9}}$ $= \frac{4}{\sqrt{14}}$ units OR

	$\overrightarrow{AB}=\begin{pmatrix}3\\1\\1\end{pmatrix}$ shortest distance between B and $\prod$ $=\dfrac{\left\lvert\begin{pmatrix}3\\1\\1\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-3\end{pmatrix}\right\rvert}{\sqrt{4+1+9}}$ $=\dfrac{4}{\sqrt{14}}$ units $=\dfrac{2\sqrt{14}}{7}$ units **Method 2: Find Foot of perpendicular first** Let F be the foot of perpendicular of B on $\prod$. $l_{BF}:\quad \mathbf{r}=\begin{pmatrix}1\\1\\2\end{pmatrix}+\mu\begin{pmatrix}2\\1\\-3\end{pmatrix}, \mu\in\mathbb{R}$ F lies on l_{BF} $\Rightarrow \overrightarrow{OF}=\begin{pmatrix}1+2\mu\\1+\mu\\2-3\mu\end{pmatrix}$ for some $\mu\in\mathbb{R}$ F lies on Π $\Rightarrow\begin{pmatrix}1+2\mu\\1+\mu\\2-3\mu\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-3\end{pmatrix}=-7$ $2+4\mu+1+\mu-6+9\mu=-7$ $14\mu=-4$ $\mu=-\dfrac{2}{7}$ $\therefore \overrightarrow{OF}=\begin{pmatrix}\frac{3}{7}\\\frac{5}{7}\\\frac{20}{7}\end{pmatrix}$ $\therefore \overrightarrow{BF}=-\dfrac{2}{7}\begin{pmatrix}2\\1\\-3\end{pmatrix}$

	shortest distance between B and $\prod$ $=\left\|\overrightarrow{BF}\right\|=\frac{2}{7}\sqrt{4+1+9}=\frac{2\sqrt{14}}{7}$ units
(iv)	Method 1: Hence $\overrightarrow{BC}=\frac{1}{5}\begin{pmatrix}-14\\-7\\-5\end{pmatrix}$  Required angle $=\sin^{-1}\dfrac{\frac{4}{\sqrt{14}}}{\frac{1}{5}\sqrt{14^2+7^2+5^2}}$ $=19.0°$ (1dp) Method 2: Otherwise $\overrightarrow{BC}=\frac{1}{5}\begin{pmatrix}-14\\-7\\-5\end{pmatrix}$ Required angle $=\sin^{-1}\dfrac{\left\|\begin{pmatrix}14\\7\\5\end{pmatrix}\cdot\begin{pmatrix}2\\1\\-3\end{pmatrix}\right\|}{\sqrt{14^2+7^2+5^2}\sqrt{4+1+9}}$ $=\sin^{-1}\dfrac{20}{\sqrt{270}\sqrt{14}}$ $=19.0°$ (1dp)
8 (i)	From $y=e^{\tan^{-1}(x)}$, Differentiate with respect to x, $\frac{dy}{dx}=e^{\tan^{-1}(x)}.\frac{d}{dx}\left[\tan^{-1}(x)\right]$ $\frac{dy}{dx}=e^{\tan^{-1}(x)}.\left(\frac{1}{1+x^2}\right)$

	Upon multiplying both sides by $\left(1+x^2\right)$: $\left(1+x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{e}^{\tan^{-1}(x)}$, recall $y=\mathrm{e}^{\tan^{-1}(x)}$ $\left(1+x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}=y \qquad (1)$ Differentiate (1) with respect to x, $2x\frac{\mathrm{d}y}{\mathrm{d}x}+\left(1+x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}x}$ Upon rearrangement: $\left(1+x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left(1-2x\right)\frac{\mathrm{d}y}{\mathrm{d}x} \qquad (2)$ <u>Alternative Solution</u> From $y=\mathrm{e}^{\tan^{-1}(x)}$, $\ln y=\tan^{-1}\left(x\right)$ Differentiate with respect to x, $\frac{1}{y}\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{1+x^2}$ $\left(1+x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x}=y \qquad (1)$ Differentiate (1) with respect to x, $2x\frac{\mathrm{d}y}{\mathrm{d}x}+\left(1+x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\frac{\mathrm{d}y}{\mathrm{d}x}$ Upon rearrangement: $\left(1+x^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left(1-2x\right)\frac{\mathrm{d}y}{\mathrm{d}x} \qquad (2)$
(ii)	Differentiate (2) with respect to x, $\left(1+x^2\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+2x\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left(1-2x\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}-2\frac{\mathrm{d}y}{\mathrm{d}x} \qquad (3)$ Using $y=\mathrm{e}^{\tan^{-1}x}$ and $x=0$, $y=\mathrm{e}^{\tan^{-1}(0)}=1$

	From (1): Sub $x=0$ and $y=1$, $(1+0^2)\frac{\mathrm{d}y}{\mathrm{d}x}=1$ $\frac{\mathrm{d}y}{\mathrm{d}x}=1$ From (2): Sub $x=0$, $y=1$ and $\frac{\mathrm{d}y}{\mathrm{d}x}=1$, $\left(1+0^2\right)\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left[1-2(0)\right](1)$ $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=1$ From (3): Sub $x=0,\ y=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=1$ and $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=1$, $\left(1+0^3\right)\frac{\mathrm{d}^3y}{\mathrm{d}x^3}+2(0)(1)=\left[1-2(0)\right](1)-2(1)$ $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}=1-2=-1$ Overall, when $x=0$: $y=1,\ \frac{\mathrm{d}y}{\mathrm{d}x}=1,\ \frac{\mathrm{d}^2y}{\mathrm{d}x^2}=1,\ \frac{\mathrm{d}^3y}{\mathrm{d}x^3}=-1$ Hence, $y=1+x(1)+\frac{x^2}{2}(1)+\frac{x^3}{6}(-1)+\ldots$ $\quad=1+x+\frac{1}{2}x^2-\frac{1}{6}x^3+\ldots$
(iii)	$\mathrm{e}^{\frac{\pi}{6}}=\mathrm{e}^{\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)}$ Upon using the Maclaurin Series found in (ii): $\mathrm{e}^{\tan^{-1}(x)}=1+x+\frac{1}{2}x^2-\frac{1}{6}x^3+\ldots$

	Therefore, $e^{\frac{\pi}{6}}=e^{\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)}\approx 1+\left(\frac{1}{\sqrt{3}}\right)+\frac{1}{2}\left(\frac{1}{\sqrt{3}}\right)^2-\frac{1}{6}\left(\frac{1}{\sqrt{3}}\right)^3$ $=1+\frac{1}{6}+\frac{1}{\sqrt{3}}-\frac{1}{18}\left(\frac{1}{\sqrt{3}}\right)$ $=\frac{7}{6}+\left(1-\frac{1}{18}\right)\left(\frac{1}{\sqrt{3}}\right)$ $=\frac{7}{6}+\frac{17}{18}\left(\frac{\sqrt{3}}{3}\right)$ $=\frac{7}{6}+\frac{17}{54}\sqrt{3}$ $p=7,\ q=6,\ r=17$ and $s=54$
9 (i)	$(2\underset{\sim}{a}+\underset{\sim}{b})\cdot(2\underset{\sim}{a}+\underset{\sim}{b})=4(\underset{\sim}{a}\cdot\underset{\sim}{a})+2(\underset{\sim}{b}\cdot\underset{\sim}{a})+2(\underset{\sim}{a}\cdot\underset{\sim}{b})+(\underset{\sim}{b}\cdot\underset{\sim}{b})$ $=4\lvert\underset{\sim}{a}\rvert^2+4(\underset{\sim}{a}\cdot\underset{\sim}{b})+\lvert\underset{\sim}{b}\rvert^2$ $\lvert 2\underset{\sim}{a}+\underset{\sim}{b}\rvert^2=4\lvert\underset{\sim}{a}\rvert^2+4(\underset{\sim}{a}\cdot\underset{\sim}{b})+\lvert\underset{\sim}{b}\rvert^2$ $\left(2\sqrt{74}\right)^2=4(9^2)+4(\underset{\sim}{a}\cdot\underset{\sim}{b})+(1^2)$ $296=4(9^2)+4(\underset{\sim}{a}\cdot\underset{\sim}{b})+(1^2)$ $4(\underset{\sim}{a}\cdot\underset{\sim}{b})=296-4(9^2)-(1^2)$ $\underset{\sim}{a}\cdot\underset{\sim}{b}=-\frac{29}{4}$
(ii)	$\lvert\underset{\sim}{a}\cdot\underset{\sim}{b}\rvert$ is the length of projection of $\overrightarrow{OA}$ onto $\overrightarrow{OB}$
(iii)	If P, Q and R are collinear, then $k\overrightarrow{PQ}=\overrightarrow{PR}$ $k\left[(6\underset{\sim}{a}+5\underset{\sim}{b})-(7\underset{\sim}{a}-5\underset{\sim}{b})\right]=\left[(9\underset{\sim}{a}+\lambda\underset{\sim}{b})-(7\underset{\sim}{a}-5\underset{\sim}{b})\right]$ $k[-\underset{\sim}{a}+10\underset{\sim}{b}]=\left[2\underset{\sim}{a}+(\lambda+5)\underset{\sim}{b}\right]$ since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are non-parallel, non-zero vectors, $k=-2$ $(\lambda+5)=10k \Rightarrow (\lambda+5)=-20$ $\therefore\ \lambda=-25$

	Alternative If P, Q and R are collinear, then $k\overrightarrow{PQ} = \overrightarrow{QR}$ $k\left[(6\underset{\sim}{a}+5\underset{\sim}{b})-(7\underset{\sim}{a}-5\underset{\sim}{b})\right]=\left[(9\underset{\sim}{a}+\lambda\underset{\sim}{b})-(6\underset{\sim}{a}+5\underset{\sim}{b})\right]$ $k[-\underset{\sim}{a}+10\underset{\sim}{b}]=\left[3\underset{\sim}{a}+(\lambda-5)\underset{\sim}{b}\right]$ since $\underset{\sim}{a}$ and $\underset{\sim}{b}$ are non-parallel, non-zero vectors, $k=-3$ $(\lambda-5)=10k \;\Rightarrow\; (\lambda-5)=-30$ $\therefore\; \lambda=-25$
(b)	$\overrightarrow{UX}=\overrightarrow{OX}-\overrightarrow{OU}=\frac{1}{16}\mathbf{u}+\frac{3}{4}\mathbf{v}-\mathbf{u}=\frac{3}{4}\mathbf{v}-\frac{15}{16}\mathbf{u}$ $\overrightarrow{UW}=\overrightarrow{OW}-\overrightarrow{OU}=\frac{1}{2}\mathbf{u}-\mathbf{u}=-\frac{1}{2}\mathbf{u}$ **Area of triangle** $UWX=\frac{1}{2}\left\|\overrightarrow{UW}\times\overrightarrow{UX}\right\|=\frac{1}{2}\left\|-\frac{1}{2}\mathbf{u}\times\left(\frac{3}{4}\mathbf{v}-\frac{15}{16}\mathbf{u}\right)\right\|$ $=\frac{3}{16}\|\mathbf{u}\times\mathbf{v}\|\qquad \because \mathbf{u}\times\mathbf{u}=\mathbf{0}$ $m=\frac{3}{16}$

	ALTERNATIVE [Not recommended] $\overrightarrow{UX}=\overrightarrow{OX}-\overrightarrow{OU}=\frac{1}{16}\mathbf{u}+\frac{3}{4}\mathbf{v}-\mathbf{u}=\frac{3}{4}\mathbf{v}-\frac{15}{16}\mathbf{u}$ $\overrightarrow{XW}=\overrightarrow{OW}-\overrightarrow{OX}=\frac{1}{2}\mathbf{u}-\left(\frac{1}{16}\mathbf{u}+\frac{3}{4}\mathbf{v}\right)=\frac{7}{16}\mathbf{u}-\frac{3}{4}\mathbf{v}$ **Area of triangle** UWX $=\frac{1}{2}\left\|\overrightarrow{UX}\times\overrightarrow{XW}\right\|$ $=\frac{1}{2}\left\|\left(\frac{3}{4}\mathbf{v}-\frac{15}{16}\mathbf{u}\right)\times\left(\frac{7}{16}\mathbf{u}-\frac{3}{4}\mathbf{v}\right)\right\|$ $=\frac{1}{2}\left\|\frac{21}{64}\mathbf{v}\times\mathbf{u}+\frac{45}{64}\mathbf{u}\times\mathbf{v}\right\|$ $\qquad\because \mathbf{u}\times\mathbf{u}=\mathbf{v}\times\mathbf{v}=\mathbf{0}$ $=\frac{1}{2}\left\|-\frac{21}{64}\mathbf{u}\times\mathbf{v}+\frac{45}{64}\mathbf{u}\times\mathbf{v}\right\|$ $=\frac{1}{2}\left\|\frac{24}{64}\mathbf{u}\times\mathbf{v}\right\|$ $=\frac{12}{64}\left\|\mathbf{u}\times\mathbf{v}\right\|$ $=\frac{3}{16}\left\|\mathbf{u}\times\mathbf{v}\right\|$ $\therefore m=\frac{3}{16}$
10 (i)	$\frac{\mathrm{d}x}{\mathrm{d}t}=v\cos\theta$ $\frac{\mathrm{d}y}{\mathrm{d}t}=v\sin\theta-10t$ $\Rightarrow \frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\frac{\mathrm{d}y}{\mathrm{d}t}}{\frac{\mathrm{d}x}{\mathrm{d}t}}=\frac{v\sin\theta-10t}{v\cos\theta}$
(ii)	For maximum height, $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{v\sin\theta-10t}{v\cos\theta}=0$ $\Rightarrow t=\frac{v\sin\theta}{10}.$

When $t = \dfrac{v\sin\theta}{10}$,

$$y = (v\sin\theta)\left(\frac{v\sin\theta}{10}\right) - 5\left(\frac{v\sin\theta}{10}\right)^2 = \frac{v^2\sin^2\theta}{20}$$

Hence, maximum height $= \dfrac{v^2\sin^2\theta}{20} + 1.5$

$= \dfrac{v^2\sin^2\theta + 30}{20}$ metres.

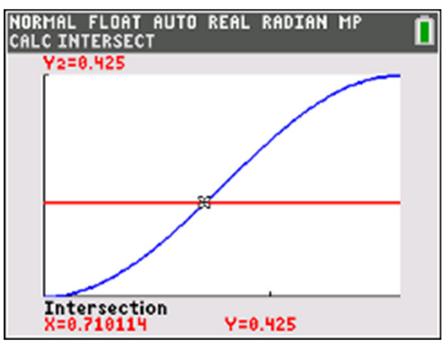
A=30

(iii) Let $v = 20$,

$$0 < \frac{400\sin^2\theta + 30}{20} < 10$$

$0 < \sin^2\theta < 0.425$

Using GC



Hence, $0 < \theta < 0.710$

(iv) At a height of 1.5, $y = 0$:

$0 = (20\sin\theta)t - 5t^2$

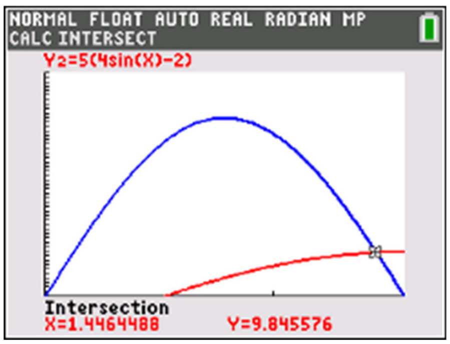
$0 = t\left[20\sin\theta - 5t\right]$

$\therefore t = 0$ or $20\sin\theta - 5t = 0$

(rejected since it is the position of the ball initially)

$\therefore t = 4\sin\theta$

When $t = 4\sin\theta$,

	$x=(20\cos\theta)t$ $=(20\cos\theta)4\sin\theta$ $=80\sin\theta\cos\theta$ $=40\sin2\theta$
(v)	x coordinate of Isabelle at $t=4\sin\theta$ $=5\times(4\sin\theta-2)$ For Isabelle to intercept the ball, $40\sin2\theta=5\times(4\sin\theta-2)$ Using GC  $\theta=1.45$ For the ball not to hit the ceiling, $0<\theta<0.710$. But $\theta=1.45>0.710$, it is not possible for Isabelle to intercept the ball thrown by Nicholas