2024 Year 5 Math Practice Paper 2 Suggested Solutions

<table>
<tr><th>Qn</th><th>Suggested Solution</th></tr>
<tr><td>1</td><td>
Let the price of a desktop monitor, keyboard and mouse be $x, $y and $z respectively.
$x+y+z=317.90$
$0.9x+0.85y+0.9z=282.66$
$0.95x+0.75y+0.8z=283.72$
$\underline{x=219\ ,\ y=69\ ,\ z=29.9}$

Employees of Company B will pay $[0.95(219)+0.75(69)+0.78(29.9)] = $283.12 > $282.66.
No, it will not be more attractive for employees from Company B to purchase all the three items from their own company since they will still have to pay more than the sale by Company A .
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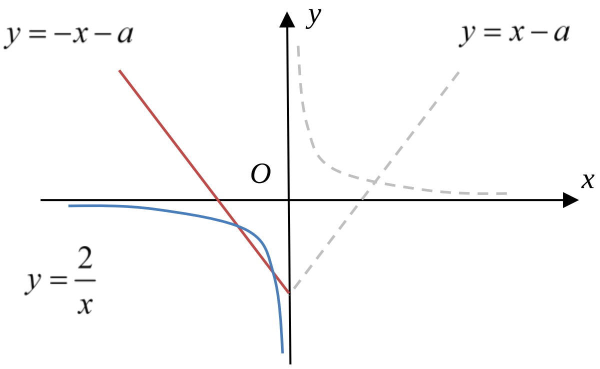
<tr><th>Qn</th><th>Suggested Solution</th></tr>
<tr><td>2</td><td>


For $x<0$, solve $\frac{2}{x}=-x-a$ to determine the x-coordinate of the point of intersection.

$x^2+ax+2=0$
$\left(x+\frac{a}{2}\right)^2+2-\frac{a^2}{4}=0$
$x=-\frac{a}{2}\pm\sqrt{\frac{a^2}{4}-2}=-\frac{a}{2}\pm\sqrt{\frac{a^2-8}{4}}$

Since $a^2>9$, then $a^2-8>0$.

From the graph, the soln is
$-\frac{a}{2}+\sqrt{\frac{a^2-8}{4}}<x<0$ or $x<-\frac{a}{2}-\sqrt{\frac{a^2-8}{4}}$.
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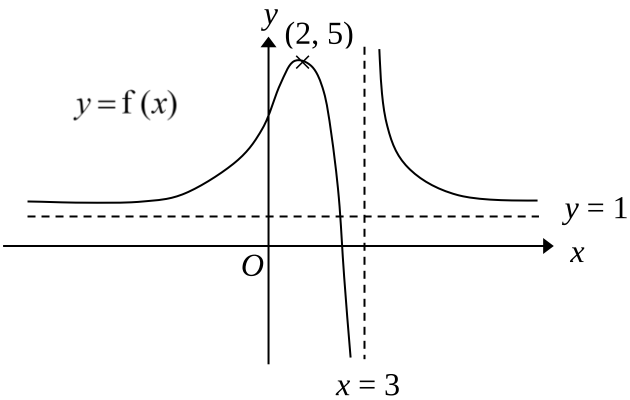
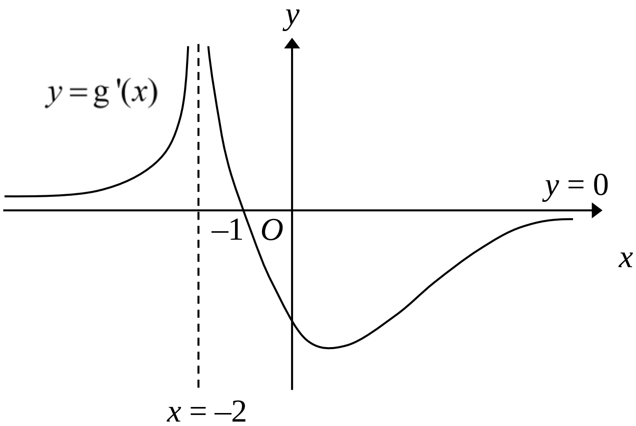
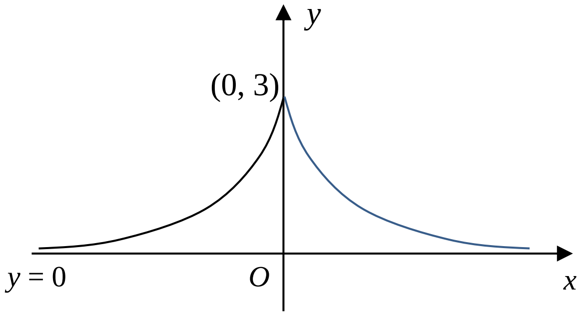
Qn	Suggested Solution
3(a)	
(b)	

Qn	Suggested Solutions
4(a)	$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{3}{(1+t^2)^2}\left(2t\right) = \frac{-6t}{(1+t^2)^2}$ $\frac{\mathrm{d}x}{\mathrm{d}t} = 9t^2$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-6t}{(1+t^2)^2} \times \frac{1}{9t^2} = \frac{-2}{3t(1+t^2)^2}$ There are no real solutions for t when $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$. Curve C has no stationary points. Alternatively, $\frac{-2}{3t(1+t^2)^2} = 0 \Rightarrow -2 = 0$ (Inconsistent) . Thus C has no stationary points.

(b)	

Qn	Suggested Solution
5(a)	$\mathrm{e}^y = 2 + \mathrm{e}^x$ Differentiate with respect to x $\mathrm{e}^y \dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x$ Differentiate with respect to x $\mathrm{e}^y \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)\left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right) + \mathrm{e}^y \dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \mathrm{e}^x$ $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \dfrac{\mathrm{e}^x}{\mathrm{e}^y}$ $\dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \left(\dfrac{\mathrm{d}y}{\mathrm{d}x}\right)^2 = \dfrac{\mathrm{d}y}{\mathrm{d}x}$ (shown) When $x = 0,\ y = \ln 3,\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{3},\ \dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{2}{9}$ $y = \ln 3 + x\left(\dfrac{1}{3}\right) + \dfrac{x^2}{2!}\left(\dfrac{2}{9}\right) + \ldots$ $\quad = \ln 3 + \dfrac{x}{3} + \dfrac{x^2}{9} + \ldots$
(b)	$y = \ln\left(2 + \mathrm{e}^x\right)$ $\quad = \ln\left(2 + 1 + x + \dfrac{x^2}{2!} + \ldots\right)$ $\quad = \ln 3\left[1 + \dfrac{1}{3}\left(x + \dfrac{x^2}{2!} + \ldots\right)\right]$ $\quad = \ln 3 + \ln\left[1 + \dfrac{1}{3}\left(x + \dfrac{x^2}{2!} + \ldots\right)\right]$

	$y=\ln\left(2+e^{x}\right)=\ln 3+\ln\left[1+X\right]$ $\left[\text{let } X=\frac{1}{3}\left(x+\frac{x^2}{2!}+\ldots\right)\right]$ $=\ln 3+\left[X-\frac{1}{2}X^2+\ldots\right]$ (apply std series of $\ln(1+X)$) $=\ln 3+\left[\frac{1}{3}\left(x+\frac{x^2}{2!}+\ldots\right)-\frac{1}{2}\left(\frac{1}{3}\left(x+\frac{x^2}{2!}+\ldots\right)\right)^2+\ldots\right]$ $=\ln 3+\left[\frac{x}{3}+\frac{x^2}{6}-\frac{x^2}{18}+\ldots\right]$ $=\ln 3+\frac{x}{3}+\frac{x^2}{9}+\ldots$ (verified)

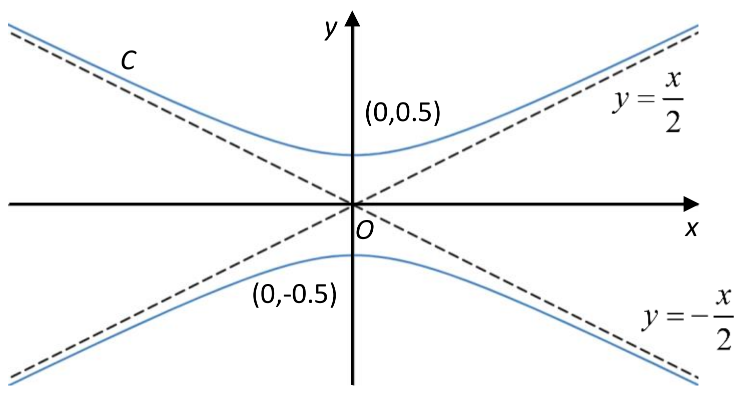
Qn	**Suggested Solutions**
6(a)	(table below)

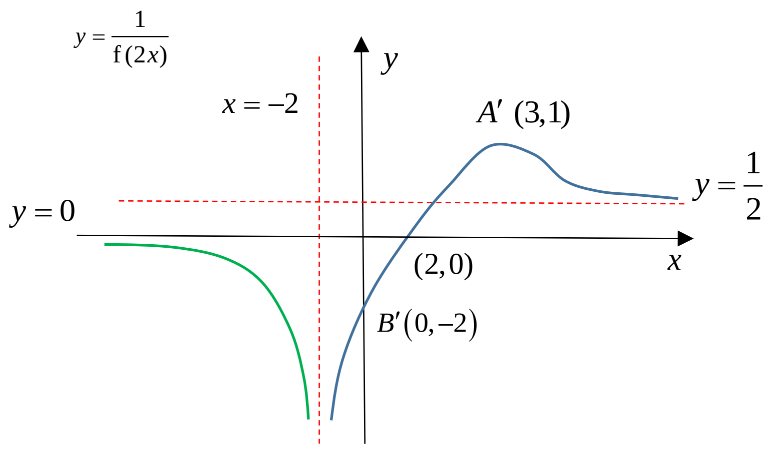
n	**Loan balance at beginning of month (after interest)**	**Loan balance at end of month (after repayment)**
1	847500	$847500-M$
2	$(847500-M)(1.0025)$ $=847500(1.0025)-1.0025M$	$847500(1.0025)-1.0025M-M$
3	$847500(1.0025)^2$ $-1.0025^2M-1.0025M$	$847500(1.0025)^2$ $-M(1.0025^2+1.0025+1)$
$\vdots$	$\vdots$	$\vdots$
n		$847500(1.0025)^{n-1}$ $-M(1.0025^{n-1}+\ldots+1.0025+1)$

Loan at the end of second month
$=847500(1.0025)-1.0025M-M$

Loan at the end of nth month
$=847500(1.0025)^{n-1}-M(1.0025^{n-1}+\ldots+1.0025+1)$
$=847500(1.0025)^{n-1}-M\left(\frac{1-1.0025^n}{1-1.0025}\right)$
$=847500(1.0025)^{n-1}+400M\left(1-1.0025^n\right)$

(b)	For loan to be repaid in 25 years, $847500(1.0025)^{300-1}+400M\left(1-1.0025^{300}\right)=0$ Monthly repayment $=M=4008.9185=\$4008.92$ $\frac{4008.92}{7300}\times 100=54.916\le 55$ Thus the bank will approve Jane's loan.
(c)	Assume that the housing loan is the only debt that Jane has.

<table>
<tr><th>Qn</th><th>Suggested Solution</th></tr>
<tr><td>7(a)(i)</td><td>$4y^2 - x^2 = 1$
Standard eqn of hyperbola:
$\left(\frac{y}{1/2}\right)^2 - x^2 = 1$
To find equations of oblique asymptotes:
as $x, y \to \infty$,
$4y^2 \to x^2$
$y \to \pm\frac{x}{2}$
ie. Equation of asymptotes: $y = \pm\frac{x}{2}$</td></tr>
<tr><td></td><td>
</td></tr>
<tr><td>(ii)</td><td>$(2y)^2 - x^2 = 1 \xrightarrow{y \text{ by } \frac{y}{2}} y^2 - x^2 = 1 \xrightarrow{y \text{ by } (y-1)} (y-1)^2 + x^2 = 1$

1) Scaling parallel to y axis by factor of 2
2) Translation by 1 unit in the positive y- axis direction</td></tr>
<tr><td>(b)</td><td>
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<table>
<tr><th>Qn</th><th>Suggested Solution</th></tr>
<tr><td>8a(i)</td><td>

$$\sum_{r=1}^{n}\left(2^{r+1}+3r-r^2\right)$$

$$=2\sum_{r=1}^{n}2^r+3\sum_{r=1}^{n}r-\sum_{r=1}^{n}r^2$$

$$=2\left[\frac{2\left(2^n-1\right)}{2-1}\right]+3\left[\frac{n}{2}(n+1)\right]$$

$$-\frac{1}{6}(n)(n+1)(2n+1)$$

$$=4\left(2^n-1\right)+\frac{n(n+1)}{6}(9-(2n+1))$$

$$=4\left(2^n-1\right)+\frac{1}{3}n(n+1)(4-n)$$

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<tr><td>a(ii)</td><td>

Replace r with $r+1$

$$\sum_{r=4}^{N}\left(2^r+3r-(r-1)^2\right)$$

$$=\sum_{r+1=4}^{r+1=N}\left(2^{r+1}+3(r+1)-(r+1-1)^2\right)$$

$$=\sum_{r=3}^{N-1}\left(2^{r+1}+3r-r^2\right)+\sum_{r=3}^{N-1}3$$

$$=\sum_{r=1}^{N-1}\left(2^{r+1}+3r-r^2\right)-\sum_{r=1}^{2}\left(2^{r+1}+3r-r^2\right)+3(N-1-3+1)$$

$$=4\left(2^{N-1}-1\right)+\frac{1}{3}(N-1)(N-1+1)(4-(N-1))-(16)+(3N-9)$$

$$=2\left(2^N\right)+\frac{1}{3}N(N-1)(5-N)+3N-29$$

where $B=2, C=\frac{1}{3}, D=3, E=-29$

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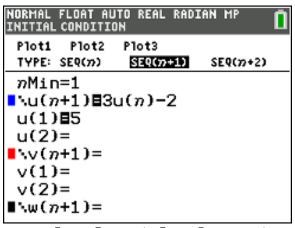
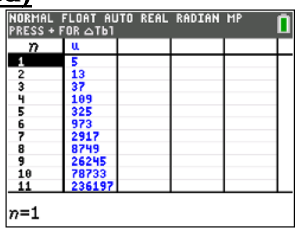
<tr><td>8b
(i, ii)</td><td>

<u>$p=5$</u>

<u>Method 1(GC)</u> <u>(preferred)</u>





<u>Method 2 (algebraic)</u>

$$v_2=3v_1-2=3(5)-2=13$$

$$v_3=3v_2-2=3(13)-2=37$$

$$v_4=3v_3-2=3(37)-2=109$$

… & so on

$\therefore$ The sequence <u>increases & diverges</u>.

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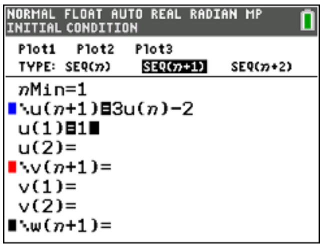
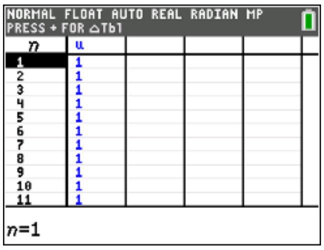
$p = 1$

Method 1(GC) **(preferred)**





Method 2 (algebraic)

$v_2 = 3v_1 - 2 = 3(1) - 2 = 1$

$v_3 = 3v_2 - 2 = 3(1) - 2 = 1$

$v_4 = 3v_3 - 2 = 3(1) - 2 = 1$

… & so on

$\therefore$ It is a **constant** sequence which **converges to 1**.

Qn	Suggested Solution

9(a)

$$\int \frac{5}{(2x-3)(x+1)}\,\mathrm{d}x$$

$$= \int \left(\frac{2}{2x-3} - \frac{1}{x+1}\right)\mathrm{d}x$$

$$= \ln|2x-3| - \ln|x+1| + C \text{ where } C \text{ is an arbitrary constant}$$

(b)

$$\int_{\sqrt{2}}^{2} \frac{\sqrt{x^2-1}}{x}\,\mathrm{d}x$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sqrt{\sec^2\theta - 1}}{\sec\theta}(\sec\theta\tan\theta)\,\mathrm{d}\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \tan^2\theta\,\mathrm{d}\theta$$

$$= \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (\sec^2\theta - 1)\,\mathrm{d}\theta$$

$$= [\tan\theta - \theta]_{\frac{\pi}{4}}^{\frac{\pi}{3}}$$

$$= \left(\tan\frac{\pi}{3} - \frac{\pi}{3}\right) - \left(\tan\frac{\pi}{4} - \frac{\pi}{4}\right)$$

$$= \sqrt{3} - 1 - \frac{\pi}{12}$$

$$\mathrm{d}x = (\sec\theta\tan\theta)\,\mathrm{d}\theta$$

When $x = 2,\ \sec\theta = 2$

$$\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$

When $x = \sqrt{2},\ \sec\theta = \sqrt{2}$

$$\cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}$$

(c)

$$\int \frac{x}{\sqrt{1-m^2x^2}}\,\mathrm{d}x = -\frac{1}{2m^2}\int -2m^2x\left(1-m^2x^2\right)^{-\frac{1}{2}}\,\mathrm{d}x$$

$$= -\frac{1}{2m^2}\frac{\left(1-m^2x^2\right)^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + c$$

$$= -\frac{1}{m^2}\sqrt{1-m^2x^2} + c$$

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$$\int\left(\sin^{-1} mx\right)\frac{x}{\sqrt{1-m^2x^2}}\ \mathrm{d}x$$
$$=\left(-\frac{\sin^{-1} mx}{m^2}\sqrt{1-m^2x^2}\right)-\int -\frac{1}{m}\,\mathrm{d}x$$
$$=\left(-\frac{\sin^{-1} mx}{m^2}\sqrt{1-m^2x^2}\right)+\frac{x}{m}+c$$

$$\boxed{\begin{array}{ll} u=\sin^{-1} mx & \dfrac{\mathrm{d}v}{\mathrm{d}x}=\dfrac{x}{\sqrt{1-m^2x^2}} \\ \dfrac{\mathrm{d}u}{\mathrm{d}x}=\dfrac{m}{\sqrt{1-m^2x^2}} & v=-\dfrac{1}{m^2}\sqrt{1-m^2x^2} \end{array}}$$

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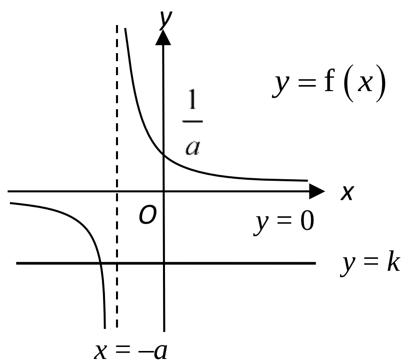
<tr><td>10(a)(i)</td><td>



Every horizontal line $y = k$, $k \in \mathbb{R}$ cuts the graph of f at most once hence f is one-one and f^{-1} exist.

Let $y=\dfrac{1}{x+a}$

$\Rightarrow xy+ay=1$

$\Rightarrow xy=1-ay$

$\Rightarrow x=\dfrac{1-ay}{y}=\dfrac{1}{y}-a$

Therefore $\mathrm{f}^{-1}: x \mapsto \dfrac{1}{x}-a, \quad x\in\mathbb{R}, \quad x\neq 0.$

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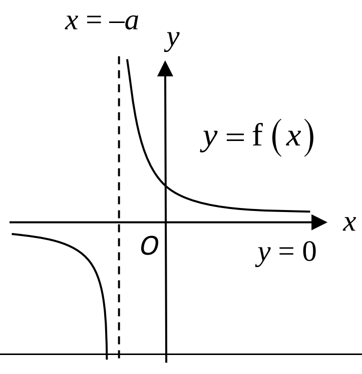
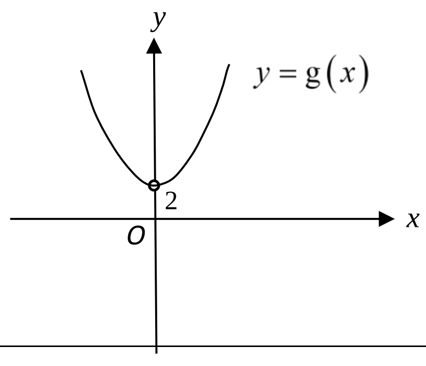
<tr><td>(a)(ii)</td><td>

Since $\mathrm{R_f}=\mathbb{R}\setminus\{0\}$ and $\mathrm{D_g}=\mathbb{R}$

$\mathrm{R_f}\subseteq \mathrm{D_g}$, thus gf exists.





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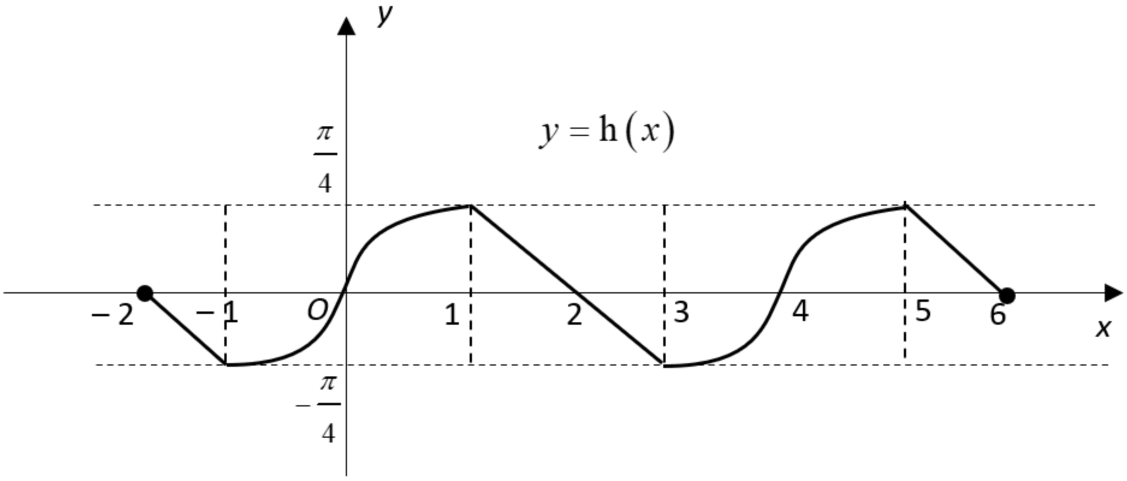
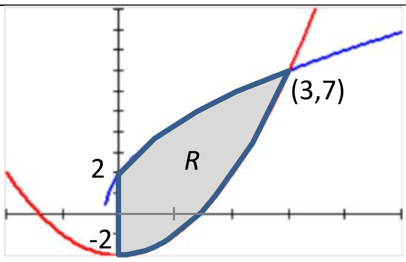
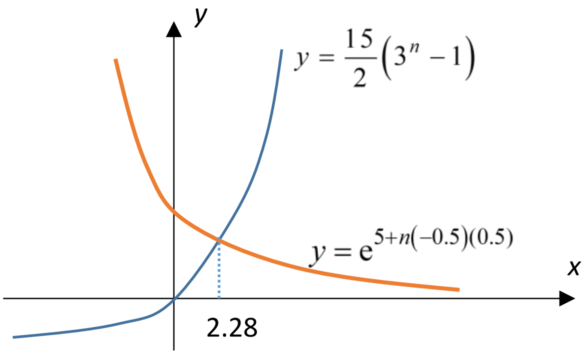
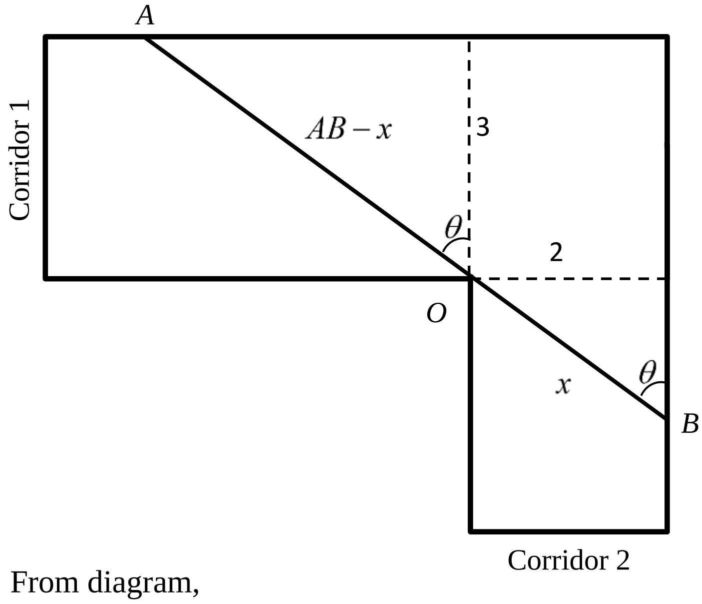
	Using the graphs of f and g $\mathrm{D_f} \rightarrow R_\mathrm{f} \rightarrow R_\mathrm{gf}$ $\mathbb{R}\setminus\{-a\} \rightarrow \mathbb{R}\setminus\{0\} \rightarrow (2,\infty)$ $\mathrm{R_{gf}} = (2,\infty)$
b(i)	
b(ii)	$\int_{-2}^{6} \mathrm{h}(x)\ \mathrm{d}x = 0$

Qn	Suggested Solution
11(a)	$\sqrt{15x+4} = x^2 - 2$ $15x+4 = (x^2-2)^2$ $15x+4 = x^4 - 4x^2 + 4$ $x(x^3 - 4x - 15) = 0$ $x^3 - 4x - 15 = 0$ $\quad \because x \neq 0$ (does not satisfy original eqn) $(x-3)(x^2+3x+5) = 0$ Since $x^2+3x+5 = \left(x+\frac{3}{2}\right)^2 + \frac{11}{4} > 0$ for all x, $\therefore x = 3, y = 7$ $\therefore (3,7)$ is the only point of intersection between the 2 curves.
(b)	 Area of the region R

	$=\int_0^3 \sqrt{15x+4}-(x^2-2)\,\mathrm{d}x$ $=\left[\dfrac{(15x+4)^{\frac{3}{2}}}{\frac{3}{2}(15)}-\dfrac{x^3}{3}+2x\right]_0^3$ $=\left[\dfrac{2}{45}(45+4)^{\frac{3}{2}}-\dfrac{27}{3}+6-\dfrac{2}{45}(4)^{\frac{3}{2}}\right]$ $=\left[\dfrac{686}{45}-3-\dfrac{16}{45}\right]$ $=\dfrac{551-16}{45}$ $=\dfrac{535}{45}$ $=\dfrac{107}{9}$ unit^2
11(c)	Volume of revolution $=\pi\left[\int_{-2}^{7} y+2\,\mathrm{d}y-\dfrac{1}{225}\int_2^7\left(y^2-4\right)^2\,\mathrm{d}y\right]$ $=91.7$ units^3 **<u>Alternative (markers' reference only)</u>** Volume of revolution $=\pi\left[\int_{-2}^{7} y+2\,\mathrm{d}y-\dfrac{1}{225}\int_2^7\left(y^2-4\right)^2\,\mathrm{d}y\right]$ $=\pi\left[\left[\dfrac{y^2}{2}+2y\right]_{-2}^{7}-\dfrac{1}{225}\int_2^7 y^4-8y^2+16\,\mathrm{d}y\right]$ $=\pi\left[\dfrac{81}{2}-\dfrac{1}{225}\left[\dfrac{y^5}{5}-\dfrac{8y^3}{3}+16y\right]_2^7\right]$ $=\pi\left[\dfrac{81}{2}-\dfrac{1}{225}\left(\dfrac{38381}{15}-\dfrac{256}{15}\right)\right]$ $=\pi\left[\dfrac{81}{2}-\dfrac{305}{27}\right]$ $=\dfrac{1577}{54}\pi$ units^3

Qn	Suggested Solutions
12(a)	$S_n = 3n(n+2)$ $u_n = S_n - S_{n-1}$ $\quad = 3n(n+2) - 3(n-1)(n+1)$ $\quad = 6n+3$ $u_n - u_{n-1} = 6n+3-\left(6(n-1)+3\right)$ $\qquad\qquad = 6n+3-6n+3$ $\qquad\qquad = 6 \ (\text{constant})$ Since the difference between two consecutive terms is a constant, the series is an arithmetic progression. The common difference is 6.
(b)	$v_1 = u_2 = 6(2)+3 = 15$ $v_2 = u_7 = 6(7)+3 = 45$ common ratio, $r = \dfrac{45}{15} = 3$ $v_3 = 15(3)^2 = 135$ The m^{th} term of the series in (i), $135 = 6(m)+3$ $m = \dfrac{135-3}{6} = 22$ Since $r = 3$ does not lie within $-1 < r < 1$, the sum to infinity of v_n does not exist.
(c)	common ratio $= \dfrac{w_n}{w_{n-1}}$ $= \dfrac{e^{5+nx(x+1)}}{e^{5+(n-1)x(x+1)}}$ $= \dfrac{e^5 e^{nx(x+1)}}{e^5 e^{(n-1)x(x+1)}}$ $= e^{nx(x+1)-(n-1)x(x+1)}$ $= e^{x(x+1)}$ For the series to converge, $\left\lvert e^{x(x+1)} \right\rvert < 1$, $x(x+1) < 0$ The range of values of x is $-1 < x < 0$.

12(d)	Sum of first n terms of v_n, S_{v_n} $= \frac{15\left(3^n - 1\right)}{3-1} = \frac{15}{2}\left(3^n - 1\right)$ $S_{v_n} > w_n$ using $x = -0.5$, $\frac{15}{2}\left(3^n - 1\right) > \mathrm{e}^{5+n(-0.5)(0.5)}$  From the graph, the least value of n is 3. **Alternative (table method)**

Qn	Suggested Solution
13(a)	Let OB be x m.  From diagram, $\cos\theta=\dfrac{3}{AB-x}$ and $\sin\theta=\dfrac{2}{x}$. So we have $AB=\left(AB-x\right)+x=\dfrac{3}{\cos\theta}+\dfrac{2}{\sin\theta}$, where $\alpha=3, \beta=2$.
(b)	Differentiating the result in part (a) with respect to t, $\dfrac{\mathrm{d}\left(AB\right)}{\mathrm{d}t}=\left(3\sec\theta\tan\theta-2\operatorname{cosec}\theta\cot\theta\right)\left(\dfrac{\mathrm{d}\theta}{\mathrm{d}t}\right)\cdots(*)$ When $OB=\dfrac{2}{\sin\theta}=4,\ \ \theta=\frac{1}{6}\pi.$ Subst $\theta=\frac{1}{6}\pi$ and $\dfrac{\mathrm{d}\theta}{\mathrm{d}t}=-0.1$ into (*): Using GC, $\dfrac{\mathrm{d}\left(AB\right)}{\mathrm{d}t}=0.493$ m/s (to 3 s.f.) $\therefore$ Rate of increase in AB when $OB=4$ is $0.493\,\text{m/s}$
(c)	$p\leq AB$ for all possible lengths p and angles θ, i.e. max $p=$ min AB. $AB=\dfrac{3}{\cos\theta}+\dfrac{2}{\sin\theta}.$ $\dfrac{\mathrm{d}(AB)}{\mathrm{d}\theta}=\dfrac{3\sin\theta}{\cos^2\theta}-\dfrac{2\cos\theta}{\sin^2\theta}=\dfrac{3\sin^3\theta-2\cos^3\theta}{\cos^2\theta\sin^2\theta}$ At min AB, $\dfrac{\mathrm{d}\left(AB\right)}{\mathrm{d}\theta}=0$ $\Rightarrow 3\sin^3\theta=2\cos^3\theta$ $\tan^3\theta=\dfrac{2}{3}\Rightarrow\tan\theta=\left(\dfrac{2}{3}\right)^{\frac{1}{3}}$ $\therefore\theta=\tan^{-1}\left(\dfrac{2}{3}\right)^{\frac{1}{3}}.$

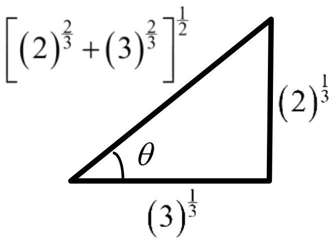
θ	0.717	$\tan^{-1}\left(\frac{2}{3}\right)^{\frac{1}{3}}$	0.719
$\frac{dAB}{d\theta}$	–0.0216	0	0.0205
Tangent	\	-	/

$\therefore AB$ is a minimum when $\theta = \tan^{-1}\left(\frac{2}{3}\right)^{\frac{1}{3}}$.

Alternative

$$\frac{d(AB)}{d\theta} = \frac{3\sin^3\theta - 2\cos^3\theta}{\cos^2\theta\sin^2\theta} = \frac{3\cos^3\theta\left(\tan^3\theta - \frac{2}{3}\right)}{\cos^2\theta\sin^2\theta}$$

θ	$\left(\tan^{-1}\left(\frac{2}{3}\right)^{\frac{1}{3}}\right)^{-}$	$\tan^{-1}\left(\frac{2}{3}\right)^{\frac{1}{3}}$	$\left(\tan^{-1}\left(\frac{2}{3}\right)^{\frac{1}{3}}\right)^{+}$
$\frac{dAB}{d\theta}$	negative	0	positive



Thus we have

$$AB = \frac{3\left[(2)^{\frac{2}{3}} + (3)^{\frac{2}{3}}\right]^{\frac{1}{2}}}{(3)^{\frac{1}{3}}} + \frac{2\left[(2)^{\frac{2}{3}} + (3)^{\frac{2}{3}}\right]^{\frac{1}{2}}}{(2)^{\frac{1}{3}}}$$

$$= \left[(2)^{\frac{2}{3}} + (3)^{\frac{2}{3}}\right]^{\frac{1}{2}}\left[(3)^{\frac{2}{3}} + (2)^{\frac{2}{3}}\right]$$

$$= \left[(2)^{\frac{2}{3}} + (3)^{\frac{2}{3}}\right]^{\frac{3}{2}}$$

max $p = \left[(2)^{\frac{2}{3}} + (3)^{\frac{2}{3}}\right]^{\frac{3}{2}}$, where $k = \frac{2}{3}$.