Superposition

1 Superposition

When 2 particles collide, they bounce off each other or sometimes stick together. However, when waves meet they interact in a very unique way called the principle of *superposition*:

> When two waves of the *same kind meet* at a point,
> the *resultant displacement* at the point is a
> *vector sum* of their individual displacements.

Fig. 1.1 shows how 2 wave pulses on a string interact. Notice how they pass right through each other and thereafter continue their journey as if nothing has happened. Such behaviour distinguishes waves from particles.

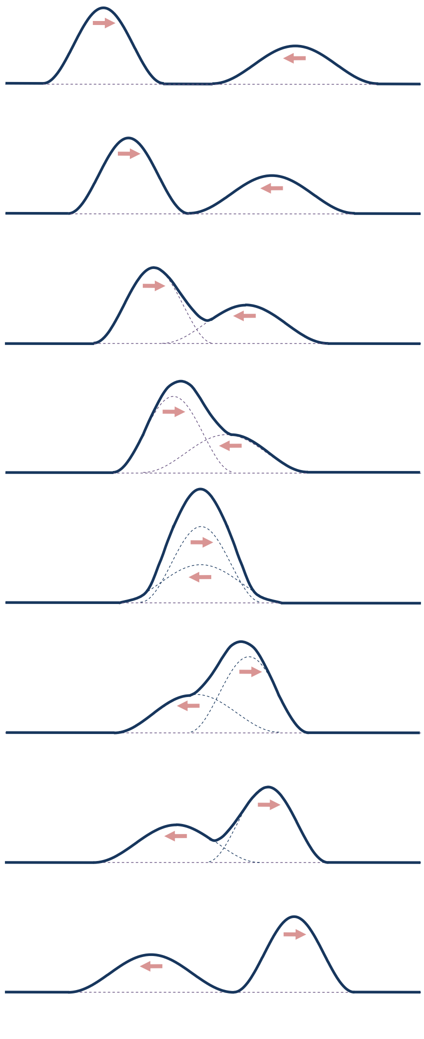


Fig. 1.1a

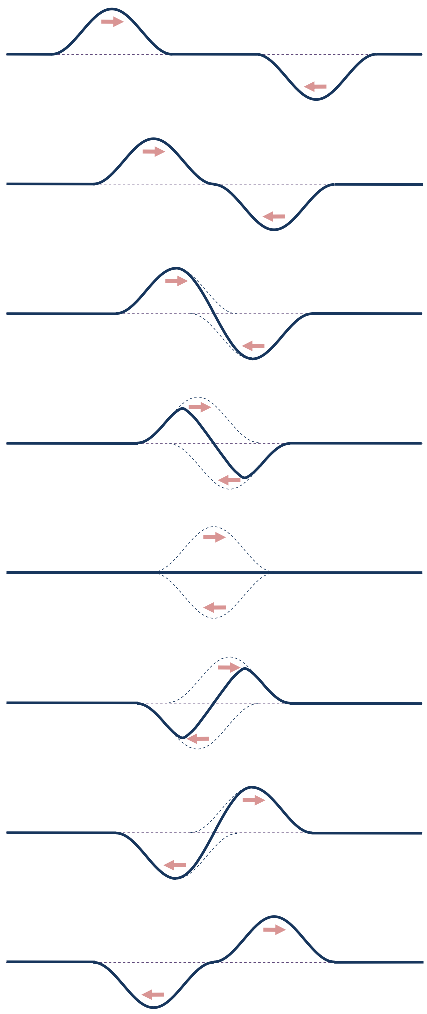


Fig. 1.1b

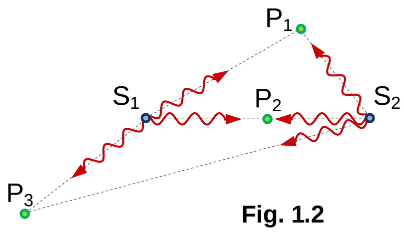


Fig. 1.2

Waves from two sources do not meet by travelling in opposite directions only. Also, instead of an interaction of pulses, the waves can be continuous. Fig. 1.2 shows 2 sources S_1 & S_2 continuously emitting waves meeting at points P_1, P_2 & P_3 among many other possible points. For the rest of this topic, we will keep things simple by considering the sources and meetings points to be all on the same plane.

When two waves of the *same kind meet* at a point, the *resultant displacement* at the point is a *vector sum* of their individual displacements.

Superposition of Water Waves

S_1 & S_2 In Phase

In Fig. 1.3, 1.4 & 1.5, sources S_1 & S_2 oscillate in phase and continuously radiate water waves of the same frequency. P is a selected point where we consider how the waves meet and superpose. To simplify things, we assume that as the waves travel from S_1 & S_2 to P, the respective amplitudes A_1 and A_2 do not decrease. $| S_2P - S_1P |$ is called the *path difference* (*PD*) which is the difference in distance travelled by the waves from S_1 and S_2. We shall see that *PD* is one factor in determining the result of superposition.

PD, *path difference* is a factor determining if waves from 2 sources meet in phase or otherwise.

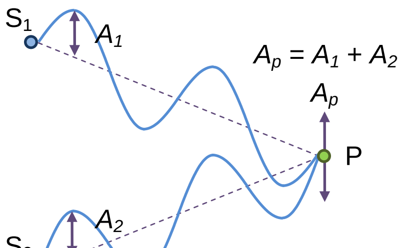


Fig. 1.3

$PD = S_2P - S_1P = 2\lambda - 2\lambda = 0$

Constructive Interference

In Fig. 1.3, P is a point equidistant from the sources. As the waves *originated in phase* and travelled the *same distance* to meet at P, they will meet in phase i.e. the trough of one wave will always meet the trough of the other and similarly for crests.

The result of superposition is an oscillation at P with amplitude $= A_1 + A_2$.

When 2 wave trains meet *in phase continuously*, the resulting *amplitude* is the sum of the individual wave amplitudes and the situation is called *constructive interference*.

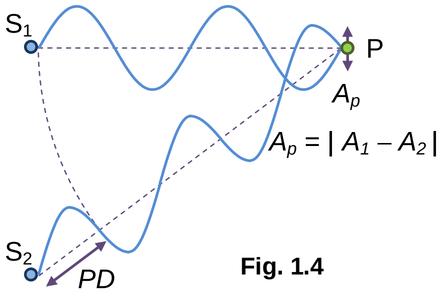


Fig. 1.4

$PD = S_2P - S_1P = 2½\lambda - 2\lambda = ½\lambda$

Destructive Interference

In Fig. 1.4, waves originated from S_1 and S_2 *in phase* but waves from S_2 always travelled an extra $½\lambda$ to reach P. The *different path lengths* cause the waves to always meet at P in anti-phase(crest meet trough), resulting in cancellation of displacements by the principle of superposition.

The amplitude at P is $= | A_1 - A_2 |$ which equals zero if the individual amplitudes are equal.

When 2 wave trains meet *in anti-phase continuously*, the resulting *amplitude* is the difference between the individual wave amplitudes and the situation is called *destructive interference*.

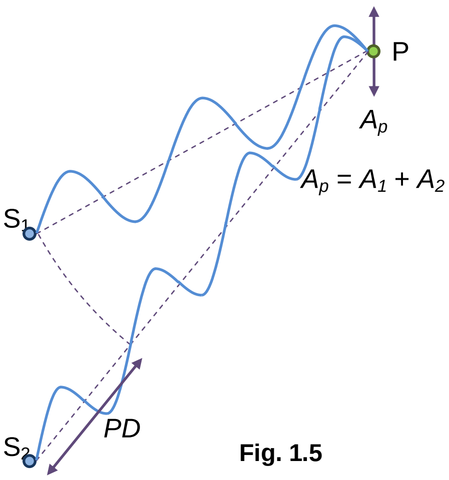


Fig. 1.5

$PD = S_2P - S_1P = 3½\lambda - 2½\lambda = \lambda$

Constructive Interference

In Fig. 1.5, S_1 and S_2 are in phase again but the *PD* is now λ compared to $½\lambda$ in Fig. 1.4.

Now the result of superposition is an oscillation at P with amplitude $= A_1 + A_2$.

Whenever the amplitudes add, we say that the waves from S_1 and S_2 interfere *constructively* and whenever the amplitudes subtract, we say they interfere *destructively*. There is no special term for situations in between constructive and destructive interference.

To generalise:

For sources S_1 and S_2 in phase, when
- $PD = n\lambda$, waves meet in phase $\leftrightarrow$ constructive interference
- $PD = (n + ½)\lambda$, waves meet in anti-phase $\leftrightarrow$ destructive interference

where n is integer.

When 2 sources are in phase, the conditions for constructive or destructive interference are shown in the box on the left.

S_1 & S_2 In Anti-Phase

The difference between Fig. 1.6, 1.7 & 1.8 compared to Fig. 1.3, 1.4 & 1.5 respectively is that the sources S_1 & S_2 now oscillate in anti-phase instead. We shall see that the *initial phase difference* when the waves are produced is the other factor that will also determine the result of superposition.

Initial phase difference between 2 wave sources is another factor determining if waves from 2 sources meet in phase or otherwise.

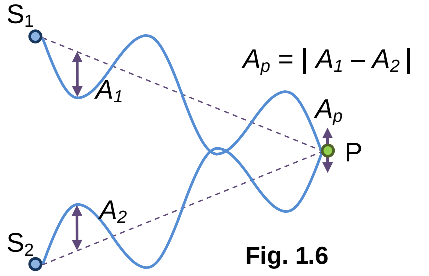


Fig. 1.6

$PD = S_2P - S_1P = 2\lambda - 2\lambda = 0$

Destructive Interference

In Fig. 1.6, the initial phase difference between the waves generated at S_1 and S_2 is maintained as the waves travelled to meet at P because the paths travelled have the same distance. Hence the waves meet in anti-phase, resulting in cancellation of displacements.

Resulting amplitude $A_p = |\,A_1 - A_2\,|$.

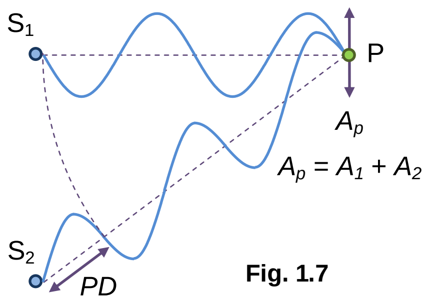


Fig. 1.7

$PD = S_2P - S_1P = 2\tfrac{1}{2}\lambda - 2\lambda = \tfrac{1}{2}\lambda$

Constructive Interference

In Fig. 1.7, waves from S_1 and S_2 have initial phase difference 180° or π rad. Recall in the topic Waves that λ corresponds to a phase difference of 360° or 2π rad. The waves from S_2 travelled an extra distance of $\tfrac{1}{2}\lambda$ to meet waves from S_1 thus introducing an extra 180° phase difference and allowing the waves to meet in phase.

Resulting amplitude $A_p = A_1 + A_2$.

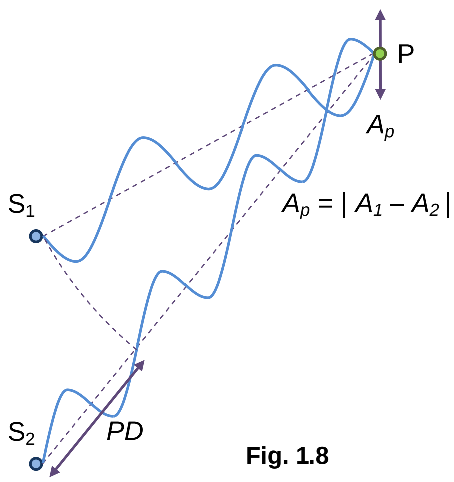


Fig. 1.8

$PD = S_2P - S_1P = 3\tfrac{1}{2}\lambda - 2\tfrac{1}{2}\lambda = \lambda$

Destructive Interference

In Fig. 1.8, the *PD* is now λ compared to $\tfrac{1}{2}\lambda$ in Fig. 1.7. Due to the PD, the extra phase difference introduced is 360° or one cycle, which *in effect* is equivalent to a phase difference of 0. Taking into account the initial phase difference of 180° due to the sources, the waves would meet in anti-phase.

Resulting amplitude $A_p = |\,A_1 - A_2\,|$.

Again, to generalise:

For sources S_1 and S_2 in anti-phase, when $PD = n\lambda$, waves meet in anti-phase $\leftrightarrow$ destructive interference $PD = (n + \tfrac{1}{2})\lambda$, waves meet in phase $\leftrightarrow$ constructive interference where n is integer.

When 2 sources are in anti-phase, the conditions for constructive or destructive interference are shown in the box on the left.

Ripple Tank

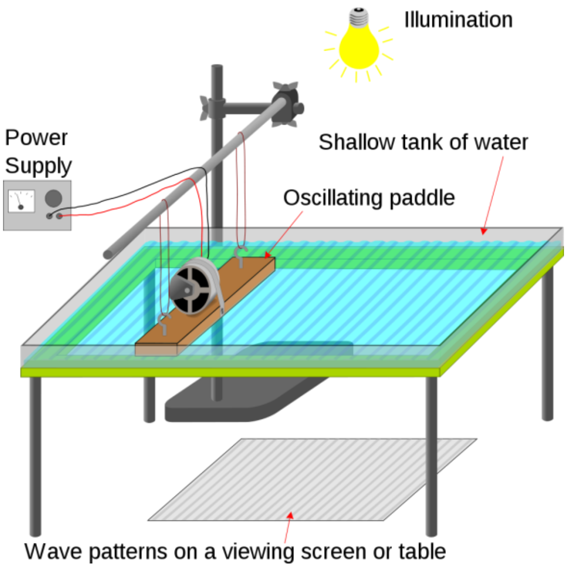


Fig. 1.9

A ripple tank can be used to create circular or plane waves depending on set-up. Fig. 1.9 shows plane waves being created.

To see the interference of circular waves from two point sources S_1 and S_2, the paddle will be suspended above the water surface and 2 vertical rods attached to the paddle. As the paddle oscillates, the rods will dip in and out of the water surface, creating circular ripples.

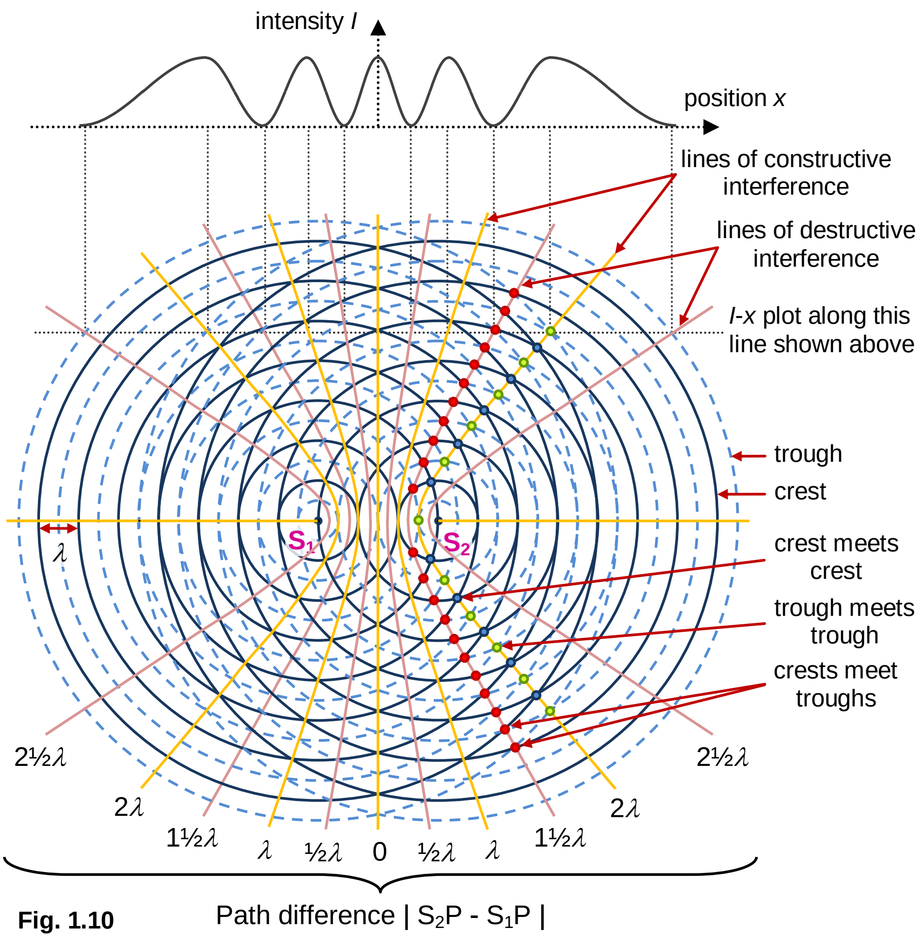


Fig. 1.10

When waves of the same λ from 2 sources meet, an *interference pattern* made up of lines of constructive and destructive interference is formed.

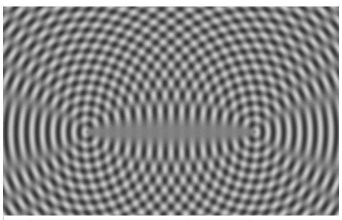


Fig. 1.11

Fig. 1.10 and 1.11 show how the water waves from 2 *in phase* sources produce an *interference pattern* made up of lines of constructive and destructive interference. For the points where crest meets trough, crest meets crest or trough meets trough, the path difference can be worked out by counting the difference in the number of rings away from each source.

2 Coherence & Interference

Coherence

Two *oscillations* are coherent if they have a *constant* phase difference.

Two *oscillations* are coherent if they have a *constant* phase difference.

Coherent

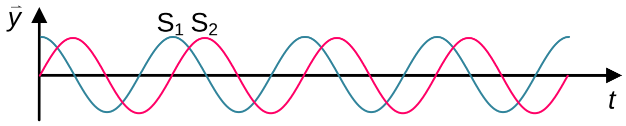


Fig. 2.1

The 2 oscillations have a phase difference that is constant in time.

Incoherent

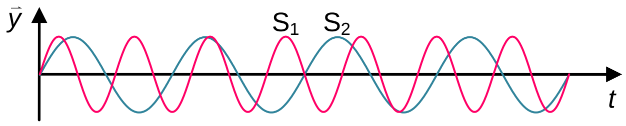


Fig. 2.2

The 2 oscillations have different frequencies and hence a variable phase difference.

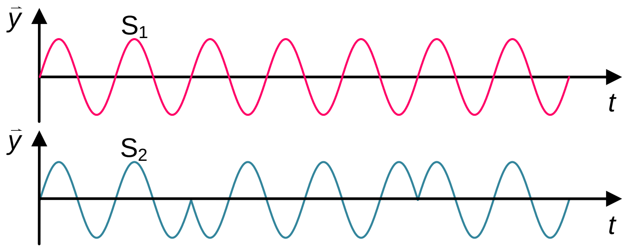


Fig. 2.3

The 2 oscillations have same frequency but a variable phase difference.

Fig. 2.1 & 2.2 show that coherent oscillations must have the same frequency while Fig. 2.3 shows that same frequency does not imply coherence.

For 2 oscillations, *coherence* ⇒ *same frequency*.

But *same frequency* ⇏ *coherence*

If the sources S_1 and S_2 in the previous section were incoherent like in Fig. 2.2 and 2.3, the variable phase difference means that for *every* point P, the interference may be constructive at one moment, destructive another moment and in between constructive and destructive at another moment. Thus there will not be a stable interference pattern where there are *fixed* lines that are *always* at constructive interference or *fixed* lines that are *always* at destructive interference like in Fig. 1.10.

On pages 2 and 3, the sources S_1 and S_2 are either in phase or in anti-phase which means phase difference of 0° and 180° respectively. As far as producing a *stable interference pattern* is concerned, the phase difference can be any value as long as it is constant. Of course the conditions at the bottom of pages 2 and 3 for constructive and destructive interference would not apply for phase difference between sources that is neither 0° nor 180°.

Coherence is necessary for a *stable* interference pattern.

Interference

To tie up all the key concepts:

- When waves meet at a point they interfere by superposition.
- *Constructive* and *destructive interference* refer to the superposition result of *maximum and minimum amplitudes* respectively.
- When sources are *coherent*, the amplitudes of constructive and destructive interference will be *stable*.
- When waves from coherent sources interfere *region-wide*, a stable *interference pattern* is formed.

Interference is the interaction of waves by superposition to form regions where the amplitudes add or subtract.

3 Stationary Waves

> A stationary wave is formed by superposition of
> two coherent waves
> of the same type and
> same amplitude
> travelling in opposite directions.

Examples of *types* - both sound waves or both EM waves.

Fig. 3.1 to 3.5 show how at intervals of $T/8$, the 2 opposite travelling waves superpose to produce a resultant wave that has some points which will not oscillate at all times. These points are called *nodes* (N). On the other hand, between the nodes are the *anti-nodes* (AN) which are points with maximum amplitude of oscillation.

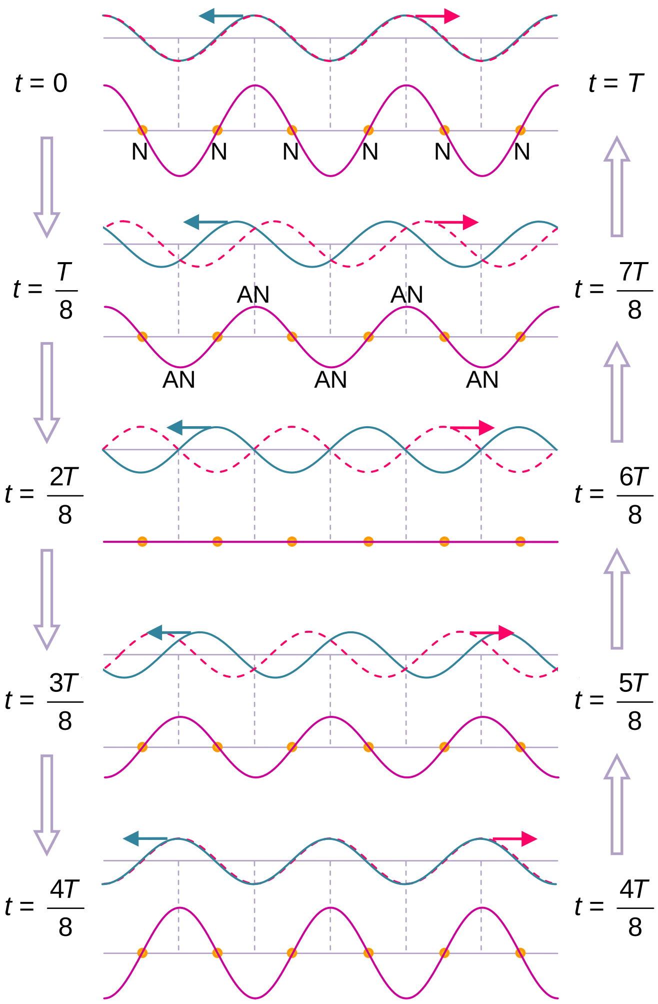


Fig. 3.1

Fig. 3.2

Fig. 3.3

Fig. 3.4

Fig. 3.5

[Link to video of formation of standing waves]

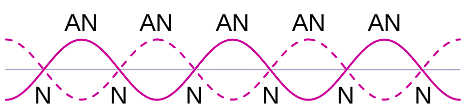


Fig. 3.6

Stationary waves are usually represented with a solid line and a dotted line which mark out the 2 extreme positions of the oscillations in the wave.

Characteristics of stationary waves:

1. Distance between consecutive nodes or anti-nodes is $\lambda/2$.
2. Oscillations between consecutive nodes are always in phase.
3. Oscillations on the 2 sides of each node are always in anti-phase.

A stationary wave is formed by superposition of two coherent waves of the same type and same amplitude travelling in opposite directions.

Nodes are points in a stationary wave that have *minimum* amplitudes while anti-nodes are points which have *maximum* amplitudes *at all times*.

Standing wave characteristics:

1 Distance between consecutive nodes or anti-nodes is $\lambda/2$.

2 Oscillations between consecutive nodes are always in phase.

3 Oscillations on the 2 sides of each node are always in anti-phase.

Fixed L Variable f

In Fig. 3.7 a motor drives a rod to oscillate vertically and sends waves down the string. These waves repeatedly reflect off the fixed end as well as the end fixed to the oscillating rod. At *certain* frequencies f_0, f_1, f_2 etc, the opposite waves superpose to form one, two or more points of constructive interference called anti-nodes. Though the rod oscillates, its amplitude is small compared to that of the anti-nodes and it is effectively a node. At other frequencies, the many reflected waves could not meet consistently at fixed points to produce constructive or destructive interference so resulting in an unstable waveform. The stable stationary waveforms are called *resonant modes* and the lowest frequency mode is called fundamental while subsequent higher frequency modes are called *overtones*.

As the motor's frequency is increased the wavelength is decreased according to $v = f\lambda$ where v is fixed for the same string at the same tension. Recall that distance between consecutive nodes is $\lambda/2$ so λ can be expressed in terms of the easily measured string length L. We can thus show that there is a certain pattern to the resonant frequencies.

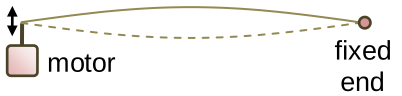
A string of fixed length and tension can only have standing waves of certain frequencies.

These standing waveforms are called *resonant modes*.

The lowest frequency mode is called the *fundamental* while higher frequency modes are called *overtones*.

The set of resonant frequencies follow a pattern.

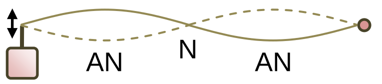
Fundamental, f_1



$L = \lambda_1/2$ and $v = f_1\lambda_1$

$\therefore f_1 = v/2L$

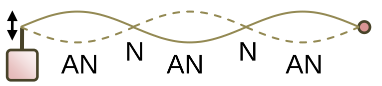
1st overtone, f_2



$L = \lambda_2$ and $v = f_2\lambda_2$

$\therefore f_2 = v/L$

2nd overtone, f_3



$L = 3\lambda_3/2$ and $v = f_3\lambda_3$

$\therefore f_3 = 3v/2L$

Fig. 3.7 Thus, $f_2 = 2f_1$, $f_3 = 3f_1$, . . . , $f_n = nv/2L$ for this set-up

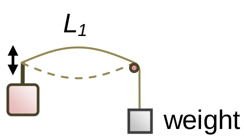
Fixed f Variable L

The motor's frequency f_m is fixed and a fixed weight provides a constant tension, thus resulting in constant v, f and λ. As length L is increased slowly from a small value, resonant modes are encountered at length L_1, L_2, L_3 etc.

A string with variable length and constant tension has variable resonant frequencies.

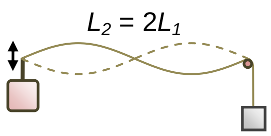
As length increases, the fundamental and overtone frequencies decrease.

String of length L_1



Fundamental frequency is f_m

String of length L_2

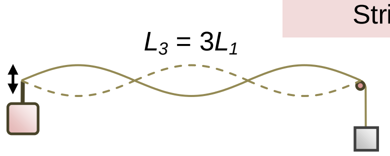


1st overtone frequency is f_s

$\Rightarrow$ from variable f case above:

Fundamental frequency is $f_m/2$

String of length L_3



2nd overtone frequency is f_m

$\Rightarrow$ from variable f case above:

1st overtone frequency is $2f_m/3$

Fundamental frequency is $f_m/3$

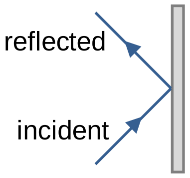
Fig. 3.8

This case shows strings of different L have different fundamental frequencies.

Reflection of Waves

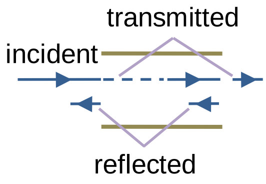
Waves get reflected all the time. Reflection occurs whenever there is a change in the properties of the medium (Fig. 3.9). Sometimes the reflection is almost 100% but sometimes it is partial with a portion of the incident energy being transmitted.

Reflection of waves occurs whenever there is a change in the property of the medium.



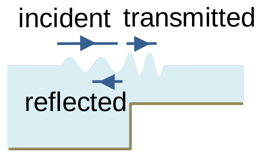
Light reflects off shiny metal plate. Medium change - density, conductivity.

Fig. 3.9a



Sound reflects at pipe openings. Medium change - dimension of vibrating air mass.

Fig. 3.9b



Water waves reflect at step. Medium change - depth of water.

Fig. 3.9c

Stationary Sound Wave

Fixed *L* Variable *f*

Sound waves can be reflected off both a closed end as well as the open end of a tube. At the closed end, air cannot oscillate so it must be a displacement node which is a pressure anti-node (recall from Waves section 4 that positions of zero longitudinal displacement have largest deviations from atmospheric pressure while the positions of greatest displacement are at atmospheric pressure). The pressures at open ends tend to equalise with atmospheric pressure, so the open ends are always displacement anti-nodes (AN). It turns out that the exact positions of the *displacement* ANs at the ends are always at a small distance c outside the ends of the pipe. c is called the *end correction*.

Just like a string with fixed length, v is constant so increasing f leads to decreasing λ. Gradually increasing the speaker frequency will lead to a series of rise and fall in loudness from the tube. The loudness maxima correspond to the resonant modes when there are ANs at the tube openings.

A tube (closed end or open ends) of fixed length has a specific set of resonant frequencies.

Whenever a resonant mode is set up, a loud sound can be heard from the tube.

A little distance (end correction) outside the open end must be a *displacement* anti-node while the closed end must be a displacement node. The *displacement* nodes are *pressure* anti-nodes and displacement anti-nodes are pressure nodes.

<u>**Closed End Tube**</u> <u>**Open Ends Tube**</u>

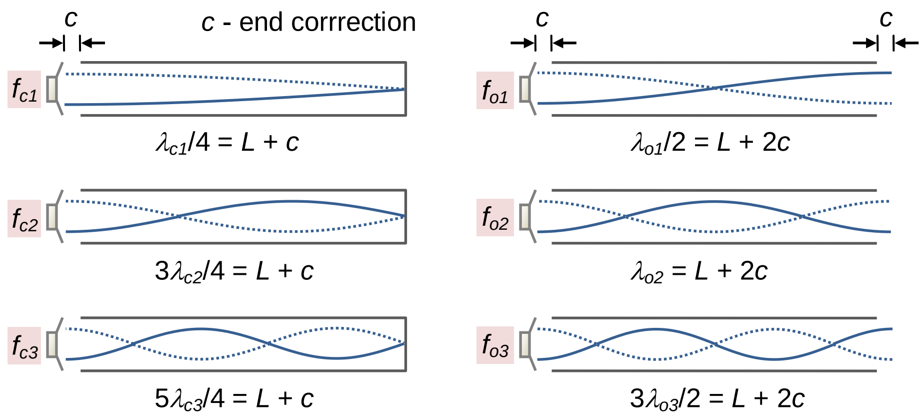


Fig. 3.10

From the above, deduce that

$\lambda_{c2} = \lambda_{c1}/3 \;\; \therefore \; f_{c2} = 3f_{c1}$

$\lambda_{c3} = \lambda_{c1}/5 \;\; \therefore \; f_{c3} = 5f_{c1}$

. . .

$\lambda_{cn} = \lambda_{c1}/(2n-1) \;\; \therefore \; f_{cn} = (2n-1)f_{c1}$

From the above, deduce that

$\lambda_{o2} = \lambda_{o1}/2 \;\; \therefore \; f_{o2} = 2f_{o1}$

$\lambda_{o3} = \lambda_{o1}/3 \;\; \therefore \; f_{o3} = 3f_{o1}$

. . .

$\lambda_{on} = \lambda_{o1}/n \;\; \therefore \; f_{on} = nf_{o1}$

Just like for a string, the resonant frequencies follow a pattern.

Fixed f Variable L

With fixed f, λ becomes fixed so the distance between consecutive nodes becomes fixed. Varying the length of the air column will thus produce resonant or stationary wave modes at specific lengths separated by half a wavelength. Hence as the length of air column is increased from zero, the first loud sound will be heard at L_1, then at L_2, L_3 etc.

Closed End Tube

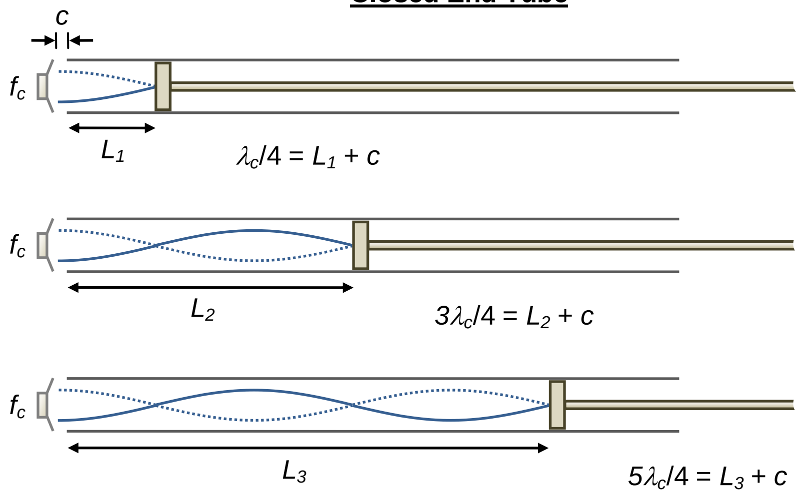


Fig. 3.11

Open Ends Tube

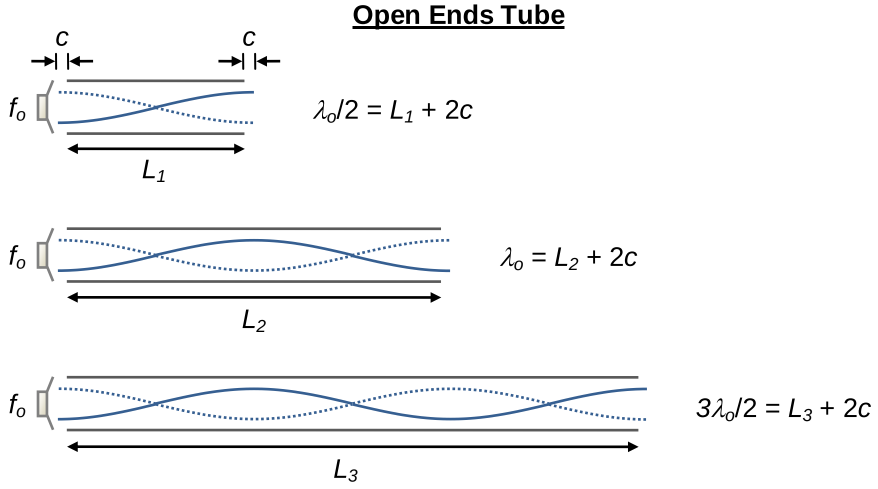


Fig. 3.12

A tube (closed end or open ends) with fixed frequency source and variable length is useful for determining the wavelength of sound since the change in length between hearing a loud sound and the next loud sound is $\lambda/2$.

These set-ups are useful for finding the wavelength of sound. By listening for the occurrence of loudness maxima, one can then measure the lengths L_1, L_2, L_3 etc. These lengths are separated by $\lambda/2$ so λ can be found easily. If the frequency is known from the signal generator, the speed of the sound can be calculated (see Waves, end of section 6 for a slightly different set-up).

4 Diffraction

Diffraction is the *spreading or bending* of waves as they pass through an aperture or around an obstacle.

Diffraction is the *spreading or bending* of waves as they pass through an aperture or around an obstacle.

All waves diffract and this phenomenon is most easily observed in a ripple tank. Plane waves are created using a long straight edge and allowed to pass through an aperture. Fig. 4.1 shows the diffraction effect. After spreading, the wave amplitude and intensity is not uniform in all directions. Most of the wave energy is concentrated in the forward direction and there are some directions which do not get any wave energy.

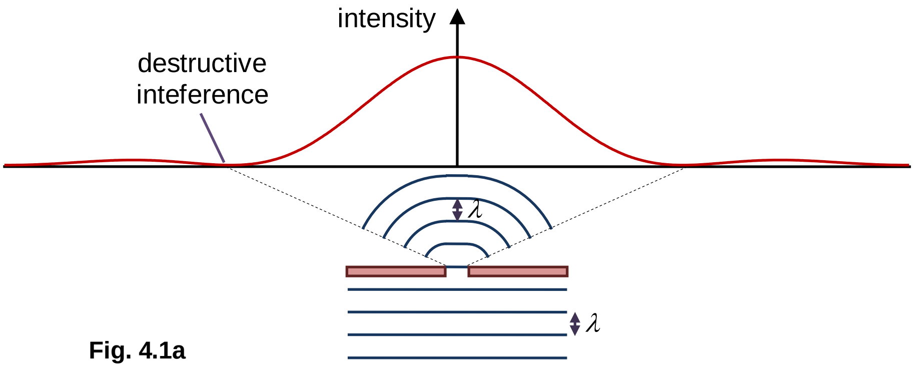


Fig. 4.1a

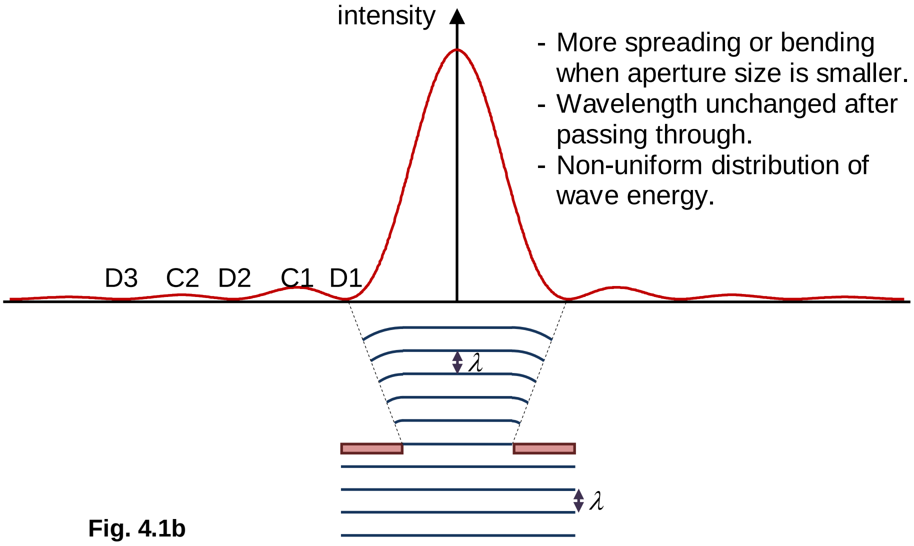


Fig. 4.1b

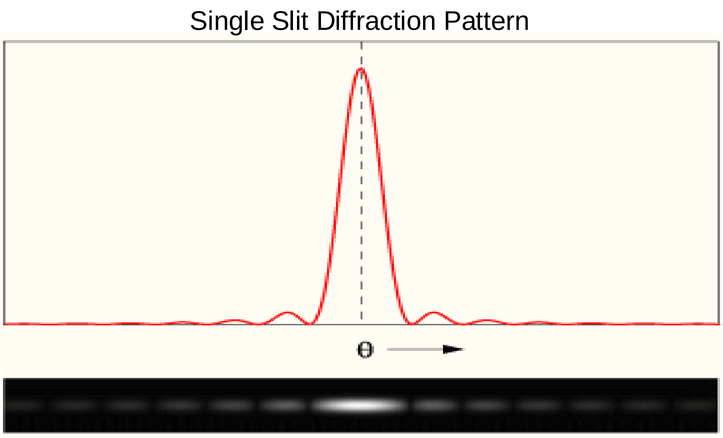


Fig. 4.2 by Rosarinagazo Wikimedia Commons CC-BY-SA-3.0 Unported License

In diffraction, the distribution of wave energy is not uniform in all directions.

For the same wavelength, as aperture increases, diffraction decreases.

When a laser beam is directed at a small slit of about 0.25 mm wide, it will spread out with an intensity pattern as shown in Fig. 4.2 and form a series of bright and dark fringes on a screen in a dark room.

Diffraction can be explained by treating the whole medium as made up of oscillators. When plane waves reach the aperture, the aperture can be seen as a line of oscillators emitting circular waves spreading out from each oscillator. The waves from these oscillators will *all* meet constructively only along the central line, leading to maximum intensity. In some directions, *all* the waves meet destructively (D1, D2, D3 in Fig. 4.1b). In other directions (C1 & C2 in Fig. 4.1b), of the many waves contributed by the many oscillators, there is incomplete cancellation of displacements, resulting in an intensity between maximum and zero.

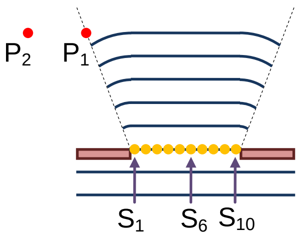


Fig. 4.3

As seen earlier, the path difference (PD) in relation to the wavelength determines whether 2 waves interfere constructively or destructively. To get a rough idea of why the degree of spreading depends on the wavelength, we consider a point P_1 behind the obstacle. At P_1 waves from S_1 and S_2 will meet almost in phase because the path difference is very small. When waves from S_3, S_4, . . . etc are also involved in the superposition at P_1, there will come a point when waves from one of the sources, perhaps S_6, will travel an extra distance of $\lambda/2$ compared to waves from S_1. Then waves from S_7 will also travel an extra distance of $\lambda/2$ compared to waves from S_2 and so on. The result is destructive interference or zero intensity at P_1 as waves from S_1 to S_5 are cancelled by waves from S_6 to S_{10}.

Next, if we switch to using waves with bigger λ in Fig. 4.3, the destructive interference at P_1 will happen with pairs of sources which are spaced further apart, say between S_1 and S_9. This will mean P_1 is no longer a point of destructive interference. However, there will be a point P_2 further to the left that will now be a destructive interference point because $S_6P_2 - S_1P_2$ is now $\lambda/2$ instead. Therefore,

For same aperture or obstacle size, as wavelength increases the diffraction increases.

as wavelength increases, the spreading increases.

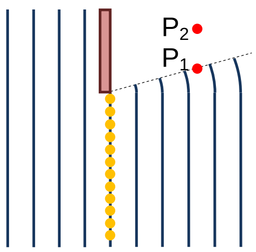


Fig. 4.4

Fig. 4.4 is like a 'semi-infinite' aperture with many more oscillators compared to Fig. 4.3. Taking into account that sources further away will contribute waves with smaller amplitude, the result that using waves of larger wavelength will cause the waves to spread further behind the obstacle (from P_1 to P_2) is still true. Hence sound waves which have much greater wavelengths than light will easily spread around a pillar and be heard behind it whereas light will cast shadows behind the pillar.

Fig. 4.5 shows single frequency waves (monochromatic) spreading behind an obstacle to give a line of constructive interference. This constructive interference behind the obstacle can be seen when a laser beam is directed at a small disc and a bright spot is produced in the centre of the circular shadow.

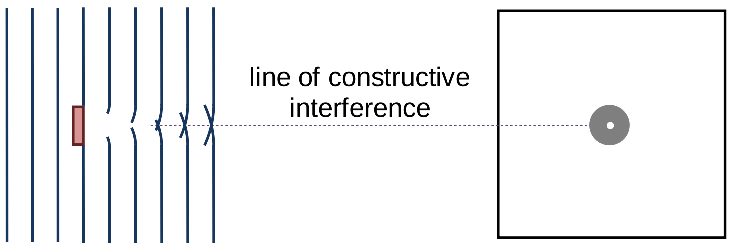


Fig. 4.5

See diffraction of water waves near coast http://bit.ly/1xqwKU8

Diffraction and Interference

Interference of waves by superposition is the most fundamental phenomenon from which we can understand how waves from two sources superpose to form an *interference pattern* made up of points of *constructive* and *destructive* interference and how diffraction is a result of superposition of many waves along an aperture.

In other words, diffraction is not a new and separate phenomenon. However, it is convenient to still use the word 'diffraction' to refer to the spreading or bending of waves.

Young's Double Slit

In late 1700s and early 1800s, scientists were debating whether light was made up of particles or waves. Thomas Young demonstrated the interference pattern of water waves and subsequently showed that light could produce similar interference pattern and thus provided key evidence for the wave nature of light. The set-up he used was similar to Fig.4.6.

Young's double slit experiment provides key evidence for wave nature of light.

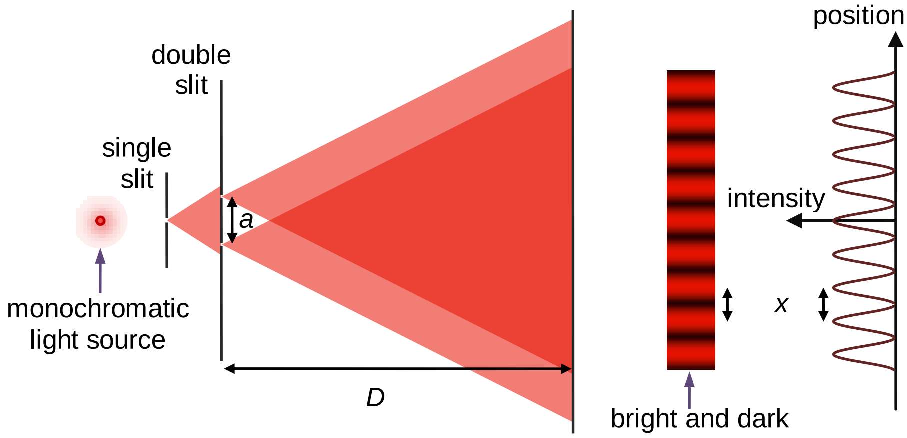


Fig. 4.6

Light spreading from single slit acts a *point* source to ensure double slit sources are coherent.

The single slit supplied light to the double slit by diffraction. Since the two slits received light from the *same point* source, the two slits behave as *coherent* sources. When light from the double slits overlap, an interference pattern is formed. The pattern at any chosen plane can be made visible by placing a screen there.

When the fringes are close to the central axis and D is much larger than a, the spacing between the bright fringes (or dark fringes) is given by

$$\boxed{x \approx \frac{\lambda D}{a}}$$

Spacing of fringes is:

$x \approx \dfrac{\lambda D}{a}$

- for fringes near the central axis
- & for $D >> a$

Typically, the slit *separation a* is about 1 to 2 mm, D is a few metres and for visible light λ is about 400 nm to 750 nm. Fig. 1.10 shows that the spacing between constructive interference points is not even when D is small and when they are positioned far away from the central axis.

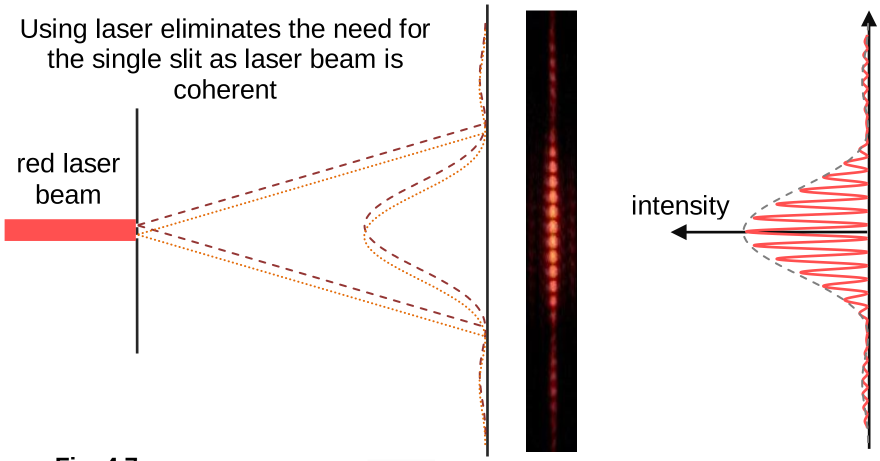


Fig. 4.7 Photo by Jordgette Wikimedia Commons CC-BY-SA-3.0 Unported License

The distribution of light in diffraction is more uniform as slit width gets smaller but the overall intensity will decrease.

In Fig. 4.6 the fringes are shown as equally bright. In reality diffraction or spreading of light from each slit is not uniform (see Fig. 4.1a & b). This leads to higher intensity for fringes nearer the central axis than those further away. When the slit *widths* are narrower, the light energy is more spread out (compare Fig. 4.1a & b) but overall intensity is lower. This will allow more fringes near the central axis to be visible but they will be less bright than before.

If white light which has all the colours in it is used instead of monochromatic light, different wavelengths will have fringes of different spacing according to the formula $x = \lambda D/a$. Violet has the smallest spacing while red has the largest (Fig. 4.8). The central or 0^{th} order(PD is zero) bright fringe will appear more or less white but higher order fringes will be more coloured as the different colours are more separated.

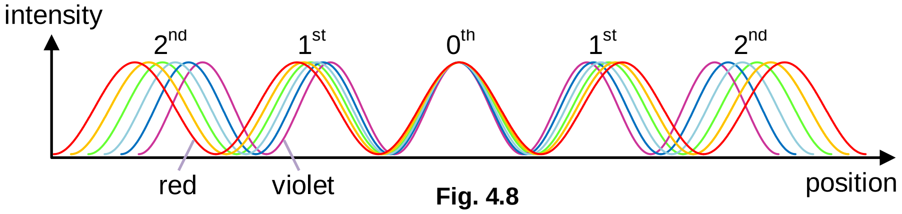


Fig. 4.8

Two Source Interference

So far we have seen a number of *two source* interference set-ups:

1. Two source water waves (Fig. 1.3 to 1.11).
2. Formation of standing waves including set-ups using reflector. Reflector creates a virtual source.
3. Young's double slit (ideas apply to microwaves too).

They may look different but they are all two source interference set-ups. Sometimes one source can be substituted by a reflector which creates a virtual source like in Fig. 4.9.

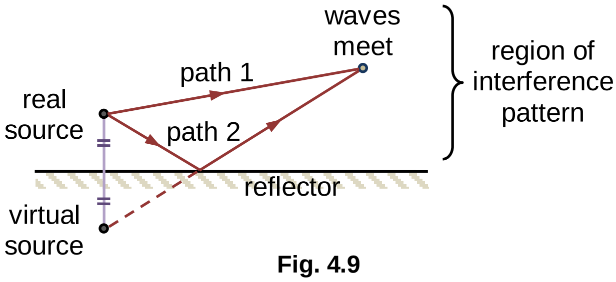


Fig. 4.9

Recall that for the formation of *stable* interference pattern the sources need to be coherent. Furthermore the waves from the sources must either be *polarised in the same plane* or both *unpolarised*. If the polarisations of the waves are perpendicular to each other, they cannot give rise to points of destructive interference.

Stable interference pattern needs
- coherent sources
- waves from sources to be *polarised in the same plane* or both *unpolarised*

Diffraction Grating

Diffraction grating is usually a plastic sheet that has hundreds of slits or lines mm^{-1} engraved on it. Whether 2 or hundreds of slits, the mechanism is the same. Firstly, diffraction occurs when light passes through the slits, then the overlapping waves interfere to produce lines of constructive and destructive interference. When the light is intercepted by a screen, a series of bright dots/lines can be seen. These dots/lines are far more spaced out than the bright fringes of the double slit because the slits are far closer in the grating.

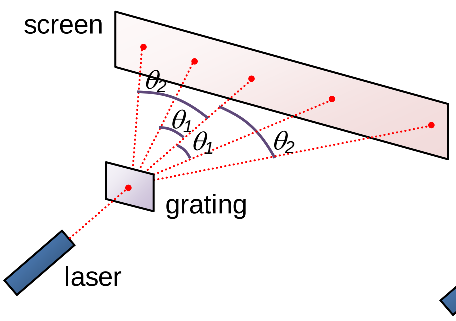


Fig. 4.10a

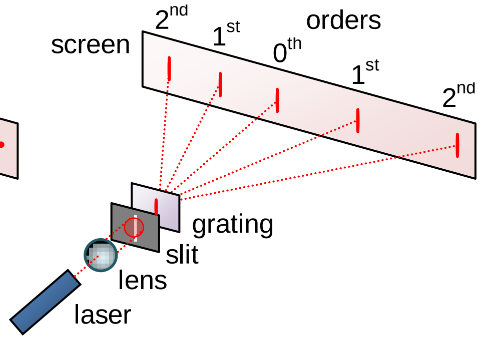


Fig. 4.10b

As a laser beam is usually narrow the screen shows a series of dots (Fig. 4.10a). When the laser beam is expanded using a lens and a slit is used to direct a thin line of light on the grating, the screen shows a series of lines (Fig. 4.10b).

The grating splits the original beam into several beams. The undeviated beam is called the 0^{th} order beam followed by the 1^{st} and 2^{nd} order beams as shown in Fig. 4.9b. The angle of deviation θ is related to the spacing of the slits d, wavelength λ and the order of the beam n:

$$\boxed{d \sin\theta = n\lambda}$$

Both the double slit set-up and grating are useful for determining the wavelength of light by measuring the other parameters.

Diffraction grating: $d \sin\theta = n\lambda$

Maximum Order

Consider a typical grating that has 600 lines per mm. Using 635 nm red laser, what is the maximum number of orders visible?

Slit separation d = 0.001/600

Theoretical maximum deviation angle = 90°

$\therefore$ putting the numbers into the formula $d \sin\theta = n\lambda$

$$0.001/600 \sin(90°) = n_{max} (635 \times 10^{-9})$$

$$n_{max} = 2.6$$

Since n must be integer, the maximum order is = 2

White Light On Grating

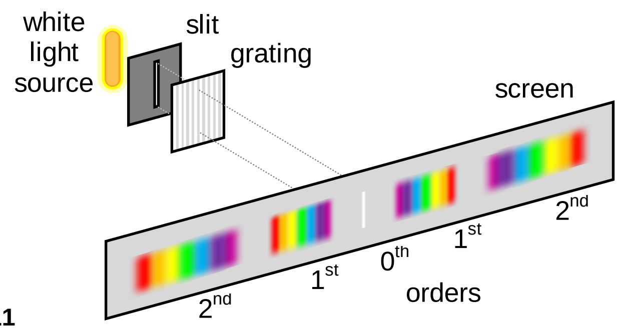


Fig. 4.11

When white light which has all the colours in it is directed at a grating, from 1^{st} order onwards, the different colours(λ) are separated into different angles(θ) as shown in Fig. 4.11.