H

millennia institute

CANDIDATE NAME

CLASS

ADMISSION NUMBER

2022 End-of-Year Examination
Pre-University 2

MATHEMATICS **8865/01**

Paper 1 **12 September 2022**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

READ THESE INSTRUCTIONS FIRST

Write your admission number, name and class on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.

Answer **all** the questions.
Give your answers in the spaces provided in the Question Paper.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
You are expected to use an approved graphing calculator.
Unsupported answers from a graphing calculator are allowed unless a question specifically states otherwise.
Where unsupported answers from a graphing calculator are not allowed in a question, you are required to present the mathematical steps using mathematical notations and not calculator commands.
You are reminded of the need for clear presentation in your answers.

The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 100.

Qn No.	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	*	Total
Score														
Max Score	**4**	**7**	**8**	**9**	**12**	**5**	**5**	**6**	**8**	**10**	**13**	**13**		**100**

Section A: Pure Mathematics [40 marks]

1 At a salad stall, patrons can customise their salads by choosing the number of servings of beans, vegetables and nuts. Dressings are free of charge. Amy, Betty and Cathy go for lunch at the salad stall. Amy pays \$9.10 for her salad which consists of two serving of beans, three servings of vegetables and one serving of nuts. Betty pays \$14 for her salad which consists of three serving of beans, four servings of vegetables and two serving of nuts. Cathy, who has nut allergy, orders what Amy orders for her salad but replaces the one serving of nuts with another serving of beans. Cathy pays \$8.10 for her salad.

(i) How much does one serving of beans, vegetables and nuts each cost at the salad stall? [3]

(ii) The salad stall offers a 20% discount for any purchase. Daisy pays \$6.32 for her salad which consists of 1 serving of beans, a serving(s) of vegetables and 1 serving of nuts. Find the value of a. [1]

2 **(a)** One of the solutions for the following pair of simultaneous equations

$$p^2x - 5y = 8,$$
$$x^2 + 9y^2 + 3x = 30 + xy.$$

is $x = 3$ and $y = p$, where p is an integer.

(i) Show that $p = -1$. [2]

(ii) Using $p = -1$, find the other solution for this pair of simultaneous equations. [3]

(b) Solve $x^2 > 5$ algebraically. [2]

3 A curve C has equation $y = \frac{1}{3}x^3 - \frac{7}{2}x^2 + 10x + 1$.

(i) Using a non-calculator method, show that the equation of the tangent to C at the point P where $x = 5$ is

$$y = \frac{31}{6}.$$ [3]

(ii) Sketch the graphs of C and the tangent to C at P on the same diagram, stating the coordinates of any points of intersection with the axes. [3]

(iii) Hence or otherwise, find the finite area of the region bounded by C and the tangent to C at P. [2]

4 **(a)** Differentiate the following expressions with respect to x.

(i) $\dfrac{1}{3(2x-1)^2}$ [2]

(ii) $4e^{x^3}$ [2]

(b) Integrate the following expressions with respect to x.

(i) $\left(x-\dfrac{2}{x}\right)^2$ [3]

(ii) $e^{\frac{x+1}{4}}$ [2]

5 Codling moth is a pest that infests apple trees. Scientists are interested to study the effect on the population of codling moth in an apple orchard after the introduction of a natural bird predator of the moth.

The population, P thousand, of codling moth in the apple orchard is modelled by the equation

$$P = 15\ln(t+1) - 5t + 10,$$

where t is the time in years after the natural bird predators are introduced in the orchard.

(i) Find $\dfrac{dP}{dt}$ when $t = 5$. In the context of the question, what does this value represent? [2]

(ii) Use differentiation to find the maximum value of P, justifying that this value is a maximum. [5]

(iii) Sketch the graph of P against t, stating clearly the coordinates of any points of intersection with the axes. [2]

The scientists are also interested to model the rate of change of the population, Q hundred per year, of the natural bird predator of codling moth in the apple orchard t years after they are introduced. The model used is

$$Q = b(3t+1) - a(2^t).$$

It is known that the population of the natural bird predator remains constant initially.

(iv) Explain why $a = b$. [2]

(v) It is known that $a = b = 5$. The population, in hundreds, of the natural bird predator *m* years after they are introduced can be found by

$$\int_0^m 5(3t+1) - 5(2^t)\,\mathrm{dt}\,.$$

Find the population of the natural bird predator 4 years after they are introduced. [1]

Section B: Probability and Statistics [60 marks]

6 Find the number of different arrangements of the 9 letters of the word FAVOURITE if

(i) the vowels and the consonants alternate, [2]

(ii) at most 3 of the consonants are together. [3]

7 On average, 7% of mechanical pencils made by a factory are faulty. A random sample of 15 mechanical pencils is selected.

(i) Find the probability that fewer than a quarter of the mechanical pencils in the sample are faulty. [2]

(ii) Given that fewer than a quarter of the mechanical pencils in the sample are faulty, find the probability that at least one of them is faulty. [3]

8 Two independent events *A* and *B* are such that $\mathrm{P}(A' \cap B') = 0.2$ and $\mathrm{P}(A \cap B') = 0.5$.

(i) Show that the probability *B* occurs is 0.3. [2]

(ii) Find the probability *A* occurs. [3]

(iii) State the probability *A* does not occur, given that *B* occurs. [1]

9 A researcher recorded the maximum ambient temperature, x ^{o}C and the mean ambient temperature, y ^{o}C of a room on seven randomly chosen days. The data is summarised in the table below.

x (^{o}C)	34.0	32.2	30.8	32.3	29.4	32.4	31.8
y (^{o}C)	28.8	27.8	27.4	28.0	26.7	27.5	27.7

(i) Give a sketch of the scatter diagram for the data, as shown on your calculator. [2]

(ii) Find the value of the product moment correlation coefficient and comment on its value in the context of the data. [2]

(iii) Find the equation of the regression line of y on x, giving your answer in the form $y = mx + c$, with the values of m and c correct to 2 decimal places. Sketch this line on your scatter diagram. [2]

(iv) Use the equation of your regression line in part **(iii)** to estimate the mean ambient temperature of the room when the maximum ambient temperature is 31 ^{o}C. Give a reason why you would expect this to be a reliable estimate. [2]

10 A study of the number of male and female offspring in a particular species is being carried out. The model states that the first offspring in any family of this species is equally likely to be male or female. For any subsequent offspring, the probability that it will be of the same gender as the previous offspring is 0.65.

A randomly chosen family has 3 offspring.

(i) Draw a tree diagram to show all possible outcomes. [3]

(ii) Find the probability that all 3 offspring are of the same gender. [2]

(iii) Find the probability that the first offspring is a male given that there are at least 2 female offspring in the family. [3]

Three families are randomly selected. Each family has 3 offspring.

(iv) Find the probability that exactly 2 of the 3 families have all 3 offspring of the same gender. [2]

11 A coffee roaster sells two kinds of roasted coffee beans, Espresso Roast and Filter Roast. The masses, in grams, of packets of both kinds of roasted coffee beans follow independent normal distributions with means and standard deviations as shown in the table.

	Mean	Standard deviation
Espresso Roast	μ	σ
Filter Roast	205	8

(i) It is given that 15% of the packets of Espresso Roast coffee beans weigh less than $(\mu - 5)$ grams.

(a) Find the probability that the mass of a randomly chosen packet of Espresso Roast coffee beans weigh less than $(\mu + 5)$ grams. [1]

(b) Show that $\sigma = 4.8$, correct to 1 decimal place. [3]

For the rest of the question, use $\mu = 200$ and $\sigma = 5$.

(ii) The probability that the total mass of three randomly chosen packets of Espresso Roast coffee beans weighs less than *k* grams is 0.3. Find the value of *k*. [3]

(iii) Find the probability that a randomly chosen packet of Espresso Roast coffee beans weighs less than a randomly chosen packet of Filter Roast coffee beans. [3]

(iv) Before selling the coffee beans to customers, the coffee roaster grinds the coffee beans in the packets into ground coffee bean. The grinding process reduces the mass of coffee beans in a packet by 5%. Find the probability that the total mass of ground coffee in 2 randomly chosen packets of Espresso Roast coffee beans and 1 randomly chosen packet of Filter Roast coffee beans is more than 580 grams. [3]

12 Based on historical data, the mid-day temperature at a particular place during the period of May to July has a mean value of 33.9 ^{o}C. A group of researchers sets up an experiment to collect data to test whether the mid-day temperature at this place is getting hotter. A random sample of 60 days between May to July for the past three years is selected and the mid-day temperature is recorded. The data is summarised as follows.

$$\sum x = 2104.2 \qquad \sum x^2 = 74698.8$$

(i) Explain, in context, what is meant by the term 'random sample'. [2]

(ii) Find the unbiased estimates of the population mean and variance. [2]

(iii) State the null and alternative hypotheses that the group of researchers should use. [1]

(iv) Carry out the test at the 5% level of significance. [3]

(v) A test at the α% level of significance does not indicate that the mean mid-day temperature at this place is getting hotter, find the range of values of α [2]

(vi) Assuming that the mid-day temperature between May and July at this place is normally distributed with a standard deviation of 2.3 ^{o}C. The group of researchers selected another random sample of 60 days and its average mid-day temperature is *k*. A test at 5% level of significance indicated that the mid-day temperature at this place is hotter. Find the range of values of *k*. [3]

End of Paper