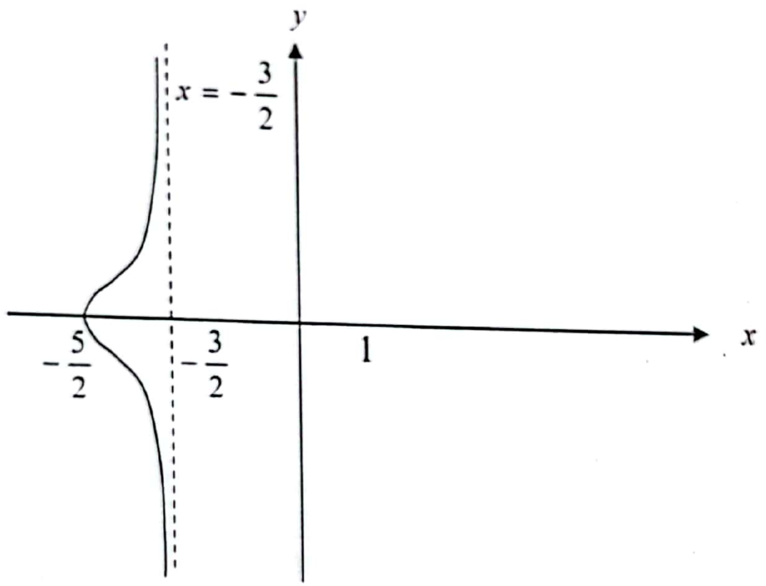
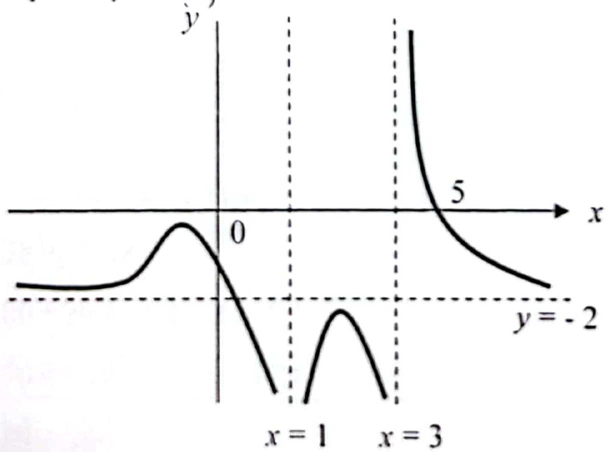
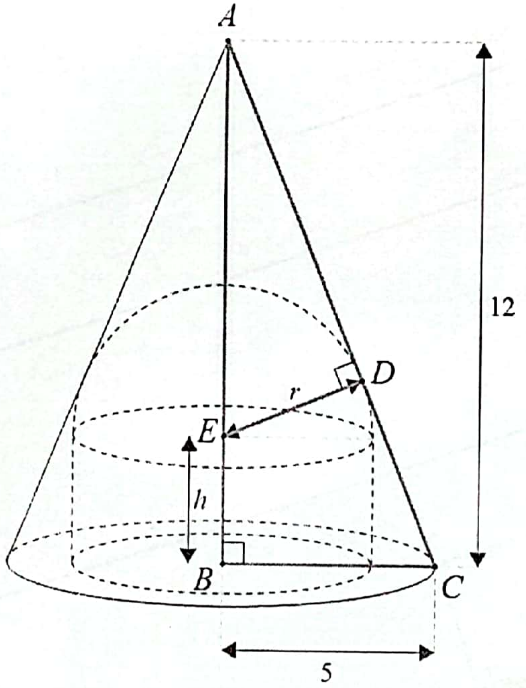
1	Topic: Transformation of Curves					
			Solution			
			(a) Graph of $y^2 = f(-2x)$ 			
			(b) Graph of $y = f(x)$ 			

2.	Topic: Vectors					
			Solution			
			(i) $\lvert\mathbf{a}\rvert = 2\lvert\mathbf{b}\rvert$ $\sqrt{16+36p^2+64} = 2\sqrt{4+9+16p^2}$ $80+36p^2 = 4\left(13+16p^2\right)$ $28p^2 = 28$ $p=1$ or $p=-1$ (Reject $\because p>0$) $\therefore p=1$			
			(ii) Length of projection of **a** on **b**			
			(iii) $\frac{1}{\lvert\mathbf{b}\rvert}\lvert\mathbf{b}\cdot\mathbf{a}\rvert = \frac{\left\lvert\begin{pmatrix}2\\-3\\4\end{pmatrix}\cdot\begin{pmatrix}4\\6\\-8\end{pmatrix}\right\rvert}{\sqrt{2^2+(-3)^2+4^2}} = \frac{42}{\sqrt{29}}$			

3	Topic: Complex Numbers					
			Solution			
		(i)	The assumption is that a, b and c are all real.			
		(ii)	Let $x^3+ax^2+bx+c=(x-(3+\mathrm{i}))(x-(3-\mathrm{i}))(x-2)$ $=(x^2-6x+10)(x-2)$ $=x^3-8x^2+22x-20$ By comparing coefficients, we have $a=-8$, $b=22$ and $c=-20$.			

			Solution			
			(i) Using the diagram provided,			



Identify that ΔABC is similar to ΔADE.

$$\therefore \frac{AE}{DE} = \frac{AC}{BC}$$

Since $AC = \sqrt{5^2 + 12^2} = 13$, $BC = 5$, $DE = r$,

$\therefore \frac{AE}{r} = \frac{13}{5} \Rightarrow AE = \frac{13}{5}r$

$h = AB - AE = 12 - \frac{13}{5}r$

Since r and h are lengths,

$r \geq 0$ and $h \geq 0$ (i.e. $12 - \frac{13}{5}r \geq 0$, $r \leq \frac{60}{13}$).

$0 \leq r \leq \frac{60}{13}$

Solution

(ii) Volume of inscribed container,

$V = \pi r^2 h + \frac{1}{2}\left(\frac{4}{3}\pi r^3\right)$

$= \pi r^2\left(12 - \frac{13}{5}r\right) + \frac{2}{3}\pi r^3$

$= 12\pi r^2 - \frac{29}{15}\pi r^3$

Differentiating this,

$\frac{dV}{dr} = 24\pi r - \frac{29}{5}\pi r^2$.

Consider $\frac{dV}{dr} = 0$.

i.e. $24\pi r - \frac{29}{5}\pi r^2 = 0$,

$\pi r(24 - \frac{29}{5}r) = 0$,

$r = 0$(rejected as $r \neq 0$) or $r = \frac{120}{29}$.

First derivative test

$\frac{dV}{dr} = 24\pi r - \frac{29}{5}\pi r^2$

r	$r = \frac{120}{29}^-$ e.g. $r = \frac{119}{29}$	$r = \frac{120}{29}$	$r = \frac{120}{29}^+$ e.g. $r = \frac{121}{29}$
$\frac{dV}{dr}$	$\frac{119}{145}\pi > 0$	0	$-\frac{121}{145}\pi < 0$

Maximum volume at $r = \frac{120}{29}$

4	Topic: Application of Differentiation					
			Solution			
			Second derivative test $\frac{dV}{dr} = 24\pi r - \frac{29}{5}\pi r^2$ $\frac{d^2V}{dr^2} = 24\pi - 2\left(\frac{29}{5}\right)\pi r$ $= 24\pi - 2\left(\frac{29}{5}\right)\pi r$ $= 24\pi - \frac{58}{5}\pi\left(\frac{120}{29}\right)$ $= -24\pi < 0$ (maximum volume)			

5	Mathematical Induction				
			Solution		
			Let P_n be the statement that $u_n = \frac{2n}{(n-1)!}$ for $n \in \mathbf{Z}^+$, $n \geq 1$. When $n=1$, LHS= $u_1 = 2$ (given) $\text{RHS} = \frac{2}{(0)!} = 2 = \text{LHS}$, $\therefore P_1$ is true. Assume that P_k is true for some k, where $k \in \mathbf{Z}^+, k \geq 1$ i.e. $u_k = \frac{2k}{(k-1)!}$ To prove P_{k+1} is also true ,i.e., $u_{k+1} = \frac{2(k+1)}{k!}$ LHS = $u_{k+1} = \frac{k+1}{(k)^2} u_k$ $= \frac{k+1}{(k)^2} \frac{2k}{(k-1)!}$ $= \frac{2(k+1)}{k(k-1)!}$ $= \frac{2(k+1)}{k!} = \text{RHS}$ Since P_1 is true, and P_k is true $\Rightarrow P_{k+1}$ is true, by Mathematical Induction, P_n is true for all $n \in \mathbf{Z}^+$.		

			Solution			
			(b) $u_2 = \frac{2}{(1)^2}u_1 = \frac{2}{1}a = 2a = \frac{2}{1!}a$ $u_3 = \frac{3}{(2)^2}u_2 = \frac{3}{(2)^2}2a = \frac{3}{(2)}a = \frac{3}{2!}a$ $u_4 = \frac{4}{(3)^2}u_3 = \frac{4}{(3)^2}\frac{3}{(2)}a = \frac{4}{(3\times 2)}a = \frac{4}{3!}a$			
			"Hence" approach $u_n = \frac{n}{(n-1)!}a$ by observation from part (ii) "Otherwise" approach $u_n = \frac{n}{(n-1)^2}u_{n-1}$ $= \frac{n}{(n-1)^2}\frac{n-1}{(n-2)^2}u_{n-2}$ $= \frac{n}{(n-1)^2}\frac{n-1}{(n-2)^2}\frac{n-2}{(n-3)^2}u_{n-3}$ $= \frac{n}{(n-1)^2}\frac{n-1}{(n-2)^2}\frac{n-2}{(n-3)^2}\cdots\frac{3}{(2)^2}\frac{2}{(1)^2}u_1$ $= \frac{n}{(n-1)^1}\frac{1}{(n-2)^1}\frac{1}{(n-3)^1}\cdots\frac{1}{(2)^1}\frac{1}{(1)^1}a$ $= \frac{n}{(n-1)!}a$			

			Solution			
			(i) $1-2r = A(r+1)+Br$ $= (A+B)r + A$ $\therefore A=1, B=-3$			
			(ii) $\sum_{r=1}^{n}\frac{1-2r}{3^r} = \sum_{r=1}^{n}\frac{(r+1)-3r}{3^r}$ $= \sum_{r=1}^{n}\left(\frac{r+1}{3^r}-\frac{r}{3^{r-1}}\right)$ $= \left(\frac{2}{3^1}-\frac{1}{3^0}\right)+$ $\left(\frac{2}{3^2}-\frac{2}{3^1}\right)+$ $\vdots$ $\left(\frac{n}{3^{n-1}}-\frac{n-1}{3^{n-2}}\right)+$ $\left(\frac{n+1}{3^n}-\frac{n}{3^{n-1}}\right)$ $= \frac{n+1}{3^n}-1$			

			Solution			
			(iii) $\sum_{r=1}^{\infty}\frac{2-2r}{3^r}=\sum_{r=1}^{\infty}\frac{1-2r+1}{3^r}$ $=\sum_{r=1}^{\infty}\left(\frac{1-2r}{3^r}+\frac{1}{3^r}\right)$ $=\sum_{r=1}^{\infty}\left(\frac{1-2r}{3^r}\right)+\sum_{r=1}^{\infty}\left(\frac{1}{3^r}\right)$ $=\lim_{n\to\infty}\left(\frac{n+1}{3^n}-1\right)+\frac{\frac{1}{3}}{1-\frac{1}{3}}$ $=-1+\frac{1}{2}$ $=-\frac{1}{2}$			

7	Topic: Maclaurin's Series					
			Solution			
		(i)	For $y=\sqrt{1+\ln(1+x)}$ to be well-defined, $1+x>0$ and $1+\ln(1+x)\geq 0$ $\ln(1+x)\geq -1$ $x>-1$ and $1+x\geq e^{-1}$ $x\geq e^{-1}-1$ $\therefore x\geq e^{-1}-1$			
		(ii)	**By Implicit Differentiation,** $y=\sqrt{1+\ln(1+x)}$ $y^2=1+\ln(1+x)$ $\Rightarrow \ln(1+x)=y^2-1$ $\Rightarrow 1+x=e^{y^2-1}$ Differentiate implicitly with respect to x, $2y\frac{dy}{dx}=\frac{1}{1+x}$ $=\frac{1}{e^{y^2-1}}$ $=e^{1-y^2}$ (shown)			
		(iii)	Differentiate the above results implicitly with respect to x, $2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=\frac{d}{dx}(1-y^2)\cdot e^{1-y^2}$ $2y\frac{d^2y}{dx^2}+2\left(\frac{dy}{dx}\right)^2=-2y\frac{dy}{dx}\cdot e^{1-y^2}$ When $x=0$, $y=\sqrt{1+\ln 1}=1$, $\frac{dy}{dx}=\frac{1}{2}$.			

7	**Topic: Maclaurin's Series**					
			Solution			
			$\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=-\frac{3}{4}$, $y=1+\frac{1}{2}x+\frac{-\frac{3}{4}}{2!}x^2+\ldots$ $=1+\frac{1}{2}x-\frac{3}{8}x^2+\ldots$			
			(iv) $y=\left(1+\ln(1+x)\right)^{\frac{1}{2}}$ $=\left(1+\left(x-\frac{x^2}{2}+\ldots\right)\right)^{\frac{1}{2}}$ $=1+\frac{1}{2}\left(x-\frac{x^2}{2}+\ldots\right)+\frac{\frac{1}{2}\left(-\frac{1}{2}\right)}{2!}\left(x-\frac{x^2}{2}+\ldots\right)^2+\ldots$ $=1+\frac{1}{2}x-\frac{1}{4}x^2-\frac{1}{8}\left(x-\frac{x^2}{2}\right)\left(x-\frac{x^2}{2}\right)+\ldots$ $=1+\frac{1}{2}x-\frac{1}{4}x^2-\frac{1}{8}x^2+\ldots$ $=1+\frac{1}{2}x-\frac{3}{8}x^2+\ldots$			

8	Topic: Complex Numbers				
			Solution		
			(i) $z^2 - 6z + 36 = 0 \Rightarrow z = \dfrac{6 \pm \sqrt{36 - 4(1)(36)}}{2} = 3 \pm 3\sqrt{3}\mathrm{i}$		
			Thus, $z_1 = 6e^{\mathrm{i}\frac{\pi}{3}}$ and $z_2 = 6e^{-\mathrm{i}\frac{\pi}{3}}$		
			(ii) $\dfrac{\left(6e^{i\frac{\pi}{3}}\right)^4}{\left(6e^{i\left(\frac{\pi}{2}-\frac{\pi}{3}\right)}\right)} = 6^3 e^{i\left(\frac{7\pi}{6}\right)}$ $= 6^3\left[\cos\left(\dfrac{-5\pi}{6}\right) + i\sin\left(\dfrac{-5\pi}{6}\right)\right]$		
			(iii) $z_2 = 6e^{-\mathrm{i}\frac{\pi}{3}} \Rightarrow \quad z_2{}^n = 6^n e^{\mathrm{i}\left(-\frac{n\pi}{3}\right)}$ Since $z_2{}^n \in \mathbb{Z}^+$, $-\frac{n\pi}{3} = 2k\pi$ for some integer k. $n = -6k$. $n = \ldots,\ 12,\ 6,\ 0,\ -6,\ -12,\ \ldots$ Smallest positive integer $n = 6$.		

9	Topic: Techniques of Integration / Definite Integrals					
			Solution			
			(i) $\frac{du}{dx} = \frac{1}{2\sqrt{x+1}}$ $= \frac{1}{2u}$ $\int \frac{\sqrt{x+1}}{x-1} dx = \int \frac{u}{u^2-2} .2u du$ $= 2\int (1 + \frac{2}{u^2-2}) du$ $= 2u + \frac{2}{\sqrt{2}} \ln \left\| \frac{u-\sqrt{2}}{u+\sqrt{2}} \right\| + c$ $= 2\sqrt{x+1} + \sqrt{2} \ln \left\| \frac{\sqrt{x+1}-\sqrt{2}}{\sqrt{x+1}+\sqrt{2}} \right\| + c$			
			(ii) **(a)** Area of R $= 5(1) - \int_3^8 \frac{\sqrt{x+1}}{x-1} dx$ $= 5 - \left[2\sqrt{x+1} + \sqrt{2} \ln \left\| \frac{\sqrt{x+1}-\sqrt{2}}{\sqrt{x+1}+\sqrt{2}} \right\| \right]_3^8$ $= 5 - \{2 + \sqrt{2} \ln \left(\frac{3-\sqrt{2}}{3+\sqrt{2}} \right) - \sqrt{2} \ln \left(\frac{2-\sqrt{2}}{2+\sqrt{2}} \right) \}$ $= 3 - \left(\sqrt{2} \ln \left(\frac{3-\sqrt{2}}{3+\sqrt{2}} . \frac{2+\sqrt{2}}{2-\sqrt{2}} \right) \right)$			

9	Topic: Techniques of Integration / Definite Integrals					
			Solution			
			$= 3 - \left(\sqrt{2} \ln \left(\frac{4+\sqrt{2}}{4-\sqrt{2}} \right) \right)$ $= 3 - \sqrt{2} \ln \left(\frac{-4-\sqrt{2}}{-4+\sqrt{2}} \right)$			
			(ii) (b) Vol. generated $= \pi(1^2).5 - \pi \int_3^8 \frac{x+1}{(x-1)^2} dx$ $= 9.52830$ $= 9.53 \text{ units}^3$			

Solution

(i)

$$\overrightarrow{AB}=\begin{pmatrix}2\\-1\\4\end{pmatrix}-\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}2\\-2\\3\end{pmatrix}$$

$$\overrightarrow{AC}=\begin{pmatrix}-2\\-1\\0\end{pmatrix}-\begin{pmatrix}0\\1\\1\end{pmatrix}=\begin{pmatrix}-2\\-2\\-1\end{pmatrix}$$

$$\overrightarrow{AB}\times\overrightarrow{AC}=\begin{pmatrix}2\\-2\\3\end{pmatrix}\times\begin{pmatrix}-2\\-2\\-1\end{pmatrix}=\begin{pmatrix}8\\-4\\-8\end{pmatrix}=4\begin{pmatrix}2\\-1\\-2\end{pmatrix}$$

Choose normal vector $\underset{\sim}{n}_p$ for plane $p=\begin{pmatrix}2\\-1\\-2\end{pmatrix}$.

$$p:\ \mathbf{r}\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=\begin{pmatrix}0\\1\\1\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=-3$$

A cartesian equation of the plane p is $2x-y-2z=-3$

(ii) Let the acute angle between l and p be θ.

The angle between the normal vector $\underset{\sim}{n}_p$ (for plane p) and the direction vector $\underset{\sim}{m}_l$ (for line l),

$$\alpha=\cos^{-1}\frac{\begin{pmatrix}2\\-1\\-2\end{pmatrix}\cdot\begin{pmatrix}1\\0\\-2\end{pmatrix}}{\left|\begin{pmatrix}2\\-1\\-2\end{pmatrix}\right|\left|\begin{pmatrix}1\\0\\-2\end{pmatrix}\right|}=\cos^{-1}\frac{6}{3\sqrt{5}}=26.565^\circ$$

10	Topic: Vectors					
			Solution			
			$\therefore \theta = 90° - 26.565° = 63.4°$ (to 1d.p.) or 1.11 rad Alternative : Let the acute angle between l and p be θ. $\sin\theta = \dfrac{\left\| \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \right\|} = \dfrac{6}{3\sqrt{5}}$ $\theta = \sin^{-1}\dfrac{6}{3\sqrt{5}} = 63.4°$ (to 1d.p.) or 1.11 rad (3 s.f.)			
		2	(iii) Assume Q lies on the line l. $\begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix} \Rightarrow \begin{cases} 5 = 2 + \lambda \\ -1 = -1 \\ -2 = 4 - 2\lambda \end{cases} \Rightarrow \begin{cases} \lambda = 3 \\ \\ \lambda = 3 \end{cases}$ Since $\lambda = 3$ is consistent throughout, Q lies on the line l. Alternative 1:			

Solution

Since $\overrightarrow{OQ}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}=\begin{pmatrix}2\\-1\\4\end{pmatrix}+\begin{pmatrix}3\\0\\-6\end{pmatrix}$

$=\begin{pmatrix}2\\-1\\4\end{pmatrix}+3\begin{pmatrix}1\\0\\-2\end{pmatrix}$

Q lies on the line l.

Alternative 2:

$l: \dfrac{x-2}{1}=\dfrac{z-4}{-2},\ y=-1$

$Q=(5,-1,-2)$, i.e. $x=5,\ y=-1,\ z=4$.

$\dfrac{x-2}{1}=\dfrac{5-2}{1}=3,\ \dfrac{z-4}{-2}=\dfrac{-2-4}{-2}=3.$

Hence, Q lies on the line l.

(iv) Let F be the foot of perpendicular from the point Q to the plane p.

$l_{QF}: \mathbf{r}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}, \mu\in\mathbb{R}$

Since F lies on l_{QF}, $\overrightarrow{OF}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}$, for some $\mu\in\mathbb{R}$.

Solution

Since F also lies on plane p, $\overline{OF}\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=-3$.

$$\left[\begin{pmatrix}5\\-1\\-2\end{pmatrix}+\mu\begin{pmatrix}2\\-1\\-2\end{pmatrix}\right]\cdot\begin{pmatrix}2\\-1\\-2\end{pmatrix}=-3$$

$$2(5+2\mu)-(-1-\mu)-2(-2-2\mu)=-3$$

$$15+9\mu=-3\Rightarrow\mu=-2$$

$$\therefore\overline{OF}=\begin{pmatrix}5\\-1\\-2\end{pmatrix}-2\begin{pmatrix}2\\-1\\-2\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}$$

The foot of perpendicular from the point Q to the plane p is $(1,1,2)$.

Alternative :

$\overline{QF}=(\overline{QA}\cdot\hat{n})\,\hat{n}$, where $\underset{\sim}{n}$ is a normal vector of p

$$=\left(\begin{pmatrix}-5\\2\\3\end{pmatrix}\cdot\frac{1}{3}\begin{pmatrix}2\\-1\\-2\end{pmatrix}\right)\frac{1}{3}\begin{pmatrix}2\\-1\\-2\end{pmatrix}$$

$$=-2\begin{pmatrix}2\\-1\\-2\end{pmatrix}$$

Solution

$$\therefore \overline{OF} = \overline{OQ} + \overline{QF} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} - 2\begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

The foot of perpendicular from the point Q to the plane p is $(1,1,2)$.

$$\overline{QF} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$$

$$\left|\overline{QF}\right| = \sqrt{(-4)^2 + 2^2 + 4^2} = 6$$

$$RF = \sqrt{45 - 36} = 3$$

The locus of R is a circle that lies in plane p with centre $(1,1,2)$ and radius 3.

(v) Let Q' be the image of Q in plane p.

$$\overline{OF} = \tfrac{1}{2}(\overline{OQ} + \overline{OQ'})$$

$$\overline{OQ'} = 2\overline{OF} - \overline{OQ}$$

$$= 2\begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix}$$

Equivalent Methods

$$\overline{OQ'} = \overline{OF} + \overline{QF}$$

$$\overline{OQ'} = \overline{OQ} + \overline{QQ'}$$

$$= \overline{OQ} + 2\overline{QF}$$

Solution

$\overline{BQ'} = \overline{OQ'} - \overline{OB}$

$$= \begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix}$$

Alternative:

$\overline{BQ'} = \overline{BQ} + \overline{QQ'}$

$$= \left[\begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} - \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}\right] + 2\begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix} = \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix}$$

A vector equation of the line which is a reflection of the line l in plane p is

$$\mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} + \gamma \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix}, \gamma \in \mathbb{R}$$

$$\text{or } \mathbf{r} = \begin{pmatrix} -3 \\ 3 \\ 6 \end{pmatrix} + \gamma \begin{pmatrix} -5 \\ 4 \\ 2 \end{pmatrix}, \gamma \in \mathbb{R}$$

Solution

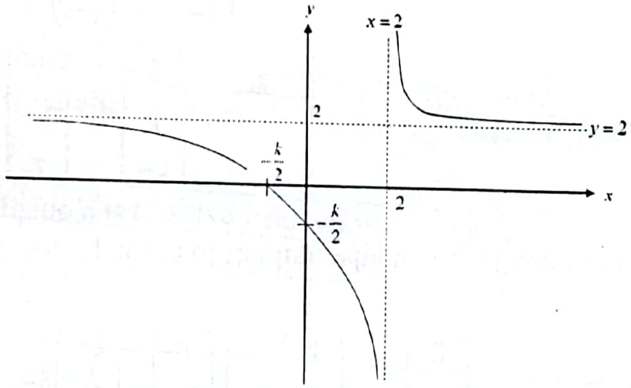
(i) $y = \dfrac{2x+k}{x-2} = 2 + \dfrac{(k+4)}{x-2}$

Vertical Asymptote: $x = 2$

Horizontal Asymptote: $y = 2$

x-intercept: $x = -\dfrac{k}{2}$

y-intercept: $y = -\dfrac{k}{2}$



Solution

$$f(x)=\frac{1}{x}$$

$$\downarrow \quad A$$

$$f(x-2)=\frac{1}{x-2}$$

$$\downarrow \quad B$$

$$(k+4)f(x-2)=\frac{(k+4)}{x-2}$$

$$\downarrow \quad C$$

$$2+(k+4)f(x-2)=2+\frac{(k+4)}{x-2}$$

A: Translation in the positive x direction by 2 units

B: Scaling parallel to the x direction (// to y-axis) by a factor of $(k+4)$

C: Translate in the positive y direction by 2 units

(iii) Let $y=\frac{2x+k}{x-2}$

$$y(x-2)=2x+k$$

$$yx-2y=2x+k$$

$$yx-2x=2y+k$$

$$x(y-2)=2y+k$$

$$x=\frac{2y+k}{y-2}$$

$$f^{-1}(y)=\frac{2y+k}{y-2}$$

Solution

$$f^{-1}(x)=\frac{2x+k}{x-2}$$

$$D_{f^{-1}}=R_f=D_f=(-\infty,2)\cup(2,\infty)$$

$$f^{-1}:x\mapsto\frac{2x+k}{x-2},\qquad x\in\square,\ x\neq 2,$$

$$R_{f^{-1}}=D_f=R_f=(-\infty,2)\cup(2,\infty)$$

(iv) $\because f(x)=f^{-1}(x)$

$$\therefore f^2=ff(x)=f\left[f^{-1}(x)\right]=x$$

$$D_{f^2}=D_f=(-\infty,2)\cup(2,\infty)$$

$$\therefore f^{2017}\left(\frac{1}{2}\right)=f\left(\frac{1}{2}\right)$$

$$=\frac{2\left(\frac{1}{2}\right)+k}{\left(\frac{1}{2}\right)-2}$$

$$=\frac{1+k}{-\frac{3}{2}}$$

$$=\frac{-2-2k}{3}$$

(v) $f(x)=\dfrac{2x+k}{x-2}$

$$D_f=(-\infty,2)\cup(2,\infty)$$

$$R_f=(-\infty,2)\cup(2,\infty)$$

11	Topic: Graphing Techniques & Functions					
			Solution			
			$g(x)=a+\sqrt{x-3}$ $D_g=(3,\infty)$ $R_g=(a,\infty)$ Since $fg(x)$ exists, $R_g \subseteq D_f$ $R_g=(a,\infty)$ $D_f=(-\infty,2)\cup(2,\infty)$ $\therefore a\geq 2$ or a subset of it.			
			For $gf(x)$ to exist, $R_f \subseteq D_g$ $R_f=(-\infty,2)\cup(2,\infty)$ $D_g=(3,\infty)$ Since $R_f \not\subset D_g$, $gf(x)$ does not exist.			