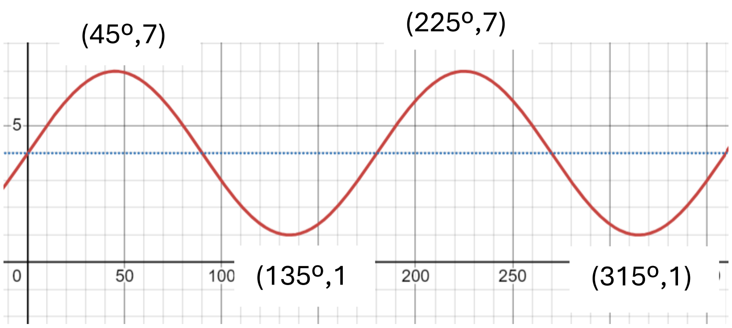
1i	$4x^2+8x-5=4(x^2+2x)-5$ $=4\left[(x+1)^2-1\right]-5$ $=4(x+1)^2-9$	M1 M1 complete square A1	AO1
ii	Turning point = (–1, –9)	B1	AO1

2	$x\sqrt{24}=x\sqrt{3}+\sqrt{6}$ $x\left(\sqrt{24}-\sqrt{3}\right)=\sqrt{6}$ $x=\dfrac{\sqrt{6}}{\left(\sqrt{24}-\sqrt{3}\right)}$ $x=\dfrac{\sqrt{6}}{\left(2\sqrt{6}-\sqrt{3}\right)}\times\dfrac{2\sqrt{6}+\sqrt{3}}{2\sqrt{6}+\sqrt{3}}$ $=\dfrac{2(6)+\sqrt{18}}{4(6)-3}$ $=\dfrac{12+3\sqrt{2}}{21}$ $=\dfrac{4+\sqrt{2}}{7}$ a = 4 and b = 2	M1 factorise x M1 Multiply by conjugate surd M1 simplification A1, A1	AO1

3i	$\angle CAE=\angle ABC$ (alt segment theorem/tangent chord thm) $\angle ACB=\angle CAE$ (alternate $\angle$s, $BC//AE$) $\angle ABC=\angle ACB$ (base $\angle$s of isos Δ) $\therefore AB=AC$ (*shown*)	B1 B1 AG1	AO3
3ii	$\angle BAC=180^\circ-2\angle ABC$ ($\angle$ sum of Δ) $\angle BDC=\angle BAC$ ($\angle$s in same segment) $\angle CDE=180^\circ-\angle BDC$ (adj $\angle$s on a straight line) $\angle CDE=180^\circ-(180^\circ-2\angle ABC)$ (adj $\angle$s on a straight line) $\angle CDE=2\angle ABC$ (shown)	M1 B1 AG1	AO3

4a i	Since the period = 8π $8\pi = \frac{\pi}{b}$ $b = \frac{1}{8}$	B1 AG1	AO3
ii	$c = 3$ $7 = a\tan\left(\frac{\pi}{4}\right) + 3$ $a = 4$ $y = 4\tan\left(\frac{x}{8}\right) + 3$	 M1 A1	AO1
4b		B1 correct sinusoidal shape with correct turning points B1 two cycles	AO1

5i	Length of rectangle = $\frac{20-3x}{2}$ Area $= \left(\frac{20-3x}{2}\right)x - \frac{1}{2}x^2\sin 60°$ $= 10x - \frac{3}{2}x^2 - \frac{1}{2}x^2\frac{\sqrt{3}}{2}$ $= 10x - \left(\frac{6+\sqrt{3}}{4}\right)x^2$	B1 M1 AG1	AO3
5ii	$\frac{dA}{dx} = 10 - 2\left(\frac{6+\sqrt{3}}{4}\right)x$ For stationary point, $10 - 2\left(\frac{6+\sqrt{3}}{4}\right)x = 0$ $\left(6+\sqrt{3}\right)x = 20$ $x = \frac{20}{\left(6+\sqrt{3}\right)}$ =2.5866 = 2.6 (2 s.f.) $A = 10(2.5866) - \left(\frac{6+\sqrt{3}}{4}\right)(2.5866)^2$ = 12.933	B1 - diff M1 – equate to 0 A1 A1	AO1

	$A=13$		
6a	Let $\dfrac{8x+13}{(1+2x)(2+x)^2}=\dfrac{A}{1+2x}+\dfrac{B}{2+x}+\dfrac{C}{(2+x)^2}$ $8x+13=A(2+x)^2+B(1+2x)(2+x)+C(1+2x)$ $x=-2 \quad \Rightarrow \quad -3=-3C$ $C=1$ $x=-\frac{1}{2} \quad \Rightarrow \quad 9=\frac{9}{4}A$ $A=4$ $x=0 \quad \Rightarrow \quad 13=4A+2B+C$ $13=4(4)+2B+1$ $B=-2$ $\therefore \dfrac{8x+13}{(1+2x)(2+x)^2}=\dfrac{4}{(1+2x)}-\dfrac{2}{(2+x)}+\dfrac{1}{(2+x)^2}$	B1 M1 – sub or compare coefficient A1 (anyone of *A*, *B* or *C* correct) A1 (all 3 values *A*, *B* & *C*) A1	AO1
6b	$\int_1^2 \dfrac{8x+13}{(1+2x)(2+x)^2}\,dx$ $=\int_1^2 \dfrac{4}{(1+2x)}-\dfrac{2}{(2+x)}+\dfrac{1}{(2+x)^2}\,dx$ $=\left[\frac{4}{2}\ln(1+2x)-2\ln(2+x)-\frac{1}{2+x}\right]_1^2$ $=\left[2\ln 5-2\ln 4-\frac{1}{4}\right]-\left[2\ln 3-2\ln 3-\frac{1}{3}\right]$ $=2\ln\frac{5}{4}+\frac{1}{12}$ or $\ln\frac{25}{16}+\frac{1}{12}$ or 0.530 (to 3 sig fig)	B2 [B1 for 2 correct ln term; B1 for the 3rd term] B1	AO2

7a	$f'(x)=18x^2+2ax-50$ Since $f'(1)=6$ $18+2a-50=6$ $a=19$ Given $f\left(\frac{3}{2}\right)=0$ $6\left(\frac{3}{2}\right)^3+19\left(\frac{3}{2}\right)^2-50\left(\frac{3}{2}\right)+b=0$ $20.25+42.75-75+b=0$	B1 M1 forming equation A1 M1 forming equation A1	AO2

	$b=12$		
7b	$f(x)=(2x-3)(3x^2+14x-4)$ When $f(x)=0$ $(2x-3)(3x^2+14x-4)=0$ $(2x-3)=0$ $x=\frac{3}{2}$ $(3x^2+14x-4)=0$ $x=\frac{-14\pm\sqrt{(14)^2-4(3)(-4)}}{2(3)}$ $x=0.270$ or $x=-4.94$ (3 sig fig)	B1 found using any valid method B1 B1	AO1

8a	$2^{3x+4}\times5^{2x-1}=16^x\times5^{3x}$ $2^{3x}\times2^4\times5^{2x}\times5^{-1}=2^{4x}\times5^{3x}$ $\frac{16}{5}=2^{4x}\div2^{3x}\times5^{3x}\div5^{2x}$ $\frac{16}{5}=2^x\times5^x$ $\frac{16}{5}=10^x$ $\lg\frac{16}{5}=\lg10^x$ $x=\lg\frac{16}{5}$ (shown)	M1: simplify base 2 and base 5 terms correctly M1: introduce log on both sides AG1	AO3
8b	$2\log_2 x-\log_2(x-4)=3$ $\log_2 x^2-\log_2(x-4)=3$ $\log_2\frac{x^2}{x-4}=3$ $\frac{x^2}{x-4}=2^3$ $x^2=8(x-4)$ $x^2-8x+32=0$ $x=\frac{-(-8)\pm\sqrt{(-8)^2-4(1)(32)}}{2(1)}$	M1: power/quotient law M1: change to index form M1: form quadratic eqn M1 solve for x OR show $b^2-4ac=-64$	AO3

	$x=\dfrac{8\pm\sqrt{-64}}{2}$ $\therefore$ No real solution (shown)	AG1: explain $\sqrt{-64}$ does not exist or $b^2-4ac<0$ $\therefore$ no real solution	

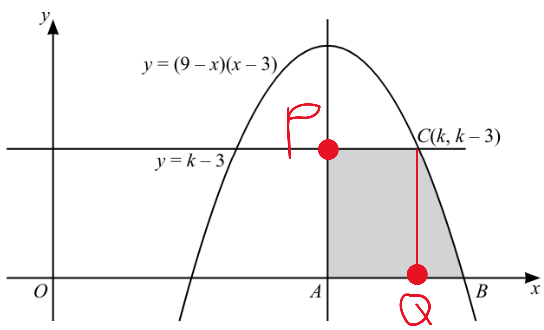
9a	$y=k\sqrt{3x+7}$, $\dfrac{\mathrm{d}y}{\mathrm{d}x}=\dfrac{3k}{2}(3x+7)^{-\frac{1}{2}}$ Given $\dfrac{\mathrm{d}y}{\mathrm{d}t}=3\dfrac{\mathrm{d}x}{\mathrm{d}t}$ $\dfrac{\mathrm{d}y}{dt}=\dfrac{dy}{dx}\times\dfrac{dx}{dt}$ $\dfrac{3\mathrm{d}x}{\mathrm{d}t}=\dfrac{\mathrm{d}y}{\mathrm{d}x}\times\dfrac{\mathrm{d}x}{dt}$ $3=\dfrac{3k}{2}(3x+7)^{-\frac{1}{2}}$ Sub x = 3 $1=\dfrac{k}{2}(3\times3+7)^{-\frac{1}{2}}$ $1=\dfrac{k}{2}\times\dfrac{1}{4}$ $k=8$	B1: correct differentiation M1: write the correct ratio M1: form chain rule correctly and sub x = 3 A1	AO2
9b i	Sub $\boldsymbol{t}=0$, $m=24$ g	B1	AO1
ii	$24\mathrm{e}^{-0.02t}=12$ $\ln\mathrm{e}^{-0.02t}=\ln 0.5$ $-0.02t=\ln 0.5$ $t=\dfrac{\ln 0.5}{-0.02}=34.7$ days	M1 A1	AO1
iii	$\dfrac{\mathrm{d}m}{\mathrm{d}t}=-0.48\mathrm{e}^{-0.02t}$ Sub $t=0.5$, $\dfrac{\mathrm{d}m}{\mathrm{d}t}=-0.48\mathrm{e}^{-0.02(0.5)}=-0.475$ (3 s.f.) Mass is decreasing at 0.475 g/day	B1 B1 (must write statement)	AO1

10 i	LHS $=\left[\frac{\sin x}{\cos x}\div\left(1+\frac{1}{\cos x}\right)\right]+\left[\left(1+\frac{1}{\cos x}\right)\div\frac{\sin x}{\cos x}\right]$ $=\left[\frac{\sin x}{\cos x}\times\left(\frac{\cos x}{\cos x+1}\right)\right]+\left[\left(\frac{\cos x+1}{\cos x}\right)\times\frac{\cos x}{\sin x}\right]$ $=\frac{\sin x}{\cos x+1}+\frac{1+\cos x}{\sin x}$ $=\frac{\sin^2 x+(1+\cos x)^2}{\sin x(1+\cos x)}$ $=\frac{\sin^2 x+1+2\cos x+\cos^2 x}{\sin x(1+\cos x)}$ $=\frac{2+2\cos x}{\sin x(1+\cos x)}$ $=\frac{2(1+\cos x)}{\sin x(1+\cos x)}$ $=\frac{2}{\sin x}$ (shown) OR LHS $=\frac{\tan^2 x+(1+\sec x)^2}{\tan x(1+\sec x)}$ $=\frac{\tan^2 x+1+2\sec x+\sec^2 x}{\tan x(1+\sec x)}$ $=\frac{2\sec^2 x+2\sec x}{\tan x(1+\sec x)}$ $=\frac{2\sec x(\sec x+1)}{\tan x(1+\sec x)}$ $=\frac{2}{\cos x}\div\frac{\sin x}{\cos x}$ $=\frac{2}{\sin x}$ (shown)	M1: $\tan x=\frac{\sin x}{\cos x}$ & $\sec x=\frac{1}{\cos x}$ M1: simplify both [ ] correctly M1: add the fractions and expand correctly M1: factorise numerator AG1 **OR** M1: add fractions correctly M1: expand and add correctly M1: factorise numerator M1: $\sec x=\frac{1}{\cos x}$ & $\tan x=\frac{\sin x}{\cos x}$ AG1	AO2

10 ii	$\frac{\tan x}{1+\sec x}+\frac{1+\sec x}{\tan x}=1+3\sin x$ $\frac{2}{\sin x}=1+3\sin x$ $3\sin^2 x+\sin x-2=0$ $(3\sin x-2)(\sin x+1)=0$	M1: form quadratic eqn M1: factorisation or general formula	AO1

	$\sin x = \frac{2}{3}$ or $\sin x = -1$ $x = 41.8^\circ, 138.2^\circ$ or $x = 270^\circ$ (reject (to 1 dp) as tan 270° is undefined)	A1, A1	

11 i	$y = (9-x)(x-3)$ Sub (k, k-3) into $y = (9-x)(x-3)$ $k-3 = (9-k)(k-3)$ * $k-3 = 9k-27-k^2+3k$ $k^2-11k+24=0$ $(k-3)(k-8)=0$ $k=3(N.A.)$ or $k=8$ *OR $(9-k)(k-3)-(k-3)=0$ $(k-3)(9-k-1)=0$ $(k-3)(8-k)=0$ $k=3(N.A.)$ or $k=8$	M1 substitution M1 form quadratic eqn M1 factorisation AG1 must state N.A. for x = 3	AO3

11 i	Let $y = 0$ $(9-x)(x-3)=0$ $x=3$ or $x=9$ x-coordinate of $B = 9$		AO2
	x-coordinate of $A = \frac{3+9}{2} = 6$ ** **OR use** $\frac{dy}{dx} = -2x+12$ **At turning point,** $\frac{dy}{dx}=0$ $-2x+12=0$ $x_A = 6$  	M1 : either x_A or x_B	
	Area of $APCQ$ $= 5\times 2 = 10$ units^2	B1 (area of rectangle)	
	Area CQB $= \int_8^9 \left(-x^2+12x-27\right)dx$ $= \left[-\frac{1}{3}x^3+6x^2-27x\right]_8^9$	M1: Integrate all terms correctly	
	$= \left[-\frac{1}{3}\left(9^3\right)+6\left(9^2\right)-27\left(9\right)\right]-\left[-\frac{1}{3}\left(8^3\right)+6\left(8^2\right)-27\left(8\right)\right]$		
	$= 2\frac{2}{3}$ units^2	A1	
	Shaded area $= 10+2\frac{2}{3} = 12\frac{2}{3}$ units^2	A1	

12 i	Gradient of PQ = gradient of OR= $\frac{1}{2}$ Eqn of PQ: $y-3=\frac{1}{2}\left(x+4\right)$ $y=\frac{1}{2}x+5$ ----- (1)	B1	AO1
	Gradient of QR = -2	M1: $m_1m_2=-1$	
	Eqn of QR: $y-2=-2(x-4)$ $y=-2x+10$	M1 (Equation of QR)	
	(1)=(2):		

	$\frac{1}{2}x+5=-2x+10$ $x=2$ $\therefore Q(2,6)$	M1: form simultaneous eqns A1	
12 ii	In eqn (1), Let $x=0$, $y=5$ $\therefore OT$ = 5 units $RT=\sqrt{(4-0)^2+(2-5)^2}$ $RT=\sqrt{25}=5$ Since OT = RT = 5 units, ΔORT is isosceles.	B1 B1 AG1	AO3
12 iii	Let S = (a, b) By inspection: S = (0–8 , 0+1) = (–8 , 1) OR Midpoint of RS = Midpoint of OP $\left(\frac{a+4}{2},\frac{b+2}{2}\right)=\left(-\frac{4}{2},\frac{3}{2}\right)$ $a+4=-4$ & $b+2=3$ $a=-8$ $b=1$ Hence coordinates of $S=(-8,1)$.	B1	AO2
12 iv	Area of trapezium OPQR $=\frac{1}{2}\begin{vmatrix}0 & -4 & 2 & 4 & 0\\0 & 3 & 6 & 2 & 0\end{vmatrix}$ $=\frac{1}{2}\lvert -24+4-24-6\rvert$ $=\frac{1}{2}\lvert -50\rvert$ $=25units^2$	M1 A1	AO1